The questions were taken from multiple previous exams.
1 Describing Sinusoids

Part 1. Let \( x_1[n] = a_1 e^{j\Omega_1 n} + a_1^* e^{-j\Omega_1 n} \) as shown in the following figure. Estimate \( a_1 \) and \( \Omega_1 \), where \( \Omega_1 \) is real-valued and \( a_1 \) may be complex. Place an "x" on the complex plane shown below (where the circle has a radius of 1) to indicate the value of \( a_1 \). Also, place an "x" on the number line shown below to indicate the value of \( \Omega_1 \).

Part 2. Let \( x_2[n] = c_2 \cos(\Omega_2 n) + d_2 \sin(\Omega_2 n) \) as shown in the following figure. Estimate the real-valued constants \( c_2 \), \( d_2 \), and \( \Omega_2 \). Place an "x" on each of the number lines shown below to indicate these values.
Part 3. Let \( x_3[n] = \cos(\Omega_3 n + \phi_3) \) as shown in the following figure.

![signal](signal.png)

Estimate the real-valued constants \( \phi_3 \) and \( \Omega_3 \). Place an "x" on each of the number lines shown below to indicate these values.

![number lines](number_lines.png)

Part 4. Let \( x_4[n] = \Re\left( a_4 e^{j\Omega_4 n}\right) \) as shown in the following figure.

![signal](signal.png)

Estimate \( a_4 \) and \( \Omega_4 \), where \( \Omega_4 \) is real-valued and \( a_4 \) may be complex. Place an "x" on the complex plane shown below (where the circle has a radius of 1) to indicate the value of \( a_4 \). Also, place an "x" on the number line shown below to indicate the value of \( \Omega_4 \).

![complex plane](complex_plane.png)
2 Related Signals

For this problem, let $x[\cdot]$ be a DT signal that is periodic in $N = 6$. One period of the Fourier series coefficients $X[\cdot]$ is $\{1, 0, j, 0, -j, 0\}$, i.e.,

$$X[k] = X[k + 6] = \begin{cases} 
1 & \text{if } k = 0 \\
j & \text{if } k = 2 \\
-j & \text{if } k = 4 \\
0 & \text{otherwise}
\end{cases}$$

**Part 1.**

Consider a new signal $y_1[\cdot]$, described by: $y_1[n] = 7 - 3x[n - 1]$. What are the DTFS coefficients of $y_1[\cdot]$?

- $Y_1[0] = \quad$ 
- $Y_1[1] = \quad$ 
- $Y_1[2] = \quad$
- $Y_1[3] = \quad$ 
- $Y_1[4] = \quad$ 
- $Y_1[5] = \quad$
Part 2. Consider another new signal $y_2[n] = 5(-1)^n x[n]$. What are the DTFS coefficients of $y_2[n]$?

\[
\begin{align*}
Y_2[0] &=  \\
Y_2[1] &=  \\
Y_2[2] &=  \\
Y_2[3] &=  \\
Y_2[4] &=  \\
Y_2[5] &=  \\
\end{align*}
\]
3 CT and DT

Part 1.
Consider taking a periodic signal \( x(\cdot) \) (with a period of \( T = 1 \) second) and sampling at a sampling rate of 8 samples per second to obtain DT signal \( x[\cdot] \) that is also periodic.

Analyzing this signal, you find that:

- \( x[\cdot] \) is an even function of \( n \).
- \( x[n] \) is positive for all values of \( n \).
- the sum of all \( x[n] \) in one period is 2.
- if you sum and subtract alternating samples in one period, you get 1; that is, \( x[0] - x[1] + x[2] - \ldots + x[N-2] - x[N-1] = 1 \).
- most of the Fourier series coefficients \( X[\cdot] \) are 0; only two (per period) are nonzero.

What are two distinct CT functions \( x(\cdot) \) that could have produced the results shown above?

\[
x(t) =
\]

or

\[
x(t) =
\]
Part 2.

When computing the DTFS coefficients of a purely real signal with a fundamental period $N$, where $N$ is an even number, two coefficients in particular ($k = 0$ and $k = N/2$) are always real-valued (whereas other values can be complex). Why is this the case? Explain briefly (1-3 sentences).

