6.003: Signal Processing

Signal Processing

- Overview of Subject
- Signals: Definitions, Examples, and Operations
- Time and Frequency Representations
- Fourier Series

February 4, 2020
Signals are functions that contain and convey information.

Examples:
- the MP3 representation of a sound
- the JPEG representation of a picture
- an MRI image of a brain

Signal Processing develops the use of signals as abstractions:
- identifying signals in physical, mathematical, computation contexts,
- analyzing signals to understand the information they contain, and
- manipulating signals to modify and/or extract information.
Signal Processing is **widely used** in science and engineering to...

- **model** some aspect of the world,
- **analyze** the model, and
- **interpret** results to gain a new or better understanding.

*Signal Processing* provides a common language across disciplines.
Signal Processing is **widely used** in science and engineering to...

- **model** some aspect of the world,
- **analyze** the model, and
- **interpret** results to gain a new or better understanding.

![Diagram showing the process of model, analyze, result, make model, world, new understanding, and interpret results.](image)

**Signal Processing** provides a common language across disciplines.

Classical analyses use a variety of maths, especially calculus. We will also use **computation** to solve real-world problems that are difficult or impossible to solve analytically.

→ *strengthens ties to the real world*
Course Mechanics

Schedule

**Lecture:** Tue. and Thu. 2-3pm in 32-141

**Recitation:** Tue. and Thu. 3-4pm in 24-121 or 26-328

**Office Hours:** Tue. and Thu. 4-5pm in 24-121 or 26-328
Wed. and Thu. 7-9pm in 36-144
Sun. 4-6pm (room TBD)

Homework – issued Tuesdays, due following Tuesday at noon

- **Drills:** focus on facts, definitions, and simple concepts
  - online with immediate feedback (not graded)
- **Problems:** focus on developing problem solving skills
  - pencil and paper problems taken from previous exams
  - simple computational extensions to real-world data
  - completely specified, unambiguous, self-contained
- **Labs:** focus on applications of 6.003 to authentic problems
  - more open-ended, multiple approaches, multiple solutions
  - deepen understanding and demonstrate wide applicability
  - issued Tuesday, required **check-in** Thursday, due following Tuesday

Two Midterms and a Final Exam
Signals

Signals are functions that are used to convey information.
– may have 1 or 2 or 3 or even more independent variables

\[ \text{sound pressure} (t) \]

\[ \text{brightness} (x, y) \]
Signals

Signals are functions that are used to convey information.
– **dependent variable** can be a scalar or a vector

**scalar:** brightness at each point \((x, y)\)

**vector:** \((\text{red}, \text{green}, \text{blue})\) at each point \((x, y)\)
Signals

Signals are functions that are used to convey information.

- **dependent variable** can be real, imaginary, or complex-valued

\[ x(t) = e^{j2\pi t} = \cos 2\pi t + j \sin 2\pi t \]
Signals

Signals are functions that are used to convey information.

– **continuous domain** versus **discrete domain**

\[
x(t)
\]

\[
x[n]
\]

Signals from physical systems are often of **continuous** domain:

- continuous time – measured in seconds
- continuous spatial coordinates – measured in meters

Computations usually manipulate functions of **discrete** domain:

- discrete time – measured in samples
- discrete spatial coordinates – measured in samples

Relating continuous and discrete representations enables application of computational methods to solve problems that are intrinsically continuous.
Signals

Sampling: converting CT signals to DT

$x(t)$

$t$

$x[n] = x(nT)$

$n$

$T =$ sampling interval

Important for computational manipulation of physical data.

- digital representations of audio signals (as in MP3)
- digital representations of images (as in JPEG)
Signals

Reconstruction: converting DT signals to CT

zero-order hold

\[ x[n] \]

\[ x(t) \]

\[ T = \text{sampling interval} \]

commonly used in audio output devices
Signals

Reconstruction: converting DT signals to CT piecewise linear

\[ x[n] \]

\[ x(t) \]

\[ T = \text{sampling interval} \]

commonly used in rendering images
Periodic signals consist of repeated cycles (periods).

Useful for modeling periodic or nearly-periodic systems
- planetary motions
- vibrating strings
Signals

**Right-sided** signals are zero before some starting time. **Left-sided** signals are zero after some ending time.

Useful for modeling systems that have a well-defined starting point:
- piano note
- striking a cymbal
Signals

Signals can be **symmetric** or **antisymmetric** about time zero.

