6.003: Signal Processing

Convolution and Filtering

time domain	frequency domain
$y(t) = (h * x)(t) = \int h(\tau)x(t-\tau)\,d\tau$	$Y(\omega) = H(\omega)X(\omega)$
$y[n] = (h * x)[n] = \sum_m h[m]x[n-m]$	$Y(\Omega) = H(\Omega)X(\Omega)$

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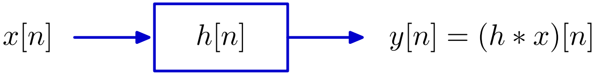
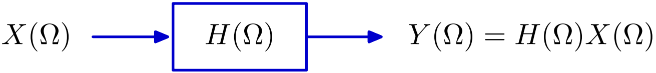
Filtering

We can view filtering in both the time and frequency domains:

Time Domain:



Frequency Domain:

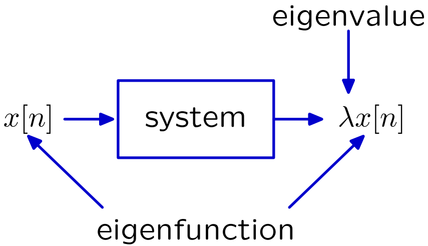


Each frequency component $X(\Omega)$ is scaled by a factor $H(\Omega)$, which can be possibly complex.

The system is completely described by the set of scale factors $H(\cdot)$, which we refer to as the **frequency response** of the system.

Eigenfunctions and Eigenvalues

If the output signal is a scalar multiple of the input signal, we refer to the signal as an **eigenfunction**, and the multiplier as the **eigenvalue**.



Complex Exponentials

Complex exponentials are eigenfunctions of LTI systems. If $h[\cdot]$ is a system's unit sample response, and $x[n] = e^{j\Omega n}$, then the system's output is:

Response to Eternal Sinusoids

If $h[n]$ is purely real, we have $H(-\Omega) = H^*(\Omega)$.

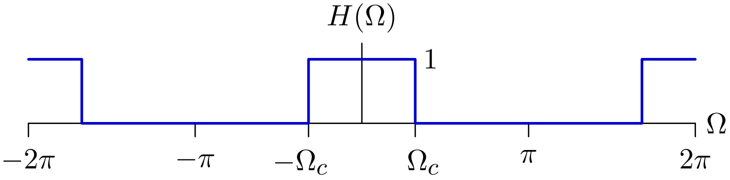
Consider $x[n] = \cos(\Omega n)$ (for all n), which can be written as:

$$x[n] = \frac{1}{2}\left(e^{j\Omega n} + e^{-j\Omega n}\right)$$

Then:

The "Ideal" Low-Pass Filter

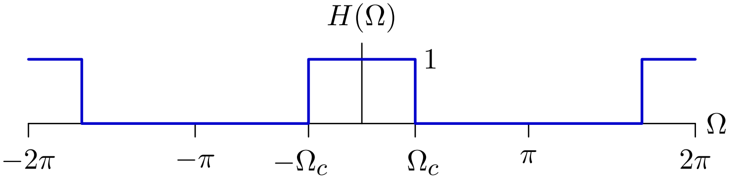
Consider a system characterized by the following purely real frequency response:



Such a system is called a **low-pass filter**, because it allows low frequencies to pass through unmodified, while attenuating high frequencies.

We could apply this filter to a signal by multiplying the DTFT of that signal by the values above. But we could also apply the filter by operating in the time domain.

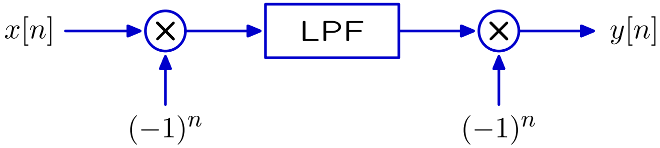
The "Ideal" Low-Pass Filter



We can apply this filter to a signal by convolving with its unit sample response. What is the unit sample response of the system whose frequency response is shown above?

Cascaded System

Consider this system, where LPF represents a lowpass filter of the form discussed on the previous slides.



How many of the following statements are true?

- The transformation from $x[n]$ to $y[n]$ is linear.
- The transformation from $x[n]$ to $y[n]$ is time invariant.
- The transformation from $x[n]$ to $y[n]$ is a high-pass filter.