Magnetic resonance images are constructed from measurements of Fourier (k-space) data.

measure $F[kr,kc]$:

\[
\log |F'| 
\]

\[
\angle (F') 
\]

image $f = \text{abs}(\text{ifft2}(F))$:
Accelerating Imaging

An important area of current research is in decreasing the time required to capture an image. One idea for accelerating imaging is to intentionally undersample the frequency representation.

What is the effect of measuring $F[k_r, k_c]$ at only even values of $k_c$?
Accelerating Imaging

Let $F[k_r, k_c]$ represent the original k-space data and $G[k_r, k_c]$ represent the k-space data after odd numbered columns are set to zero.

$$G[k_r, k_c] = F[k_r, k_c] \left( \frac{1 + (-1)^{k_c}}{2} \right) = \frac{1}{2} F[k_r, k_c] \left( 1 + e^{j\pi k_c} \right)$$

$$g[r, c] = \sum_{k_r, k_c} \frac{1}{2} F[k_r, k_c] \left( 1 + e^{j\pi k_c} \right) e^{j\frac{2\pi k_r}{R}r} e^{j\frac{2\pi k_c}{C}c}$$

$$= \frac{1}{2} f[r, c] + \frac{1}{2} \sum_{k_r, k_c} F[k_r, k_c] e^{j\pi k_c} e^{j\frac{2\pi k_r}{R}r} e^{j\frac{2\pi k_c}{C}c}$$

$$= \frac{1}{2} f[r, c] + \frac{1}{2} \sum_{k_r, k_c} F[k_r, k_c] e^{j\frac{2\pi k_r}{R}r} e^{j\frac{2\pi k_c}{C}c} (c + \frac{C}{2})$$

$$= \frac{1}{2} f[r, c] + \frac{1}{2} f \left[ r, \left( c + \frac{C}{2} \right) \mod C \right]$$

Setting odd-numbered columns of $F[k_r, k_c]$ to zero adds a half-frame circular shift to the right (or equivalently left) of the image.
Accelerating Imaging

Omitting odd numbered columns in k-space.

\[ \log |G| \quad \angle(G) \]

measure \( G[kr, kc] \):

image \( g = \text{abs}(\text{ifft2}(G)) \):

Not a good way to speed imaging, but motivates a better approach.
Multi-Coil MRI

Multiple readout coils provide additional data without increasing imaging time.

Consider two coils, one on each side of the head. The left coil will be more sensitive to the left portions of the brain, and vice versa. Characterize the sensitivity of each coil by specifying a scalar between 0 (insensitive) and 1 (sensitive) for each pixel in the brain image.

In 1D, our coil sensitivities could have the following form:

\[
c_1[c] \\
\]

\[
c_2[c] \\
\]

What would be the effect of each of these coils on the brain image?
Images From Coils 1 and 2

Since coil 1 is only sensitive to the left part of the brain, the image produced with data from coil 1 shows just the left half of the brain.

If we only measure $F_1[k_r, k_c]$ at even-numbered $k_c$, the image from coil 1 is added to a half-frame circularly shifted version (next slide).
**Images From Coils 1 and 2**

$g_1$ and $g_2$ are after omitting odd numbered columns.

<table>
<thead>
<tr>
<th>$f_1$</th>
<th>$f_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$g_1$</th>
<th>$g_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Images From Coils 1 and 2

Could you construct a full-frame full-resolution image of the brain from these data?

Yes. Combine the left part of $|g_1|$ with the right part of $|g_2|$.

Advantage:

$|g_1|$ was acquired in half the time required for a full-frame full-resolution image. Similar with $|g_2|$.

But $|g_1|$ and $|g_2|$ data were acquired simultaneously!
Why Does This Work?

Omitting odd-numbered columns in k-space is sampling in frequency. Sampling in frequency causes aliasing in space: $|g|$ contains two (overlapping) copies of the image in $|f|$.
Why Does This Work?

Results for coil 1 and 2 are similar. Sampling in frequency causes aliasing in space: but the aliased copy no longer overlaps the original, and can be removed.

Which is the aliased copy?
Why Does This Work?

Results for coil 1 and 2 are similar.
Sampling in frequency causes aliasing in space: but the aliased copy no longer overlaps the original, and can be removed.

Which is the aliased copy?

Image $|f_1|$ is from coil 1, which is only sensitive to left part of the brain. Therefore the right part of $|g_1|$ must be the aliased part.
Constructing Full-Frame Image From Coil 1 and 2 Data
What if the coils had the following sensitivities?

What would be the effect of each of these coils on the brain image?
What if the coils had the following sensitivities?

$c_3[c]$  

$c_4[c]$  

What would be the effect of each of these coils on the brain image? $c_3$ is a full-frame image. Omitting the odd number columns from $G_3$ will produce the aliased image we started with.

$c_4$ is the same as the previous $c_2$, so $|g_4|$ is the same as $|g_2|$. 
Can we create a full-frame full-resolution image from this data?
Images From Coils 3 and 4

The $|f_3|$ image can be viewed as the sum of results for the left and right sides of the brain (as in the $c_1$ and $c_2$ example).

Subtracting $|g_4|$ from $|g_3|$ would generate the previous $|g_1|$ image.

Algorithm:

Combine the left part of $|g_3| - |g_4|$ with the right part of $|g_4|$. 
What if the coils had the following sensitivities?

$c_5[c]$  

$c  

What would be the effect of each of these coils on the brain image?
What if the coils had the following sensitivities?

\[ c_5[c] \quad c \quad c_6[c] \quad c \]

What would be the effect of each of these coils on the brain image?
Multi-Coil MRI

What if the coils had the following sensitivities?

Notice that $c_6$ weights contributions from pixels in the range $-32 \leq c < 0$ exactly the same as those in $96 \leq c < 128$.

Therefore the $c_6$ image contains no information that is useful for separating these two bands of pixels.

Similar statements apply for $c_5$. 
Images From Coils 5 and 6

$g_5, g_6$ are after omitting odd numbered columns from $|f_5|, |f_6|$. 

![Images of coil outputs](image-url)
Images From Coils 5 and 6

Highlighted regions are identical: both represent sum $f[r, c] + f[r, c+128]$. 
Conclusions

Magnetic Resonance Imaging can be made faster using multiple readout coils.

Increased speed results from parallel acquisition of undersampled k-space data.

Method is only effective if the multiple coils disambiguate aliased parts of image.

Modern MRI systems can use as many as 32 coils.