**Symmetric**

\[ x(t) = x(-t) \]

**Antisymmetric**

\[ x(t) = -x(-t) \]
Listen to the following four manipulated signals:

\[ f_1(t), f_2(t), f_3(t), f_4(t). \]

How many of the following relations are true?

- \( f_1(t) = f(2t) \)
- \( f_2(t) = -f(t) \)
- \( f_3(t) = f(2t) \)
- \( f_4(t) = \frac{1}{3}f(t) \)
Check Yourself

Computer generated speech (by Robert Donovan)

Listen to the following four manipulated signals:

\( f_1(t), f_2(t), f_3(t), f_4(t). \)

How many of the following relations are true? 2

- \( f_1(t) = f(2t) \) \( \checkmark \)
- \( f_2(t) = -f(t) \) \( \times \)
- \( f_3(t) = f(2t) \) \( \times \)
- \( f_4(t) = \frac{1}{3} f(t) \) \( \checkmark \)
Musical Sounds as Signals

Signals are functions that are used to convey information. Example: a musical sound can be represented as a function of time.

Although this time function is a complete description of the sound, it does not expose many of the important properties of the sound.
Musical Sounds as Signals

Even though these sounds have the same pitch, they sound different.

It’s not clear how the differences relate to properties of the signals.

(audio clips from http://theremin.music.uiowa.edu)
Musical Signals as Sums of Sinusoids

One way to characterize differences between these signals is express each as a sum of sinusoids.

\[ f(t) = \sum_{k=0}^{\infty} \left( c_k \cos k\omega_o t + d_k \sin k\omega_o t \right) \]

Since these sounds are (nearly) periodic, the frequencies of the dominant sinusoids are (nearly) integer multiples of a fundamental frequency \( \omega_o \).
Harmonic Structure

The sum of sinusoids describes the distribution of energy across frequencies.

\[ f(t) = \sum_{k=0}^{\infty} (c_k \cos k\omega_0 t + d_k \sin k\omega_0 t) = \sum_{k=0}^{\infty} m_k \cos (k\omega_0 t + \phi_k) \]

where \( m_k^2 = c_k^2 + d_k^2 \) and \( \tan \phi_k = \frac{d_k}{c_k} \).

This distribution represents the harmonic structure of the signal.
Harmonic Structure

The harmonic structures of notes from different instruments are different.

Some musical qualities are more easily seen in time, others in frequency.
Consonance and Dissonance

Which of the following pairs is least consonant?

A1

\[
\begin{array}{c}
\text{A1} \\
\text{A2}
\end{array}
\]

\[
\begin{array}{c}
\text{B1} \\
\text{B2}
\end{array}
\]

C1

\[
\begin{array}{c}
\text{C1} \\
\text{C2}
\end{array}
\]

Obvious from the sounds ... less obvious from the waveforms.
Express Each Signal as a Sum of Sinusoids

\[ f(t) = \sum_{k=0}^{\infty} m_k \cos(k\omega_o t + \phi_k) \]

\[ = m_1 \cos(\omega_o t + \phi_1) + m_2 \cos(2\omega_o t + \phi_2) + m_3 \cos(3\omega_o t + \phi_3) + \cdots \]

Two views: as a function of time and as a function of frequency
Express Each Signal as a Sum of Sinusoids

\[ f(t) = \sum_{k=0}^{\infty} m_k \cos(k\omega_0 t + \phi_k) \]

\[ = m_1 \cos(\omega_0 t + \phi_1) + m_2 \cos(2\omega_0 t + \phi_2) + m_3 \cos(3\omega_0 t + \phi_3) + \cdots \]

The signal \( f(t) \) can be expressed as a discrete set of frequency components:

\[ \omega_0: \; m_1, \; \phi_1 \]

\[ 2\omega_0: \; m_2, \; \phi_2 \]

\[ 3\omega_0: \; m_3, \; \phi_3 \]

\[ \cdots \]
Musical Sounds as Signals

Time functions do a poor job of conveying consonance and dissonance.

- **octave (D+D’)**
- **fifth (D+A)**
- **D+E♭**

Harmonic structure conveys consonance and dissonance better.
Fourier Representations of Signals

Fourier series are sums of harmonically related sinusoids.

\[ f(t) = \sum_{k=0}^{\infty} \left( c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t) \right) \]

where \( \omega_0 = \frac{2\pi}{T} \) represents the fundamental frequency.

Basis functions:

Q1: Under what conditions can we write \( f(t) \) as a Fourier series?

