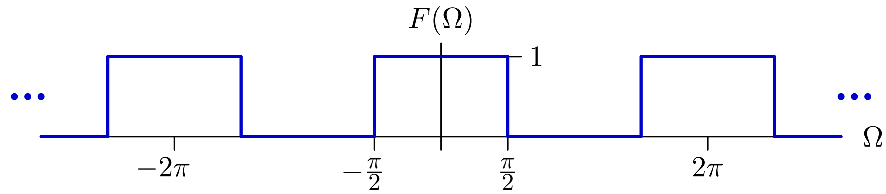
Discrete-Time Fourier Transforms

Part a.

Let $F(\Omega)$ represent the Fourier transform of a discrete-time signal $f[n]$.

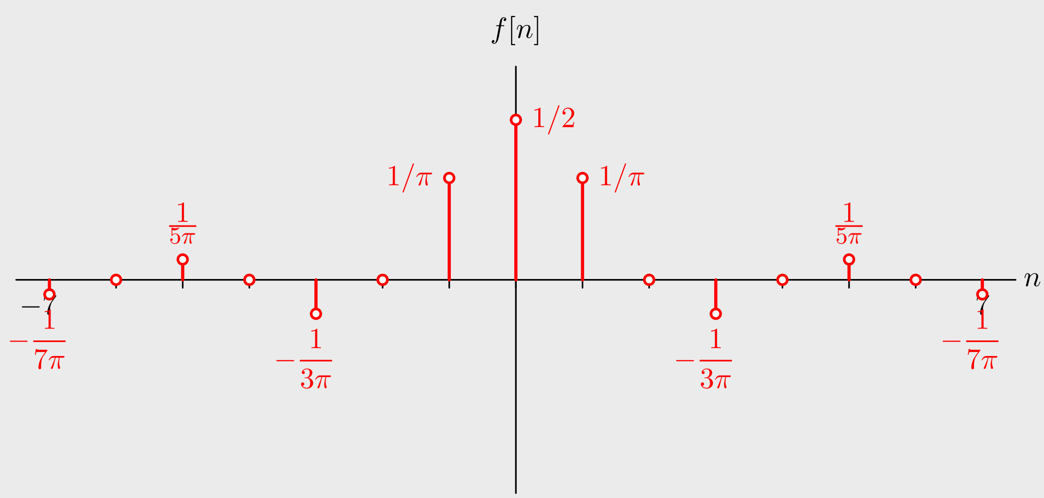
$$F(\Omega) = \begin{cases} 1 & \text{if } |\Omega| \le \pi/2 \\ 0 & \text{if } \pi/2 < |\Omega| < \pi \\ F(\Omega \bmod 2\pi) & \text{if } |\Omega| > \pi \end{cases}$$



Determine an expression for $f[n]$.
Sketch $f[n]$ versus n for $-7 \le n \le 7$ and label all important values.

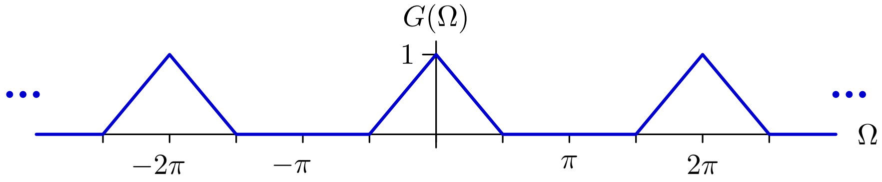
$$f[n] = \frac{1}{2\pi}\int_{2\pi} F(\Omega)e^{j\Omega n}d\Omega = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\Omega n}d\Omega = \frac{1}{2\pi}\left.\frac{e^{j\Omega n}}{jn}\right|_{-\pi/2}^{\pi/2} = \frac{1}{n\pi}\sin(\pi n/2)$$

$$= \begin{cases} \frac{1}{2} & n = 0 \\ \frac{1}{\pi n}\sin(\pi n/2) & \text{otherwise} \end{cases}$$



Part b.

Let $g[n]$ represent a discrete-time signal whose Fourier transform $G(\Omega)$ is shown below.



Define a new signal $h[n]$ as follows:

$$h[n] = \begin{cases} g[n/2] & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

Determine an expression for $H(\Omega)$ in terms of $G(\Omega)$. Sketch the magnitude and phase of $H(\Omega)$ and label all important values.

$$G(\Omega) = \sum_{n=-\infty}^{\infty} g[n]e^{-j\Omega n}$$

$$H(\Omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\Omega n} = \sum_{\substack{n=-\infty \\ n \text{ even}}}^{\infty} g[n/2]e^{-j\Omega n} = \sum_{m=-\infty}^{\infty} g[m]e^{-j\Omega 2m} = G(2\Omega)$$