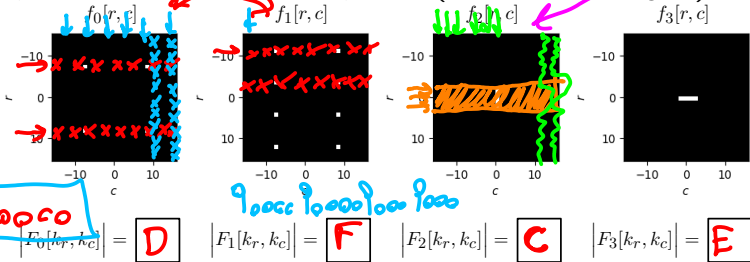


2-D Patterns

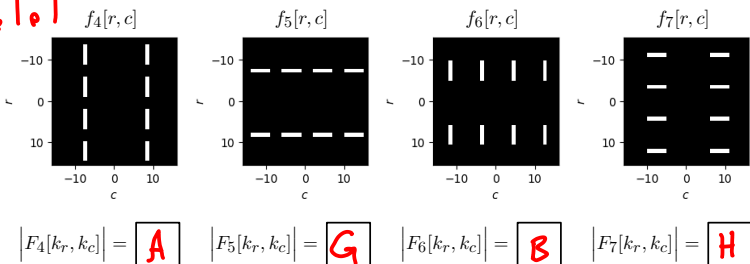
Match each 2-D signal below (each 32×32 pixels) with its DFT magnitudes. In each image, black represents 0 and white represents the most positive value in that panel (not necessarily 1).



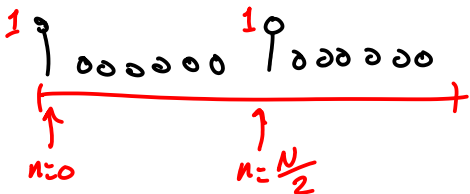
current MC question

ooooo | ooooo
 ↑
 pppppppp

ooooo | ooooo
 ↓
 p.p.p.p.p



ooooo | ppppp
 ↑



$$F[k] = \sum_{n=0}^{N-1} f[n] e^{-j \frac{2\pi k n}{N}}$$

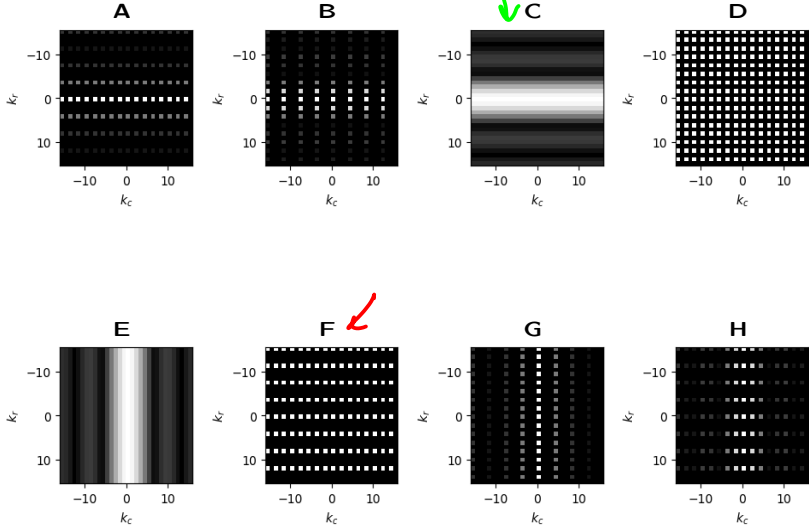
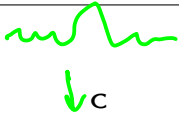
$$1 e^{-j0} + 1 e^{-j\pi k}$$

$$1 + e^{-j\pi k} = 1 + (-1)^k$$

$$2 \quad 0 \quad 2 \quad 0 \quad 2 \quad 0$$

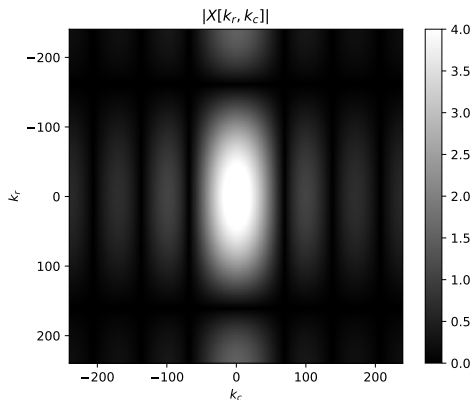
2-D Patterns

DFT Magnitude Graphs:




Mystery Photograph

Below are shown the DFT magnitudes of an image of a single small white object photographed against a black background. The image had dimensions of 480×480 pixels and represented an area that was 1.8cm wide and 1.8cm tall.