When computing the DTFS coefficients of a purely real signal with a fundamental period $N$ where $N$ is odd, are there any coefficients that are similarly guaranteed to be real? If so, specify the values of $k$. Explain briefly (1-3 sentences).
When computing the CTFS coefficients of a purely real CT signal, are there any coefficients that are similarly guaranteed to be real? If so, specify the values of $k$. Does your answer depend on the value of $T$? Explain briefly (1-3 sentences).
4 Find the Magnitude

Consider the following plots of the magnitudes of the Fourier series coefficients for four discrete-time signals, that are each periodic in $N = 20$.

For each of the following signals, determine which if any of the previous plots shows the magnitude of its Fourier series coefficients.

$x_1[n]$ Enter A to D or None:

$x_2[n]$ Enter A to D or None:

$x_3[n]$ Enter A to D or None:

$x_4[n]$ Enter A to D or None:
Worksheet (intentionally blank)
5 Trigonometric Forms

Consider a periodic signal $x(t)$ with period $T$. When $x(t)$ is expanded in a Fourier series, its trigonometric series coefficients are $c[k]$ and $d[k]$ as follows:

$$x(t) = c[0] + \sum_{k=1}^{\infty} c[k] \cos \left( \frac{2\pi kt}{T} \right) + \sum_{k=1}^{\infty} d[k] \sin \left( \frac{2\pi kt}{T} \right)$$

Each of the following parts describes a new signal $y_i(t)$ that results from a simple transformation of $x(t)$. When $y_i(t)$ is expanded in a Fourier series, its trigonometric series coefficients are $c'[k]$ and $d'[k]$ as follows:

$$y_i(t) = c'[0] + \sum_{k=1}^{\infty} c'[k] \cos \left( \frac{2\pi kt}{T} \right) + \sum_{k=1}^{\infty} d'[k] \sin \left( \frac{2\pi kt}{T} \right)$$

For each of the following parts, determine expressions for $c'[k]$ and $d'[k]$ in terms of $c[k]$ and $d[k]$.

**Part 1. Time reversal:** $y_1(t) = x(T - t)$.

\[
\begin{align*}
\text{c'}[k] &= \\
\text{d'}[k] &= 
\end{align*}
\]
**Part 2.**

**Differentiation:** \( y_2(t) = \frac{dx(t)}{dt} \).

\[
\begin{align*}
c'[k] &= \\
d'[k] &= 
\end{align*}
\]

**Part 3.**

**Symmetric part:** \( y_3(t) = \frac{x(t) + x(-t)}{2} \).

\[
\begin{align*}
c'[k] &= \\
d'[k] &= 
\end{align*}
\]
Part 4.

Delay by $\frac{T}{2}$: $y_4(t) = x(t - \frac{T}{2})$.

\[ c'[k] = \quad d'[k] = \]

Part 5.

Delay by $\frac{T}{4}$: $y_5(t) = x(t - \frac{T}{4})$.

\[ c'[k] = \quad d'[k] = \]
6 Harmonic Aliasing

Consider three periodic signals:

- \( x_1(t) \) with period \( T_1 = \frac{1}{11} \) seconds
- \( x_2(t) \) with period \( T_2 = \frac{1}{12} \) seconds
- \( x_3(t) \) with period \( T_3 = \frac{1}{13} \) seconds

Each of these signals contains a fundamental component (at frequency \( \omega \) given by \( \frac{2\pi}{\text{period}} \)) as well as harmonics 2, 3, 4, and 5, but not other frequencies.

Each of these continuous-time signals is sampled 40 times per second to generate corresponding discrete-time signals:

- \( x_1[n] = x_1(n/40) \)
- \( x_2[n] = x_2(n/40) \)
- \( x_3[n] = x_3(n/40) \)

Each of these discrete-time signals contains exactly five discrete-time sinusoidal components with frequencies in the range \( 0 \leq \Omega \leq \pi \).

Each plot on the facing page shows the frequencies found in one of these DT signals. In the circle next to each plot, write the name of the corresponding signal (either \( x_1 \), \( x_2 \), or \( x_3 \)).

