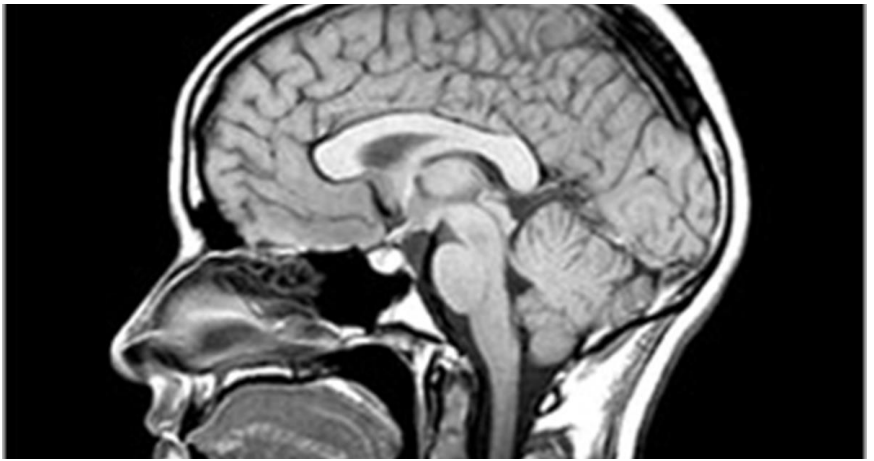
6.003: Signal Processing

Magnetic Resonance Imaging

6 May 2021

MRI



MRI is a Fourier Transform

Today's key: the raw signal $s(t)$ we measure from an MRI machine is a Fourier transform of the underlying image:

$$s(t) = \iint m(x,y) e^{-j2\pi(k_x(t)x + k_y(t)y)} \mathrm{d}x\mathrm{d}y$$

This is not a physics lecture (and I'm not a physicist), but we'll try to illustrate some of the underlying mechanisms and how they result in the key result above.

MRI was originally known as **Nuclear** Magnetic Resonance Imaging. The key is understanding the spin angular momentum of a hydrogen nucleus (i.e., a proton) and its associated magnetic dipole moment; and using these to understand how the protons behave in the presence of an external magnetic field.

Physics

The processes we're talking about today are really quantum phenomena. But we'll use a model from classical mechanics instead.

The classical model: a proton is a "charged, spinning sphere". This spinning gives rise to a current loop that creates a magnetic dipole moment $\mathbf{m}$.

Normally, the nuclei are in random orientations, and so we expect no net magnetic field. But, in the presence of a strong magnetic field, the nuclei will align.

Larmor Precession

"Kicking" our proton with an RF pulse dealigns its magnetic moment with the external field. The torque exerted by the external field tries to cause the proton to realign with the external field, but because the proton has angular momentum, it will instead *precess* about the external field.

Key relationship is the Larmor equation:

$$\omega = \gamma B$$

Precession frequency is proportional to the strength of the external magnetic field (we'll use this later). γ is the "gyromagnetic ratio", which depends on the specific nucleus we're considering.

Measurement

This precession generates a changing magnetic field. With a coil situated nearby, we can turn that magnetic flux into a voltage that changes at the same frequency:

$$\Phi(t) = \sin(\omega t)$$

$$V(t) \propto -\frac{\mathrm{d}\Phi(t)}{\mathrm{d}t} = \omega \cos(\omega t) = \gamma B \cos(\omega t)$$

This is actually a bit more complicated, as we need to consider the "flip angle", i.e., the angle by which our RF pulse rotated the proton:

$$V(t) \propto \gamma B \cos(\omega t) \sin(\theta)$$

and we're actually looking at a large number of molecules, not just one:

$$V(t) \propto N \gamma B \cos(\omega t) \sin(\theta)$$

T_1 and T_2 Relaxation

Of course, our RF "kick" doesn't last forever.

