6.003: Signal Processing

Synthetic Aperture Optics

- Fourier Relations in Physics
- Fourier Optics
- Synthetic Aperture Microscopy

Please give us feedback on 6.003:
- End-of-Term Subject Evaluations now until Friday, May 13, 9am
- http://registrar.mit.edu/subjectevaluation

Final Exam: Friday, May 13, 1:30-4:30pm in E51-376
Conflict Exams are assigned by Registrar.

May 5, 2022

Why Focus on Fourier?

What’s so special about sines and cosines?

Sinusoidal functions have interesting mathematical properties.
→ harmonically related sinusoids are orthogonal to each other over \([0, T]\).

Sines and cosines also play important roles in physics – especially the physics of waves.

Physical Example: Vibrating String

A taut string supports wave motion.

The speed of the wave depends on the tension on and mass of the string.

Reflections can interfere with excitations.

The interference can be constructive or destructive depending on the frequency of the excitation.

Physical Example: Vibrating String

The wave will reflect off a rigid boundary.

The amplitude of the reflected wave is opposite that of the incident wave.

Physical Example: Vibrating String

We get constructive interference if round-trip travel time equals the period.

\[
x = 0 \quad \text{and} \quad x = L
\]

Round-trip travel time \(= \frac{2L}{v} = T\)

\[
\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2L/v} = \frac{\pi v}{L}
\]
Physical Example: Vibrating String

In fact, we also get constructive interference if round-trip travel time is $kT$.

\[
x = 0 \quad x = L
\]

Round-trip travel time \[\frac{2L}{v} = kT\]

\[
\omega = \frac{2\pi}{T} = \frac{2\pi}{2L/kv} = \frac{k\pi v}{L} = k\omega_0
\]

Only certain frequencies persist: harmonics of \[\omega_0 = \pi v/L\].

This is the basis of stringed instruments.

Fourier Optics

Fourier relations play important roles in many branches of physics – especially those concerning wave phenomena.

Today: Fourier relations in optics.

Optical Imaging

Images from even the best microscopes are blurred. Blurring is a fundamental property of lenses.

A perfect lens transforms a spherical wave of light from a target into a spherical wave that converges to an image of the target.

Blurring is inversely related to the diameter of the lens.

Optical Imaging

Today’s lecture is on how the size of a lens affects image resolution, and how Fourier representations can be used to understand (and even overcome some of) these limitations.

If a target is located in the focal plane of a lens, light from a point on the target forms a plane wave as it passes through the lens.

If the target point lies on the axis of the lens, then the plane wave is perpendicular to the imaging plane.
Fourier Optics

If a target is located in the focal plane of a lens, light from a point on the target forms a plane wave as it passes through the lens.

If the target point lies on the axis of the lens, then the plane wave is perpendicular to the imaging plane.

If the target point lies off the axis of the lens, then the plane wave is no longer perpendicular to the image plane. The light striking the image plane has linearly increasing phase delay with distance.

Fourier Optics

The target can be described as a collection of point sources of light

\[ f(x) = \int f(x_0) \delta(x-x_0) \, dx_0 \]

and the result in the image plane is a superposition of plane waves, one for each point in the target.

\[ g(\omega) = \int f(x) e^{-j\omega x} \, dx = F(\omega) \]

Notice that \( g(\omega) = F(\omega) \) is the Fourier transform of \( f(x) \).

Fourier Optics: \( f(x) \) \text{ CFFT} \( F(\omega) \)

Now the Fourier transform relation holds for both halves of the system.

\[ F(\omega) = \int f(x) e^{-j\omega x} \, dx \]

\[ f'(x') = \frac{1}{2\pi} \int F(\omega) e^{j\omega x'} \, d\omega \]

Ideally, both limits of integration would be infinite. However the finite diameter of the lens limits the highest frequencies |\( \omega \)|.

Fourier Optics

Light emanating from the target at large angles is not captured by the lens.

\[ F(\omega) = \int f(x) e^{-j\omega x} \, dx \]

\[ f'(x') = \frac{1}{2\pi} \int F(\omega) e^{j\omega x'} \, d\omega \]

As a result, the image at \( x' \) is a lowpass version of the target at \( x \).
Microscopy with 6.003

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Michael S. Meremstein
Berthold K. P. Horn

6.003 Model of a Microscope

Microscope = low-pass filter

Phase-Modulated Microscopy

Demonstration

Poster: \( \cos(\omega_y y + f(x, y)) \)
Projector: \( \cos(\omega_y y) \)
Poster with Projector: \( \cos(\omega_y y) \cos(\omega_y y + f(x, y)) \)

Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies
Standing-wave illumination spectrum

Optical transfer function

Thanks to M. Merzelstein

2 beams

3 beams
Optical transfer function

7 beams

Aperture synthesis

Combine multiple low-NA optics to synthesize high NA

41 BEAMS IN A RING

MAKE PATTERNS LIKE THIS

M2: The first many-beam SAM
Fourier transforms are important in many branches of physics, mathematics, electrical engineering, and computer science.

Today we saw how Fourier optics helps us to understand why optical systems blur.

We also introduced Synthetic Aperture Optics as a way to overcome some limitations of conventional optics.

This new method of optical imaging is directly inspired by Fourier transforms – and especially by the application of modulation to optics.

**Summary**

Fourier transforms are important in many branches of physics, mathematics, electrical engineering, and computer science.

Today we saw how Fourier optics helps us to understand why optical systems blur.

We also introduced Synthetic Aperture Optics as a way to overcome some limitations of conventional optics.

– greatly reduced the blurring in conventional microscopy

This new method of optical imaging is directly inspired by Fourier transforms – and especially by the application of modulation to optics.