Each of the DT frequency components is associated with one of the harmonics in the original CT signal. For each DT frequency, write the number of the associated CT harmonic (1-5) in the box above that frequency. If none of these harmonics could have produced a given frequency, enter an \( X \) in its box instead.
7 Dome, Sweet Dome

Ben Bitdiddle created a signal $x_0[n]$ representing the MIT dome, but he only saved the DTFS coefficients $X_0[k]$ (and not the original signal). However, he knew that one period of the original signal (which is periodic in $N = 51$) looked like this:

Ben tried several different methods of recovering the original image based on $X_0[k]$, by applying the DTFS synthesis equation to the following sets of coefficients.

For each set of Fourier coefficients described below ($X_A$ through $X_I$), determine the corresponding signal on the following page ($x_1$ through $x_{24}$). Assume that all of the signals on the following page are purely real and are periodic in $N = 51$. If the required signal would be complex-valued, write COMPLEX in the box; otherwise, write the name of the signal from the following page.

Part 1. $X_A[k] = \text{Re}(X_0[k])$

$x_A =$ 

Part 2. $X_B[k] = \text{Im}(X_0[k])$

$x_B =$ 

Part 3. $X_C[k] = j\text{Im}(X_0[k])$

$x_C =$ 

Part 4. $X_D[0] = 0, X_D[k] = X_0[k]$ otherwise

$x_D =$ 

Part 5. $X_E[25] = 0, X_E[k] = X_0[k]$ otherwise

$x_E =$ 

Part 6. $X_F[k] = X_0[k] + 1/51$

$x_F =$ 

Part 7. $X_G[k] = e^{j\pi}X_0[k]$

$x_G =$
Part 8. $X_H[0] = X_0[0], \ X_H[k] = e^{j\pi} X_0[k]$ otherwise

$x_H = \underline{\hspace{2cm}}$

Part 9. $X_I[k] = |X_0[k]| e^{j(\angle X_0[k])}$

$x_I = \underline{\hspace{2cm}}$
\[
\begin{align*}
\{ x_n \} &\quad n = 0, 1, 2, \ldots \\
1 &\quad x_1[n] \\
-1 &\quad x_2[n] \\
1 &\quad x_3[n] \\
-1 &\quad x_4[n] \\
1 &\quad x_5[n] \\
-1 &\quad x_6[n] \\
1 &\quad x_7[n] \\
-1 &\quad x_8[n] \\
1 &\quad x_9[n] \\
-1 &\quad x_10[n] \\
1 &\quad x_{11}[n] \\
-1 &\quad x_{12}[n] \\
1 &\quad x_{13}[n] \\
-1 &\quad x_{14}[n] \\
1 &\quad x_{15}[n] \\
-1 &\quad x_{16}[n] \\
1 &\quad x_{17}[n] \\
-1 &\quad x_{18}[n] \\
1 &\quad x_{19}[n] \\
-1 &\quad x_{20}[n] \\
1 &\quad x_{21}[n] \\
-1 &\quad x_{22}[n] \\
1 &\quad x_{23}[n] \\
-1 &\quad x_{24}[n]
\end{align*}
\]
8 Missing Components

Consider a set of discrete-time signals $x_i[n]$ that are each periodic such that

$$x_i[n] = x_i[n + 8]$$

for all values of $n$. Five of these signals are defined by their Fourier series coefficients calculated for a period of $N = 8$ and illustrated in the following figure.

Use the following panels to answer questions on the following page.
Which (if any) panel shows $|x_1[n]|$? Enter A to I or None: 

Which (if any) panel shows $|x_2[n]|$? Enter A to I or None: 

Which (if any) panel shows $|x_3[n]|$? Enter A to I or None: 

Which (if any) panel shows the real part of $x_3[n]$? Enter A to I or None: 

Which (if any) panel shows the real part of $x_5[n]$? Enter A to I or None: 

Which (if any) panel shows the imaginary part of $x_2[n]$? Enter A to I or None: 

Which (if any) panel shows the imaginary part of $x_3[n]$? Enter A to I or None: 

Which (if any) panel shows the imaginary part of $x_4[n]$? Enter A to I or None: 
