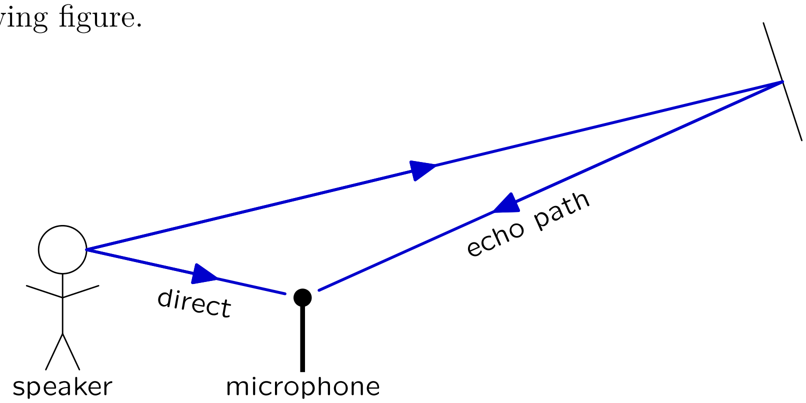
Echo Measurement

Assume that a single echo interferes with a speaker's voice that is being recorded by a microphone as illustrated in the following figure.



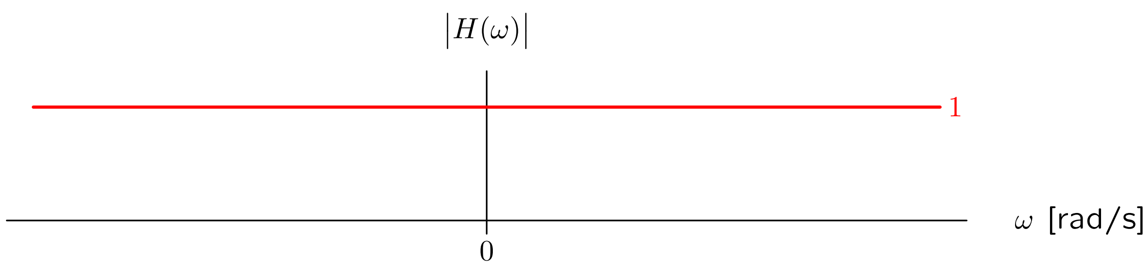
We can represent this recording situation as a linear, time-invariant system, with the speaker's voice as the input and the recorded microphone signal as the output. Assume that the impulse response of this system is

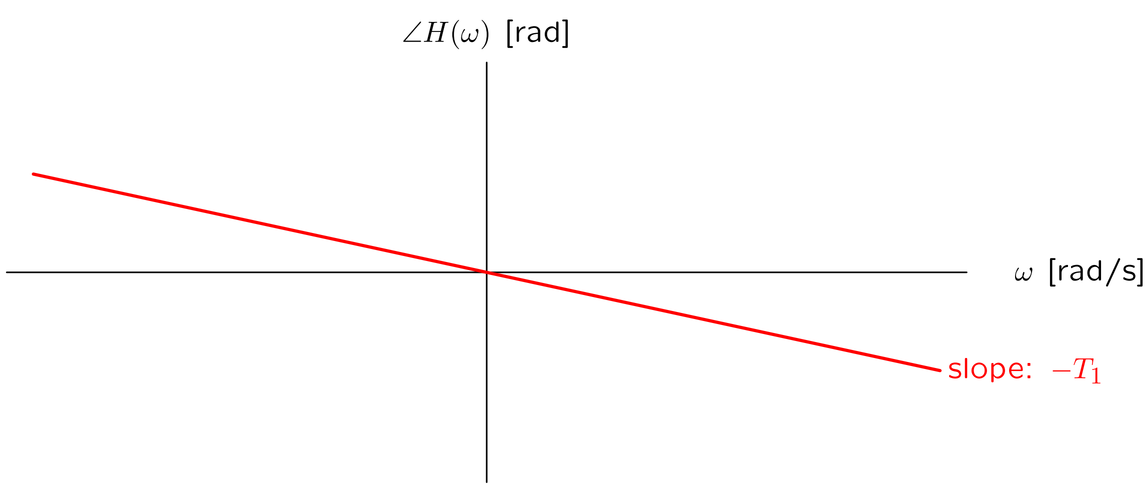
$$h(t) = \delta(t-T_1) + \epsilon\,\delta(t-T_2)$$

where T_1 represents the delay of the direct path from speaker to microphone, T_2 represents that delay through the echo path, and ϵ represents the amplitude of the echo.

Part a.

On the axes below, sketch the magnitude and angle of the frequency response of this system in the absence of an echo (i.e., when $\epsilon = 0$). Label all important values.