Our analysis on the previous slide relied on the precession of all of the protons being in phase. This will be true for a short while, but, eventually, small differences in the precession frequencies of the particles mean that the phases will dealign ("T_2 relaxation").

It is also the case that the protons will eventually realign with the external field, changing θ in the expressions on the previous page ("T_1 relaxation").

Different tissues in the body have different time constants T_1 and T_2, which provide contrast in images.

NMR: High-level Overview

NMR high-level process:

- Strong external magnetic field aligns hydrogen nuclei
- RF pulse dealigns nuclei in a predictable way
- In the process:
  - Nuclei precess, giving off an RF signal ($\omega = \gamma B$)
  - Nuclei eventually dephase (T_2 relaxation, $\sim .1$ sec)
  - Nuclei eventually realign with the external field (T_1 relaxation, ~ 1 sec)

The goal is to repeat this process a bunch of times, measuring the resulting RF signals until we've detected enough signal to produce a diagnostically useful image.

Imaging Parameters

In general, it is possible to selectively emphasize contrast based on T_1 or T_2 relaxation in an image by varying parameters of the measurement by choosing the "repetition time" (time between trials) and "echo time" (time between excitation and measurement). These different types of contrast have different diagnostic value.

Imaging

So far, we've talked (a little bit) about how to distinguish between different tissues, but we haven't yet talked about how to construct an **image** (where different spatial regions are made up of different tissues).

In general, we want to estimate the values of some true image $m(x, y, z)$.

In MRI, we encode spatial information using different frequencies by adding a *gradient field* on top of our constant magnetic field. This introduces intentional spatial variation in the precession frequency of the nuclei (nuclei in different spatial regions spin at different frequencies).

Frequency Encoding of Spatial Information

The X-gradient coil produces a linearly-varying z-directed field that is characterized by its slope, G_x. Considering only the one dimension, we have a strong uniform field from a constant magnet, and a smaller field from the X-gradient coil.

$$\vec{B}(x, y, z) = (B_0 + xG_x)\hat{z}$$

A gradient field G_x maps space to frequency.

Precession frequency ω is given by:

$$\begin{aligned}\omega &= \gamma(B_0 + \Delta B) \\ &= \omega_0 + \gamma \Delta B \\ &= \omega_0 + \gamma x G_x\end{aligned}$$

We can demodulate to find $\omega = \gamma x G_x$, i.e., the precession frequency changes with x.

Constructing the Image

The signal we measure includes information about all x and y:

$$s(t) = \iint m(x,y) e^{-j\varphi(x,y,t)} \mathrm{d}x\mathrm{d}y$$

We can note that $\frac{d\varphi(x,y,t)}{dt} = \omega(x,y,t) = \gamma B(x,y,t)$, so:

$\varphi(x,y,t) = \int_0^t \omega(x,y,\tau)d\tau = \gamma \int_0^t B(x,y,\tau)d\tau$

Plugging in, we find:

$$s(t) = \iint m(x,y) e^{-j\gamma \int_0^t B(x,y,\tau)d\tau} \mathrm{d}x\mathrm{d}y$$

If $B(x,y,t) = B_0 + xG_x(t) + yG_y(t)$, then we have:

$$s(t) = \iint m(x,y) e^{-j\gamma\left(B_0 t + \int_0^t xG_x(\tau)d\tau + \int_0^t yG_y(\tau)d\tau\right)} \mathrm{d}x\mathrm{d}y$$

After demodulation, we have:

$$s(t) = \iint m(x,y) e^{-j2\pi(k_x(t)x + k_y(t)y)} \mathrm{d}x\mathrm{d}y$$

where $k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau)d\tau$ and $k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau)d\tau$.

Constructing the Image

$$s(t) = \iint m(x,y) e^{-j2\pi(k_x(t)x + k_y(t)y)} \mathrm{d}x\mathrm{d}y$$

Our measurements of $s(t)$ are direct measurements of the Fourier transform of an unknown image $m(x,y)$. The goal is to determine $m(x,y)$.

