# 6.003: Signal Processing

## Convolution

welcome back!

we will start @ 2:05pm Eastern

DT

$$\underbrace{(f*g)[n]}= \sum_{m=-\infty}^{\infty} f[m]\,g[n-m]$$

CT

$$\underbrace{(f*g)(t)} = \int_{-\infty}^{\infty} f(x)\,g(t-x)\,dx$$

commutative

$$f*g = g*f$$

associative

$$(f*g)*h = f*(g*h)$$

distributes over addition

$$f*(g+h) = f*g+f*h$$

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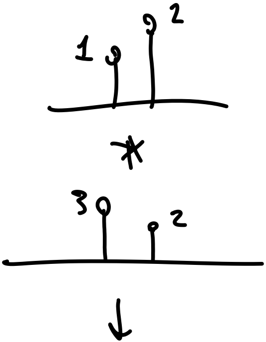
$$\begin{array}{r} 12 \\ \times\ 32 \\ \hline 64 \\ 320 \\ \hline 384 \end{array}$$



# Check Yourself

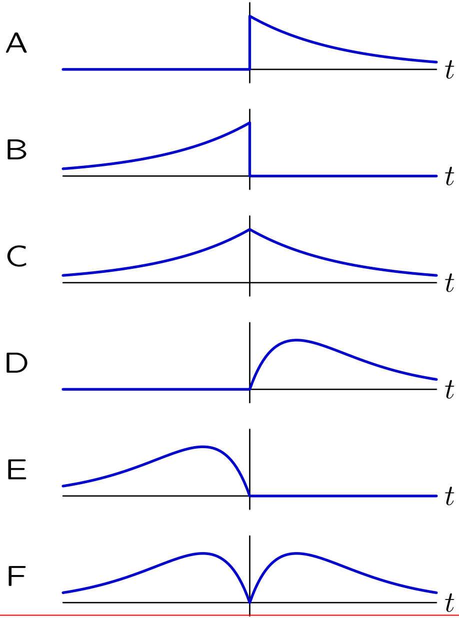
$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & \text{o.w.} \end{cases}$$

Match expressions on the left with functions on the right where

$$f(t) = e^{-t}\,u(t)$$

$$g(t) = e^{t}\,u(-t)$$

| | |
|---|---|
| $(f * f)(t)$ | D |
| $(g * g)(t)$ | E |
| $(f * g)(t)$ | C |
| $(g * f)(t)$ | C |



$$(f*f)(t) = \int_{-\infty}^{\infty} f(x)\, f(t-x)\, dx \qquad f(t) = e^{-t}u(t)$$

$$= \int_{-\infty}^{\infty} e^{-x} \underbrace{u(x)}_{\substack{0 \text{ unless} \\ x>0}} \; e^{-t+x} \underbrace{u(t-x)}_{\substack{0 \text{ unless} \\ x<t}} \, dx$$

only holds $t>0$

$$= \int_0^t \underbrace{e^{-t}}\, dx = e^{-t} \int_0^t dx = \underline{te^{-t}}\, u(t)$$

$f(t) = e^{-t}u(t)$ $\qquad$ $g(t) = f(-t)$

$g(t) = e^{t}\, u(-t)$

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$$(g*g)(t) = \int_{-\infty}^{\infty} g(x)\, g(t-x)\, dx$$

$$= \int_{-\infty}^{\infty} f(-x)\, f(x-t)\, dx$$

let $y = -x$

$$= \int_{-\infty}^{\infty} f(y)\, f(-t-y)\, dy = (f*f)(-t)$$

$$(f*g)(t) = \int_{-\infty}^{\infty} f(x)\, g(t-x)\, dx$$

$$= \int_{-\infty}^{\infty} e^{-x} \underbrace{u(x)}_{\substack{0 \text{ unless} \\ x>0}} e^{t-x} \underbrace{u(x-t)}_{\substack{0 \text{ unless} \\ x>t}} dx = e^{t} \int_{\max(0,t)}^{\infty} e^{-2x}\, dx$$

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if $t>0$

$$e^{t} \int_{t}^{\infty} e^{-2x}\, dx = \underline{e^{t} \frac{1}{-2} e^{-2x}} \Big|_{x=t}^{\infty} = 0 - \frac{1}{-2} e^{-t} = \frac{1}{2} e^{-t}$$

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if $t<0$

$$e^{t} \int_{0}^{\infty} e^{-2x}\, dx = e^{t} \frac{1}{-2} e^{-2x} \Big|_{x=0}^{\infty} = 0 - \frac{1}{-2} e^{t} = \frac{1}{2} e^{t}$$

$$\frac{1}{2} e^{-|t|}$$

# Check Yourself

Match expressions on the left with functions on the right where

$$f(t) = e^{-t}\,u(t)$$

$$g(t) = e^{t}\,u(-t)$$

$(f * f)(t)$

$(g * g)(t)$

$(f * g)(t)$

$(g * f)(t)$