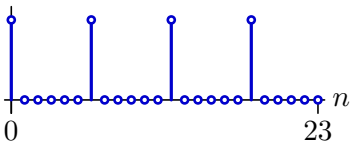
Discrete Fourier Transform Matching

Each of the following plots shows the first 24 samples of a discrete-time signal. Find the plot on the following page that corresponds to the 24-point Discrete Fourier Transform (DFT) for each of these signals. Enter the letter of the plot (A-N) in the box provided.

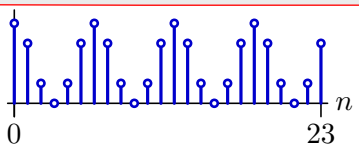


G

$x_1[n] = \delta[n] + \delta[n-6] + \delta[n-12] + \delta[n-18]$

$$X_1[k] = \frac{1}{24}\sum_{n=0}^{23} x[n]e^{-j2\pi kn/24} = \frac{1}{6}\left(1 + e^{-j2\pi k/4} + e^{-j4\pi k/4} + e^{-j6\pi k/4}\right) = \frac{1}{6}\left(1 + (-j)^k + (-1)^k + j^k\right)$$

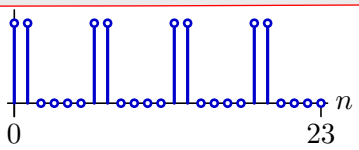
$$= \begin{cases} 1/6 & \text{if } k = 0, 4, 8, 12, 16, 20 \\ 0 & \text{otherwise} \end{cases}$$



I

$x_2[n] = 1 + \cos(2\pi n/6) = 1 + \frac{1}{2}e^{j2\pi n/6} + \frac{1}{2}e^{-j2\pi n/6}$

$X_2[k] = \delta[k] + \frac{1}{2}\delta[k-4] + \frac{1}{2}\delta[k+4] = \delta[k] + \frac{1}{2}\delta[k-4] + \frac{1}{2}\delta[k-20]$

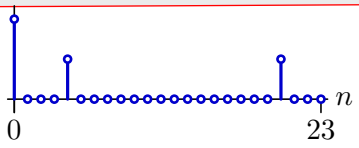


J

$x_3[n] = x_1[n] + x_1[n-1]$

$X_3[k] = \left(1 + e^{-j2\pi k/24}\right) X_1[k] = 2e^{-j\pi k/24}\cos(\pi k/24)X_1[k]$

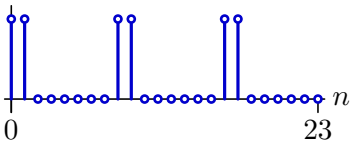
Similar to plot G except components near $k = 12$ are attenuated.



B

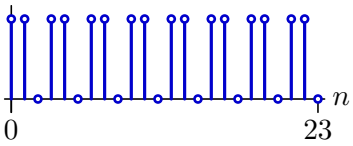
$x_4[n] = \delta[n] + \frac{1}{2}\delta[n-4] + \frac{1}{2}\delta[n-20]$

$$X_4[k] = \frac{1}{24} + \frac{1}{48}e^{-j2\pi k/6} + \frac{1}{48}e^{j2\pi k/6} = \frac{1}{24}\left(1 + \cos(2\pi k/6)\right)$$



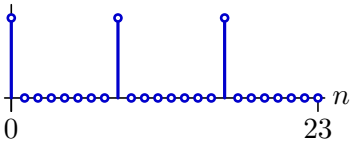
M

x_5 is imilar to x_3 except the period is 8 instead of 6. Therefore X_5 has non-zero components at $k = 0, 3, 6, ...$ and components near $k = 12$ are attenuated.



L

x_6 is similar to x_3 except the period is 3 instead of 6. Therefore X_6 has non-zero components at $k = 0, 8, 16$ and components near $k = 12$ are attenuated.