Q2: How do we find the coefficients \( c_k \) and \( d_k \).
Fourier Representations of Signals

Under what conditions can we write $f(t)$ as a Fourier series?

Fourier series can only represent **periodic** signals.

Definition: a signal $f(t)$ is periodic in $T$ if

$$f(t) = f(t+T)$$

for all $t$.

Note: if a signal is periodic in $T$ it is also periodic in $2T$, $3T$, ...

The smallest positive number $T_o$ for which $f(t) = f(t + T_o)$ for all $t$ is sometimes called the **fundamental period**.

If a signal does not satisfy $f(t) = f(t+T)$ for any value of $T$, then the signal is **aperiodic**.
Fourier Representations of Signals

Fourier series can only represent **periodic** signals.

All harmonics of \( \omega_o \) (\( \cos(k\omega_o t) \) or \( \sin(k\omega_o t) \)) are periodic in \( T = 2\pi/\omega_o \).

→ all sums of such signals are periodic in \( T = 2\pi/\omega_o \).

→ Fourier series can only represent periodic signals.
Calculating Fourier Coefficients

How do we find the coefficients $c_k$ and $d_k$ for all $k$?

Key idea: simplify by integrating over the period $T$ of the fundamental. Start with the general form:

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t))$$

Integrate both sides over $T$:

$$\int_0^T f(t) \, dt = \int_0^T c_0 \, dt + \int_0^T \left( \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t)) \right) \, dt$$

$$= T c_0 + \sum_{k=1}^{\infty} \left( c_k \int_0^T \cos(k\omega_0 t) \, dt + d_k \int_0^T \sin(k\omega_0 t) \, dt \right) = T c_0$$

All but the first term integrates to zero, leaving

$$c_0 = \frac{1}{T} \int_0^T f(t) \, dt.$$

This $k=0$ term represents the average ("DC") value.
Calculating Fourier Coefficients

Isolate the $c_l$ term by multiplying both sides by $\cos(l\omega_o t)$ before integrating.

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} (c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t))$$

$$\int_0^T f(t) \cos(l\omega_o t) \, dt = \int_0^T c_0 \cos(l\omega_o t) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_0^T c_k \cos(k\omega_o t) \cos(l\omega_o t) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_0^T d_k \sin(k\omega_o t) \cos(l\omega_o t) \, dt$$

A product of sinusoids can be expressed as sum and difference frequencies.

$$\cos(k\omega_o t) \cos(l\omega_o t) = \frac{1}{2} \cos((k-l)\omega_o t) + \frac{1}{2} \cos((k+l)\omega_o t)$$

$$\sin(k\omega_o t) \cos(l\omega_o t) = \frac{1}{2} \sin((k-l)\omega_o t) + \frac{1}{2} \sin((k+l)\omega_o t)$$
Calculating Fourier Coefficients

Isolate the $c_l$ term by multiplying both sides by $\cos(l\omega_0 t)$ before integrating.

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t))$$

$$\int_{0}^{T} f(t) \cos(l\omega_0 t) \, dt = \int_{0}^{T} c_0 \cos(l\omega_0 t) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_{0}^{T} c_k \left( \frac{1}{2} \cos((k-l)\omega_0 t) + \frac{1}{2} \cos((k+l)\omega_0 t) \right) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_{0}^{T} d_k \left( \frac{1}{2} \sin((k-l)\omega_0 t) + \frac{1}{2} \sin((k+l)\omega_0 t) \right) \, dt$$

A product of sinusoids can be expressed as sum and difference frequencies.

$$\cos(k\omega_0 t) \cos(l\omega_0 t) = \frac{1}{2} \cos((k-l)\omega_0 t) + \frac{1}{2} \cos((k+l)\omega_0 t)$$

$$\sin(k\omega_0 t) \cos(l\omega_0 t) = \frac{1}{2} \sin((k-l)\omega_0 t) + \frac{1}{2} \sin((k+l)\omega_0 t)$$
Calculating Fourier Coefficients

Isolate the $c_l$ term by multiplying both sides by $\cos(l\omega_0 t)$ before integrating.

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t))$$

$$\int_0^T f(t) \cos(l\omega_0 t) \, dt = \int_0^T c_0 \cos(l\omega_0 t) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_0^T c_k \left( \frac{1}{2} \cos((k-l)\omega_0 t) + \frac{1}{2} \cos((k+l)\omega_0 t) \right) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_0^T d_k \left( \frac{1}{2} \sin((k-l)\omega_0 t) + \frac{1}{2} \sin((k+l)\omega_0 t) \right) \, dt$$

The $c_0$ term is zero because the integral of $\cos(l\omega_0 t)$ over $T$ is zero.
Calculating Fourier Coefficients

Isolate the $c_l$ term by multiplying both sides by $\cos(l\omega_o t)$ before integrating.