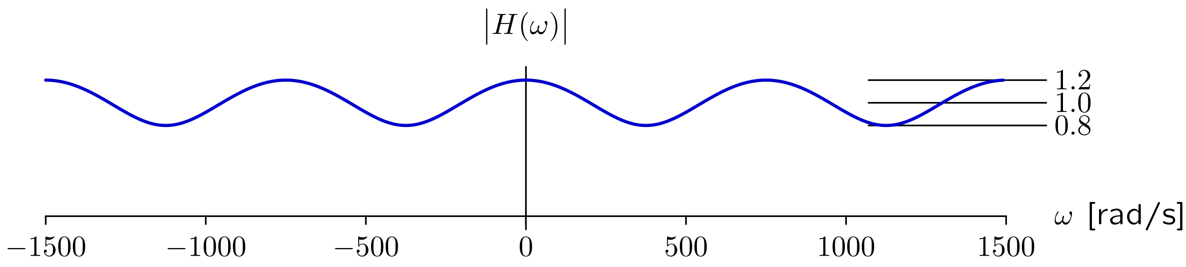


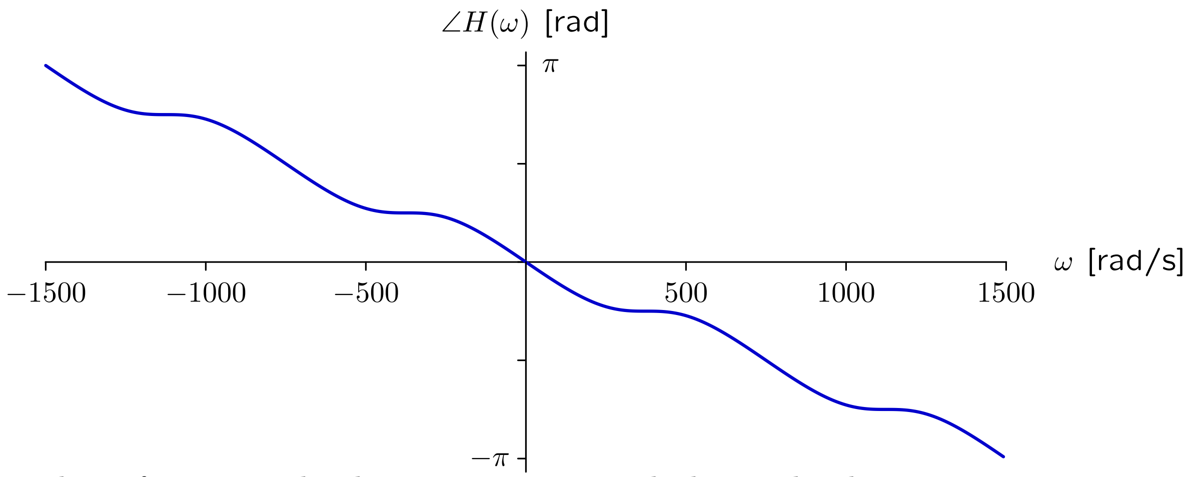


Here, we are interested in a signal $\delta(t - T_1)$. The CTFT of this signal is $e^{-j\omega T_1}$, which has a constant magnitude of 1 and an angle that varies linearly with ω.

Part b.

Now consider the case where there is an echo. The following plots show that magnitude and angle of the frequency response of this system for $|\omega| < 1500\,\text{rad/s}$, for some values of T_1, T_2, and ϵ.





Determine values of T_1, T_2, and ϵ that are consistent with the graphs above.

$T_1 = \boxed{\dfrac{\pi}{1500}}$

$T_2 = \boxed{\dfrac{\pi}{300}}$

$\epsilon = \boxed{0.2}$

Take the Fourier transform of $h(t)$ to obtain the frequency response

$$H(j\omega) = e^{-j\omega T_1} + \epsilon e^{-j\omega T_2} = e^{-j\omega T_1}\left(1 + \epsilon e^{-j\omega(T_2 - T_1)}\right).$$

The magnitude function

$$\begin{aligned} |H(j\omega)| &= \left|e^{-j\omega T_1}\right| \left|\left(1 + \epsilon e^{-j\omega(T_2 - T_1)}\right)\right| \\ &= 1\sqrt{(1 + \epsilon\cos\omega(T_2 - T_1))^2 + \epsilon^2 \sin^2\omega(T_2 - T_1)} \\ &= \sqrt{1 + 2\epsilon\cos\omega(T_2 - T_1) + \epsilon^2} \end{aligned}$$

oscillates between $1+\epsilon$ and $1-\epsilon$ with a period (in ω) of $2\pi/(T_2 - T_1)$. From the magnitude plot on the previous page, we can see that $\epsilon \approx 0.2$ and $2\pi/(T_2 - T_1) \approx 1500/2$, so that $T_2 - T_1 \approx \dfrac{4\pi}{1500}$.

The angle function is

$$\angle H(j\omega) = -\omega T_1 + \angle\Big(1 + \epsilon e^{-j\omega(T_2 - T_1)}\Big)$$

Since ϵ is small compared to 1, the first term dominates the second, which oscillates about an average value near 0. Thus we can estimate T_1 from the average slope of the angle plot on the previous page,

$$T_1 = \frac{\pi}{1500}\,.$$

Then $T_2 \approx \dfrac{4\pi}{1500} + \dfrac{\pi}{1500} = \dfrac{\pi}{300}$.