H

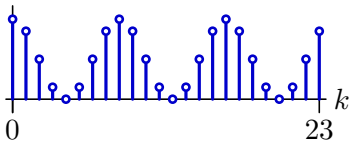
$$x_7[n] = \delta[n] + \delta[n-6] + \delta[n-12] + \delta[n-18]$$

$$X_7[k] = \frac{1}{24}\sum_{n=0}^{23} x[n]e^{-j2\pi kn/24} = \frac{1}{24}\left(1 + e^{-j2\pi k/3} + e^{-j4\pi k/3}\right)$$

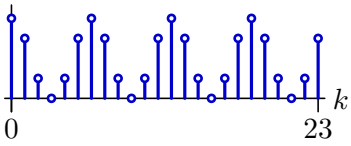
$$= \begin{cases} 1/8 & \text{if } k = 0, 3, 6, 9, 12, 15, 18, 21 \\ 0 & \text{otherwise} \end{cases}$$

Each of the following plots shows the magnitude of a DFT computed with an analysis window $N = 24$. The vertical scale for each plot is different: it has been normalized so that the peak value in each plot is 1.

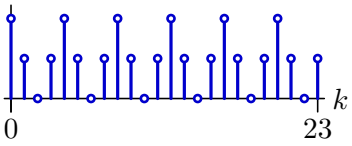
A



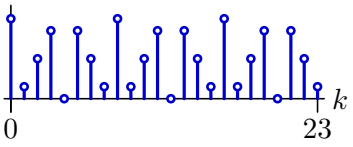
B



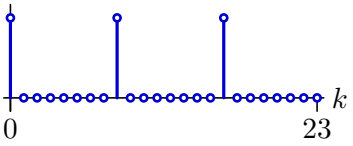
C



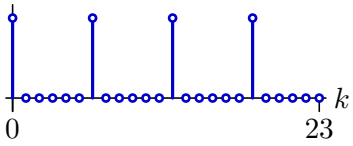
D



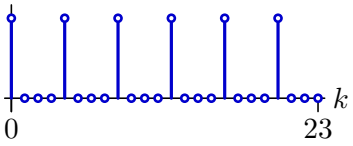
E



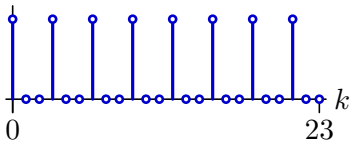
F



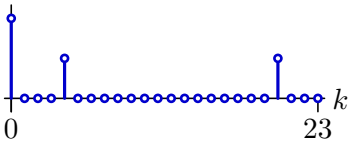
G



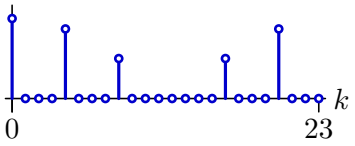
H



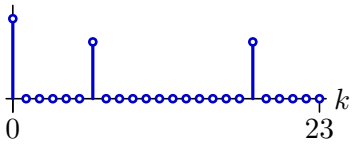
I



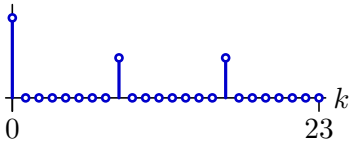
J



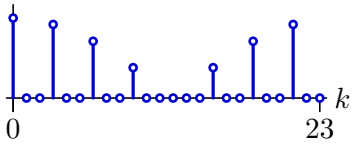
K



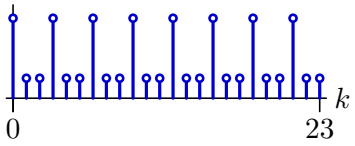
L



M



N



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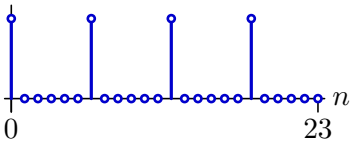