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} (c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t))$$

$$\int_0^T f(t) \cos(l\omega_o t) \, dt = \int_0^T c_0 \cos(l\omega_o t) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_0^T c_k \left( \frac{1}{2} \cos((k-l)\omega_o t) + \frac{1}{2} \cos((k+l)\omega_o t) \right) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_0^T d_k \left( \frac{1}{2} \sin((k-l)\omega_o t) + \frac{1}{2} \sin((k+l)\omega_o t) \right) \, dt$$

If $k = l$, then $\cos((k-l)\omega_o t) = 1$ and the integral is $\frac{T}{2} c_l$.

All of the other $\cos((k-l)\omega_o t)$ terms in the sum integrate to zero.

All of the $\cos((k+l)\omega_o t)$ terms in the integrate to zero.
Calculating Fourier Coefficients

Isolate the $c_l$ term by multiplying both sides by $\cos(l\omega_0 t)$ before integrating.

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t))$$

$$\int_0^T f(t) \cos(l\omega_0 t) \, dt = \int_0^T c_0 \cos(l\omega_0 t) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_0^T c_k \left( \frac{1}{2} \cos((k-l)\omega_0 t) + \frac{1}{2} \cos((k+l)\omega_0 t) \right) \, dt$$

$$+ \sum_{k=1}^{\infty} \int_0^T d_k \left( \frac{1}{2} \sin((k-l)\omega_0 t) + \frac{1}{2} \sin((k+l)\omega_0 t) \right) \, dt$$

If $k = l$, then $\sin((k-l)\omega_0 t) = 0$ and the integral is 0. All of the other $d_k$ terms are harmonic sinusoids that integrate to 0.

The only non-zero term on the right side is $\frac{T}{2} c_l$.

We can solve to get an expression for $c_l$ as

$$c_l = \frac{2}{T} \int_0^T f(t) \cos(l\omega_0 t) \, dt$$
Calculating Fourier Coefficients

Analogous reasoning allows us to calculate the $d_k$ coefficients, but this time multiplying by $\sin(l\omega t)$ before integrating.

\[
f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} (c_k \cos(k\omega t) + d_k \sin(k\omega t))
\]

\[
\int_0^T f(t) \sin(l\omega t) \, dt = \int_0^T c_0 \sin(l\omega t) \, dt \\
+ \sum_{k=1}^{\infty} \int_0^T c_k \cos(k\omega t) \sin(l\omega t) \, dt \\
+ \sum_{k=1}^{\infty} \int_0^T d_k \sin(k\omega t) \sin(l\omega t) \, dt
\]

A single term remains after integrating, allowing us to solve for $d_l$ as

\[
d_l = \frac{2}{T} \int_0^T f(t) \sin(l\omega t) \, dt
\]
Calculating Fourier Coefficients

Summarizing . . .

If $f(t)$ is expressed as a Fourier series

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} \left( c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t) \right)$$

the Fourier coefficients are given by

$$c_0 = \frac{1}{T} \int_T f(t) \, dt$$

$$c_k = \frac{2}{T} \int_T f(t) \cos(k\omega_0 t) \, dt; \, \, k = 1, 2, 3, \ldots$$

$$d_k = \frac{2}{T} \int_T f(t) \sin(k\omega_0 t) \, dt; \, \, k = 1, 2, 3, \ldots$$
Example of Analysis

Find the Fourier series coefficients for the following triangle wave:

\[ f(t) = f(t+2) \]

\[ T = 2 \]

\[ \omega_o = \frac{2\pi}{T} = \pi \]

\[ c_0 = \frac{1}{T} \int_0^T f(t) \, dt = \frac{1}{2} \int_0^2 f(t) \, dt = \frac{1}{2} \]

\[ c_k = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos \frac{2\pi kt}{T} \, dt = 2 \int_0^1 t \cos(\pi kt) \, dt = \begin{cases} -\frac{4}{\pi^2 k^2} & k \text{ odd} \\
0 & k = 2, 4, 6, \ldots \end{cases} \]

\[ d_k = 0 \quad \text{(by symmetry)} \]
Example of Synthesis

Generate $f(t)$ from the Fourier coefficients in the previous slide.