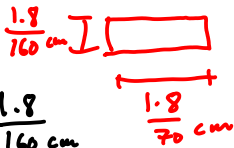


What did this image look like in the spatial domain, including dimensions?

Mystery Photograph

This 2D DFT has the form of a sinc function in both the "r" and "c" dimensions. The inverse transform of such a sinc is a rectangle. Since the sinc is broader vertically than horizontally, we expect the rectangle to be broader horizontally than vertically, like: 

The first zero crossing of the sinc horizontally is at around $k_c = 70$ px, which corresponds to $\Omega = \frac{70}{480} \cdot 2\pi$. This zero crossing happens at $\Omega = \frac{2\pi}{N}$, where N is the width of the rectangle, so $N = \frac{480}{70} \approx 7$ px. We can then figure out the width to be $\frac{480}{70} \cdot \frac{1.8}{480} = \frac{1.8}{70}$ cm.



Applying similar logic, we find the height to be a $\frac{1.8}{160}$ cm

Cats

Consider the following image, which we'll refer to as $f[r, c]$. This image has height R and width C . In this image, black corresponds to a value of 0, and white corresponds to a value of 1.



In addition, consider two other signals (and note that one is specified in the frequency domain, and the other in the spatial domain):

$$\underline{H_1[k_r, k_c]} = j \sin\left(\frac{10\pi k_c}{C}\right)$$

$$\underline{h_2[r, c]} = \sin\left(\frac{10\pi c}{C}\right)$$

Cats

For each of the expressions below, indicate which of the images on the following page is represented by that expression. In each of the images on the following page, black represents the lowest value (not necessarily 0), and white represents the highest value (not necessarily 1). The point $(r = 0, c = 0)$ is located in the center of each image; r increases downward, and c increases to the right.

current mc

$$(f \times h_1)[r, c] = \mathbf{I}$$

$$(f \times h_2)[r, c] = \mathbf{H}$$

$$(f \otimes h_1)[r, c] = \mathbf{A}$$

$$(f \otimes h_2)[r, c] = \mathbf{D}$$

$F \times H_2$

$$h_1[r, c] = \boxed{+}$$

$$H_1[k_r, k_c] = j \sin\left(\frac{10\pi k_c}{c}\right)$$

$$h_2[r, c] = \sin\left(\frac{10\pi r c}{c}\right)$$

$$H_2[k_r, k_c] = \begin{array}{c} \boxed{-+} \\ \times \\ \boxed{\text{cat}} \end{array} \rightarrow \boxed{-+}$$

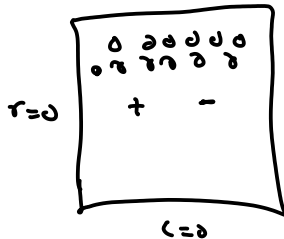
$$H_1[k_r, k_c] = j \sin\left(\frac{10\pi k_c}{c}\right) = \sum_r \sum_c f[r, c] e^{-j\left(\frac{2\pi k_r r}{R} + \frac{2\pi k_c c}{C}\right)}$$

$$= \left(\frac{1}{2}\right) \left(e^{j \frac{10\pi k_c}{c}} - \left(\frac{1}{2}\right) e^{-j \frac{10\pi k_c}{c}} \right)$$

$$h_1[r, c] = -\frac{1}{2} \delta[r] \delta[c-5] + \frac{1}{2} \delta[r] \delta[c+5]$$

$$r=0, c=5$$

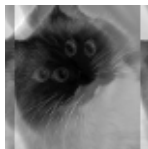
$$r=0, c=-5$$



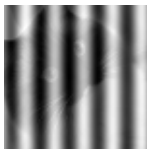
A ✓



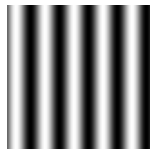
B



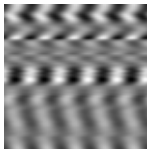
C



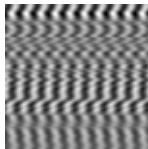
D ✓



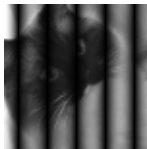
E



F



G L



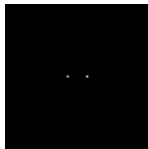
H L



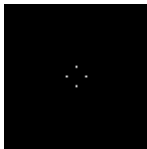
I ✓



J



K



L