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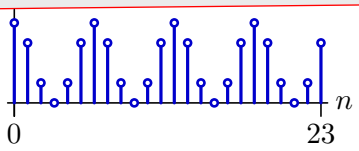

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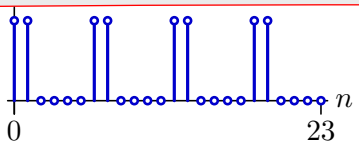

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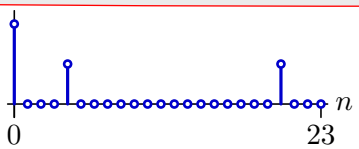

B

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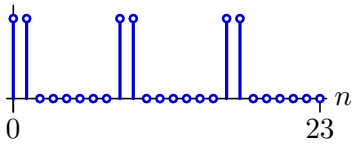

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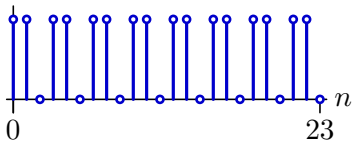

L

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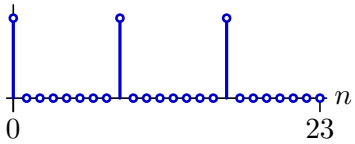

H

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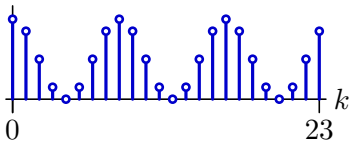
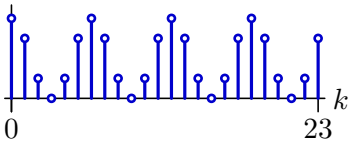
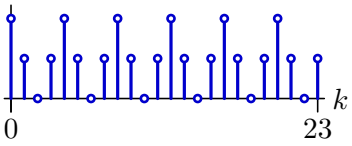
A



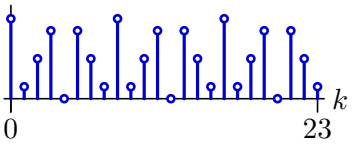
B



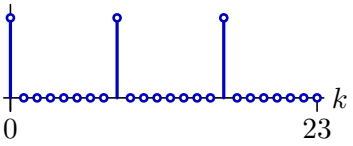
C



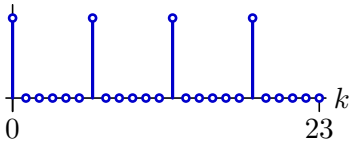
D



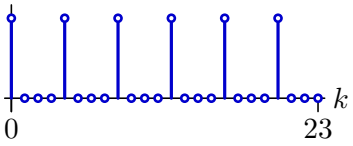
E



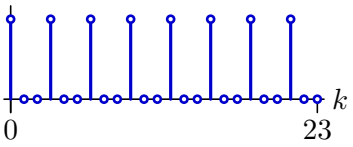
F



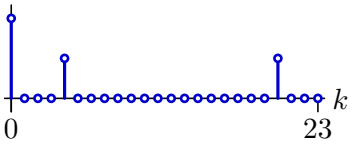
G



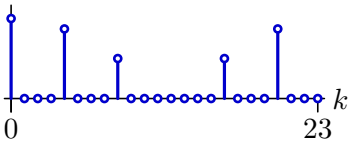
H



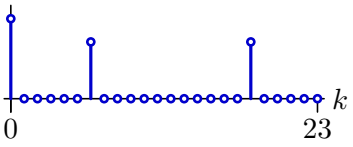
I



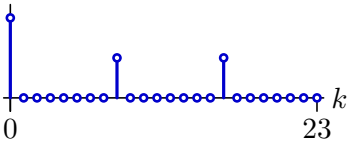
J



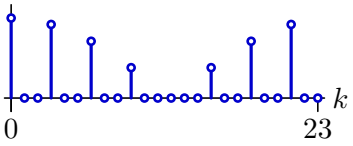
K



L



M



N