By varying G_x and G_y with time, we can select particular k_x and k_y values to measure, yielding *samples* of the Fourier transform:

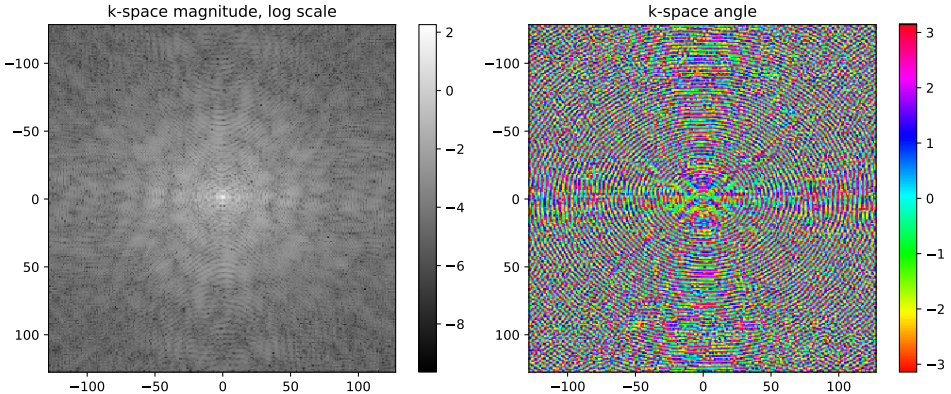
$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) d\tau$ and $k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) d\tau$

Constructing the Image

After sampling, we have something that looks like the DFT of a discretized image $m[\cdot, \cdot]$, which we can reconstruct using an inverse DFT.

Example Image

Consider the following 256×256 array of "k-space" data, gathered via the process described above:



Can we tell anything about the image from these pictures?

Check Yourself: Imaging Properties

If the image above had a field of view of 25.6cm in both the x and y dimensions,

- What is the spatial resolution in the image domain?
- What is the maximum spatial frequency f measured in these data?

Reducing Scan Time

Imagine that, for the image above, our repetition time (time between RF excitations) is 2 seconds, and we are able to make 256 measurements per readout.

Under this assumption, how long would it take to generate the image above?

Reducing Scan Time

An important problem in modern MRI research is trying to construct an accurate image using as few measurements as possible. For the remainder of the lecture, let's explore some strategies for reducing the number of measurements, and try to explain their consequences.

Reducing Scan Time: Strategy 1

We can reduce the scan time by a factor of 2 by only making 128 readouts, where each readout is a column in the matrix of our raw k-space data. If we assume the values we didn't measure directly are 0, this is equivalent to setting all the odd-numbered columns to 0 in our raw k-space data:

$$X_{1b}[k_r, k_c] = \begin{cases} X[k_r, k_c] & \text{if } k_r \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

How does this affect the spatial-domain reconstruction? Why?

Reducing Scan Time: Strategy 1b

Only sample half of the rows, but fill in the missing rows via linear interpolation:

$$X_{1b}[k_r, k_c] = \begin{cases} X[k_r, k_c] & \text{if } k_r \text{ is odd} \\ X[k_r + 1, k_c]/2 + X[k_r - 1, k_c]/2 & \text{otherwise} \end{cases}$$

How does this affect the spatial-domain reconstruction? Why?

Reducing Scan Time: Strategy 2

Same idea, but instead of only taking the even or odd readouts, only take the top half of the k-space (only sample $k_r \leq 0$). Assume the other DFT is conjugate symmetric.

How does this affect the spatial-domain reconstruction? Why?

Reducing Scan Time: Strategy 3

Same idea, but instead of only taking the even or odd readouts, only take the lowest 1/2 of frequencies $|k_r| < \frac{R}{4}$, assume other values are 0.

How does this affect the spatial-domain reconstruction? Why?