Start with the Fourier coefficients

$$f(t) = c_0 - \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t)) = \frac{1}{2} - \sum_{k=1, k \text{ odd}}^{\infty} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

$$f(t) = \frac{1}{2} - \sum_{k=1, k \text{ odd}}^{0} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$
Example of Synthesis

Generate $f(t)$ from the Fourier coefficients in the previous slide.

Start with the Fourier coefficients

$$f(t) = c_0 - \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t)) = \frac{1}{2} - \sum_{k=1}^{\infty} \frac{4}{\pi^2 k^2} \cos(k \pi t)$$

$$f(t) = \frac{1}{2} - \sum_{k=1}^{1} \frac{4}{\pi^2 k^2} \cos(k \pi t)$$
Example of Synthesis

Generate $f(t)$ from the Fourier coefficients in the previous slide.

Start with the Fourier coefficients

$$f(t) = c_0 - \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t)) = \frac{1}{2} - \sum_{k=1 \text{ odd}}^{\infty} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

$$f(t) = \frac{1}{2} - \sum_{k=1 \text{ odd}}^{3} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

![Graph of f(t)](image-url)
Example of Synthesis

Generate $f(t)$ from the Fourier coefficients in the previous slide.

Start with the Fourier coefficients

$$f(t) = c_0 - \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t)) = \frac{1}{2} - \sum_{\substack{k = 1 \\text{k odd}}}^{\infty} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

$$f(t) = \frac{1}{2} - \sum_{\substack{k = 1 \\text{k odd}}}^{5} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

![Graph of f(t)](image-url)
Example of Synthesis

Generate \( f(t) \) from the Fourier coefficients in the previous slide.

Start with the Fourier coefficients

\[
f(t) = c_0 - \sum_{k=1}^{\infty} \left( c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t) \right) = \frac{1}{2} - \sum_{\substack{k = 1 \atop k \text{ odd}}}^{\infty} \frac{4}{\pi^2 k^2} \cos(k\pi t)\]

\[
f(t) = \frac{1}{2} - \sum_{\substack{k = 1 \atop k \text{ odd}}}^{7} \frac{4}{\pi^2 k^2} \cos(k\pi t)\]

[Graph of \( f(t) \) over the interval \(-2 \leq t \leq 2\)]
Example of Synthesis

Generate $f(t)$ from the Fourier coefficients in the previous slide.

Start with the Fourier coefficients

$$f(t) = c_0 - \sum_{k=1}^{\infty} \left( c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t) \right) = \frac{1}{2} - \sum_{k = 1 \atop k \text{ odd}}^{\infty} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

$$f(t) = \frac{1}{2} - \sum_{k = 1 \atop k \text{ odd}}^{9} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$
Example of Synthesis

Generate $f(t)$ from the Fourier coefficients in the previous slide.

Start with the Fourier coefficients

$$f(t) = c_0 - \sum_{k=1}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t)) = \frac{1}{2} - \sum_{\substack{k = 1 \\text{k odd}}}^{\infty} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

$$f(t) = \frac{1}{2} - \sum_{\substack{k = 1 \\text{k odd}}}^{19} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

![Graph of $f(t)$](image-url)
Example of Synthesis

Generate $f(t)$ from the Fourier coefficients in the previous slide.

Start with the Fourier coefficients

$$f(t) = c_0 - \sum_{k=1}^{\infty} (c_k \cos(k\omega t) + d_k \sin(k\omega t)) = \frac{1}{2} - \sum_{k = 1}^{\infty} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

$$f(t) = \frac{1}{2} - \sum_{k = 1}^{99} \frac{4}{\pi^2 k^2} \cos(k\pi t)$$

The synthesized function approaches original as number of terms increases.
Two Views of the Same Signal

The harmonic expansion provides an alternative view of the signal.

\[
f(t) = \sum_{k=0}^{\infty} (c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t)) = \sum_{k=0}^{\infty} m_k \cos(k\omega_o t + \phi_k)
\]

We can view the musical signal as
- a function of time \( f(t) \), or
- as a sum of harmonics with amplitudes \( m_k \) and phase angles \( \phi_k \).

Both views are useful. For example,
- the peak sound pressure is more easily seen in \( f(t) \), while
- consonance is more easily analyzed by comparing harmonics.

This type of harmonic analysis is an example of Fourier Analysis, which is a major theme of this subject.
Recitations
Reconvene in 10 minutes for recitation.

If the second character of your kerberos username is in ’abcdefghij’:
   go to room 24-121
else:
   go to room 26-328