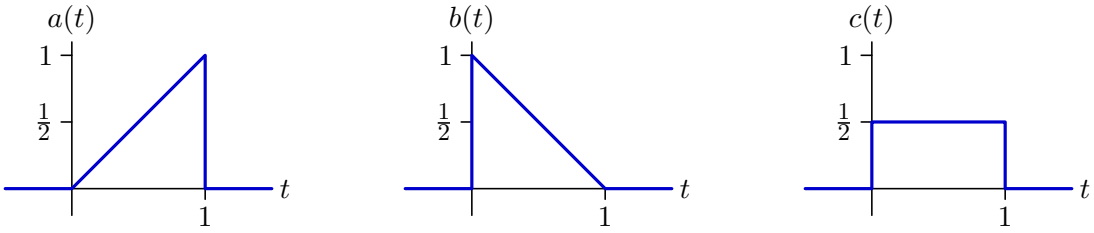
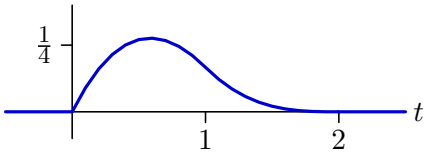
Convoluted Alternatives

Each of the following parts of this question defines a new function. Your task is to determine if that new function can be expressed as the convolution of two of the following three signals.



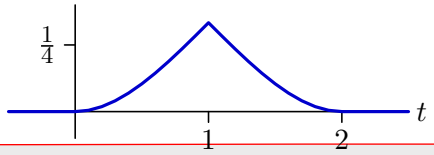
Notice that there are ten possible answers. The new function may be equal to any of the following: $(a*a)$, $(a*b)$, $(a*c)$, $(b*a)$, $(b*b)$, $(b*c)$, $(c*a)$, $(c*b)$, $(c*c)$, or **none** of these options. If the answer is not unique, you will receive full credit for any one of the correct answers.

Part a. Which two functions (if any) can be convolved to produce the following function.



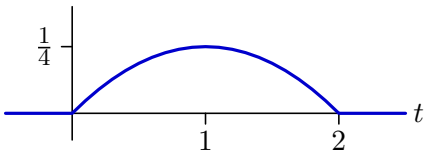
The answer must be asymmetric about $t = 1$, with larger values for $t < 1$ than for $t > 1$. The answer is $b*b$.

Part b. Which two functions (if any) can be convolved to produce the following function.



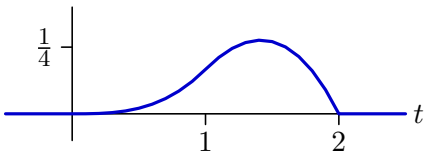
This function is symmetric about $t = 1$, which could happen if one of the inputs is flipped-in-time (about $t = 1/2$) relative to the other input. There are only a few such options. One is $c*c$ — but that would result in a triangle-shaped output. Another symmetric option is $a*b$ (or equivalently $b*a$), which fits with the first interval being concave up. The answer is $a*b$ or $b*a$.

Part c. Which two functions (if any) can be convolved to produce the following function.



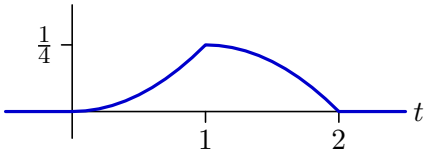
This function is symmetric about $t = 1$, which could happen if one of the inputs is flipped-in-time (about $t = 1/2$) relative to the other input. There are only a few such options. One is $c*c$ — but that would result in a triangle-shaped output. Another symmetric option is $a*b$ (or equivalently $b*a$) — but that would be concave up in first interval. None of the provided functions could result in this output.

Part d. Which two functions (if any) can be convolved to produce the following function.



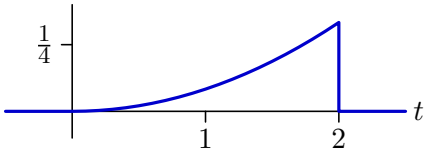
The answer must be asymmetric about $t = 1$ with small output at early times and larger output at later times. The answer is $a * a$.

Part e. Which two functions (if any) can be convolved to produce the following function.



The first interval looks like the integral of a. The second interval looks like the first interval, but shifted and flipped in time. The answer is $a * c$ or $c * a$.

Part f. Which two functions (if any) can be convolved to produce the following function.



The step discontinuity in this result could only result if one of the convolved functions contains an impulse. Therefore the answer is none.