Alternative Representations

Part a. Let c_k and d_k represent the trigonometric Fourier series coefficients for a periodic function $f_1(t)$ of continuous time t with period T:

$$f_1(t) = f_1(t+T) = c_0 + \sum_{k=1}^{\infty} c_k \cos(2\pi kt/T) + \sum_{k=1}^{\infty} d_k \sin(2\pi kt/T)$$

Determine expressions for the trigonometric Fourier series coefficients c'_k and d'_k for $f_2(t) = \frac{d}{dt} f_1(t)$:

$$f_2(t) = \frac{d}{dt} f_1(t) = c'_0 + \sum_{k=1}^{\infty} c'_k \cos(2\pi kt/T) + \sum_{k=1}^{\infty} d'_k \sin(2\pi kt/T)$$

as functions of $\ldots c_0,\ c_1,\ c_2,\ \ldots$ and/or $\ldots d_0,\ d_1,\ d_2,\ \ldots$.

Enter expressions for c'_0, c'_1, d'_1, c'_2, d'_2, c'_3, and d'_3 in the table below. Your expressions may contain any combination of the original (unprimed) coefficients ($\ldots c_0,\ c_1,\ c_2,\ \ldots$ and $\ldots d_0,\ d_1,\ d_2,\ \ldots$) but must not include $f_1(t)$ or $f_2(t)$ or any integrals or infinite sums.

Notice that d'_0 is not defined, and it's box is x'd out of the table.

	c'_k	d'_k
0:	0	X
1:	$2\pi d_1/T$	$-2\pi c_1/T$
2:	$4\pi d_2/T$	$-4\pi c_2/T$
3:	$6\pi d_3/T$	$-6\pi c_3/T$

Briefly explain.

$$f_1(t) = c_0 + \sum_{k=1}^{\infty} c_k \cos(2\pi kt/T) + \sum_{k=1}^{\infty} d_k \sin(2\pi kt/T)$$

$$f_2(t) = \frac{d}{dt} f_1(t) = \frac{d}{dt}\left(c_0 + \sum_{k=1}^{\infty} c_k \cos(2\pi kt/T) + \sum_{k=1}^{\infty} d_k \sin(2\pi kt/T)\right)$$

$$= \sum_{k=1}^{\infty} -\frac{2\pi k c_k}{T} \sin(2\pi kt/T) + \sum_{k=1}^{\infty} \frac{2\pi k d_k}{T} \cos(2\pi kt/T)$$

$$= c'_0 + \sum_{k=1}^{\infty} c'_k \cos(2\pi kt/T) + \sum_{k=1}^{\infty} d'_k \sin(2\pi kt/T)$$

Therefore

$$c'_0 = 0$$

$$c'_k = \frac{2\pi k d_k}{T}$$

$$d'_k = -\frac{2\pi k d_k}{T}$$

Part b. Let a_k represent the complex exponential Fourier series coefficients for a periodic function $f_3(t)$ of continuous time t with period T:

$$f_3(t) = f_3(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$$

Determine expressions for the complex exponential Fourier series coefficients a'_k for $f_4(t) = f_3(t)\cos(2\pi t/T)$

$$f_4(t) = f_3(t)\cos(2\pi t/T) = \sum_{k=-\infty}^{\infty} a'_k e^{j2\pi kt/T}$$

as functions of the original (unprimed) coefficients (a_k).

Enter expressions for a'_{-2}, a'_{-1}, a'_0, a'_1, and a'_2 in the table below. Your expressions may contain any combination of the original (unprimed) coefficients a_k but must not include $f_3(t)$ or $f_4(t)$ or any integrals or infinite sums.

a'_{-2}:	$(a_{-3}+a_{-1})/2$
a'_{-1}:	$(a_{-2}+a_0)/2$
a'_0:	$(a_{-1}+a_1)/2$
a'_1:	$(a_0+a_2)/2$
a'_2:	$(a_1+a_3)/2$

Briefly explain.

$$\begin{aligned}
f_4(t) &= f_3(t)\cos(2\pi t/T) \\
&= \sum_k a_k e^{j\frac{2\pi}{T}kt}\cos(2\pi t/T) \\
&= \sum_k \frac{a_k}{2} e^{j\frac{2\pi}{T}kt} e^{j\frac{2\pi}{T}t} + \sum_k \frac{a_k}{2} e^{j\frac{2\pi}{T}kt} e^{-j\frac{2\pi}{T}kt} e^{j\frac{2\pi}{T}t} \\
&= \sum_k \frac{a_k}{2} e^{j\frac{2\pi}{T}(k+1)t} + \sum_k \frac{a_k}{2} e^{j\frac{2\pi}{T}(k-1)t} \\
&= \sum_l \frac{a_{l-1}}{2} e^{j\frac{2\pi}{T}lt} + \sum_m \frac{a_{m+1}}{2} e^{j\frac{2\pi}{T}mt} \\
&= \sum_k \frac{a_{k-1}+a_{k+1}}{2} e^{j\frac{2\pi}{T}kt}
\end{aligned}$$

$$a'_k = \frac{a_{k-1}+a_{k+1}}{2}$$