Sounds as Signals

February 7, 2019
Tones and Sinusoids

A “tone” is a pressure that changes sinusoidally with time.

In 6.003, we will think of this as a “continuous-time” (CT) signal. In contrast, a “discrete-time” (DT) signal is a sequence of numbers.

Mathematically:

\[ x(t) = A \cos(\omega t) \]
\[ x[n] = A \cos(\Omega n) \]
Assume that $x[n]$ represents “samples” of $x(t)$:

$$x(t) = A \cos(\omega t)$$

$$x[n] = A \cos(\Omega n)$$

- What are the units of $\omega$, $t$, $\Omega$, and $n$?

Let $f$ represent the “frequency” of the tone in cycles/second.

- Determine $\omega$ in terms of $f$.
- Determine $\Omega$ in terms of $\omega$. [$\rightarrow f_s$]
- Determine $\Omega$ in terms of $f$. 
CT and DT Representations

Assume that $x[n]$ represents “samples” of $x(t)$:

$$x(t), x[n]$$

$$x(t) = A \cos(\omega t)$$  \hspace{1cm}  $$x[n] = A \cos(\Omega n)$$

- What are the units of $\omega$, $t$, $\Omega$, and $n$?

The product $\omega t$ is measured in units of **radians** (dimensionless ratio). Time $t$ is measured in units of **seconds**.
Therefore $\omega$ is measured in units of **radians/second**.

The product $\Omega n$ is measured in units of **radians** (domain of $\cos(\cdot)$).
Discrete time $n$ is a **dimensionless** integer.
Therefore $\Omega$ is measured in units of **radians**.

For convenience, we often think of $n$ as measured in **number of samples** and $\Omega$ in **radians/sample**.
Assume that $x[n]$ represents “samples” of $x(t)$:

$$x(t), x[n]$$

$$x(t) = A \cos(\omega t) \quad x[n] = A \cos(\Omega n)$$

Let $f$ represent the “frequency” of the tone in cycles/second.

- Determine $\omega$ in terms of $f$.
- Determine $\Omega$ in terms of $\omega$. $\rightarrow f_s$
- Determine $\Omega$ in terms of $f$.

$$\omega [\text{rad/sec}] = 2\pi [\text{rad/cycle}] f [\text{cycles/sec}]$$

$$\Omega [\text{rad/sample}] = \frac{\omega [\text{rad/sec}]}{f_s [\text{samples/sec}]} \quad \text{where } f_s = \text{sample frequency}$$

$$\Omega [\text{rad/sample}] = \frac{2\pi [\text{rad/cycle}] f [\text{cycles/sec}]}{f_s [\text{samples/sec}]}$$
Generating Sounds

Write a program to generate a tone.

We have provided some Python utilities to manipulate digital audio: \texttt{wav_utils.py} in this week’s lab.

The function \texttt{write_wav} creates a .wav file from 3 input arguments:

- \texttt{samples}: list of discrete samples
- \texttt{sample_frequency}: in samples/second
- \texttt{filename}: of resulting .wav file
Plotting

Make a plot of the numbers in list $x$.

**Use** matplotlib.

```python
import matplotlib.pyplot as plt

Line Plot

plt.plot(x)
plt.show()

Stem Plot

plt.stem(x)
plt.show()
```
Aliasing

As the frequency $\Omega$ increases, the shapes of the sampled signals deviate from those of the underlying CT signals.

$$\Omega = 1 : x[n] = \cos(n)$$

$$\Omega = 2 : x[n] = \cos(2n)$$

$$\Omega = 3 : x[n] = \cos(3n)$$
Aliasing

Worse and worse representation.

\[ \Omega = 4 : x[n] = \cos(4n) = \cos(2\pi - 4n) \approx \cos(2.283n) \]

\[ \Omega = 5 : x[n] = \cos(5n) = \cos(2\pi - 5n) \approx \cos(1.283n) \]

\[ \Omega = 6 : x[n] = \cos(6n) = \cos(2\pi - 6n) \approx \cos(0.283n) \]
Aliasing

For $\Omega > \pi$, a lower frequency $\Omega_L$ has the same sample values as $\Omega$.

$\Omega = 4 : x[n] = \cos(4n) = \cos(2\pi - 4n) \approx \cos(2.283n)$

$\Omega = 5 : x[n] = \cos(5n) = \cos(2\pi - 5n) \approx \cos(1.283n)$

$\Omega = 6 : x[n] = \cos(6n) = \cos(2\pi - 6n) \approx \cos(0.283n)$

The same DT sequence represents many different values of $\Omega$. 