Enter all answers in the boxes provided.
Other work may be considered when assigning partial credit.

You have two hours.
This quiz is closed book, but you may use two 8.5 \times 11 sheets of paper (both sides).
No calculators, computers, cell phones, music players, or other electronic devices.
1. Frequency Response [20 points]

Part 1. Let $h_1[n]$ represent the unit sample response of a linear, time-invariant system

\[ x[n] \rightarrow \text{LTI system} \rightarrow y[n] = (x * h_1)[n] \]

where

\[ h_1[n] = \begin{cases} 
1 & \text{if } n = 0 \text{ or } n = 1 \\
0 & \text{otherwise} 
\end{cases} \]

Determine the magnitude and angle of the frequency response of this filter, and enter expressions for them in the boxes below.

magnitude = $2 \left| \cos \frac{\Omega}{2} \right|$

angle = $-\frac{\Omega}{2}$

\[ H_1(\Omega) = 1 + e^{-j\Omega} = 2e^{-j\Omega/2} \left( e^{j\Omega/2} + e^{-j\Omega/2} \right) = 2e^{-j\Omega/2} \cos \frac{\Omega}{2} \]

Part 2. Let $h_2[n]$ represent an alternative unit sample response where

\[ h_2[n] = \begin{cases} 
-j & \text{if } n = 0 \\
j & \text{if } n = 1 \\
0 & \text{otherwise} 
\end{cases} \]

Calculate the frequency response of this filter and plot its magnitude and phase on the axes below. Label all important frequencies, magnitudes, and angles.

\[ H_2(\Omega) = -j + je^{-j\Omega} = 2e^{-j\Omega/2} \left( e^{j\Omega/2} - e^{-j\Omega/2} \right) = 2e^{-j\Omega/2} \sin \frac{\Omega}{2} \]

\[ |H_2(\Omega)| = \left| \sin \frac{\Omega}{2} \right| \]

\[ \angle H_2(\Omega) = \begin{cases} 
-\Omega/2 & \text{if } 0 < \Omega < 2\pi \\
-\Omega/2 - \pi & \text{if } -2\pi < \Omega < 0 
\end{cases} \]
Part 3. Let $h_3[r, c]$ represent the unit sample response of a 2D linear, time-invariant system where $h_3[r, c]$ is defined by the following diagram for $-1 \leq r \leq 1$ and $-1 \leq c \leq 1$ and is zero outside that range.

\[ \begin{pmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

Which of the following panels shows the magnitude of the 2D frequency response of this filter, where black represents 0 and white represents the most positive number in each image.

Enter A-L or none: **A**

\[
H_3(\Omega_r, \Omega_c) = e^{-j\Omega_r} e^{-j\Omega_c} - e^{-j\Omega_r} - 2e^{-j\Omega_c} - 2e^{j\Omega_c} + e^{j\Omega_r} e^{j\Omega_c} - e^{j\Omega_r} e^{-j\Omega_c}
\]

\[
= (e^{-j\Omega_r} + 2 + e^{j\Omega_r})(e^{-j\Omega_c} - e^{j\Omega_c})
\]

\[
= -4j(1 + \cos \Omega_r) \sin \Omega_c
\]
2. Interpreting Fourier Transforms [20 points]

Let $F_a(\Omega)$ represent the Discrete-Time Fourier Transform of a discrete-time signal $f_a[n]$, which is a pulse of unknown width.

$$f_a[n] = \begin{cases} 1 & -W \leq n \leq W \\ 0 & \text{otherwise} \end{cases}$$

Let $F_b[k]$ represent a sampled version of $F_a(\Omega)$,

$$F_b[k] = F_a \left( \frac{2\pi k}{9} \right)$$

where $k$ is an integer.

**Part 1.** Use an inverse Discrete Fourier Transform with length $N = 9$ to find $f_b[n]$:

$$f_b[n] \xleftarrow{\text{DFT}} N=9 F_b[k]$$

Enter your answer in the boxes below.

- $f_b[0]$: 9
- $f_b[1]$: 9
- $f_b[2]$: 9
- $f_b[3]$: 0
- $f_b[4]$: 0
- $f_b[5]$: 0
- $f_b[6]$: 0
- $f_b[7]$: 9
- $f_b[8]$: 9

**Part 2.** Let $F_c[k]$ represent the DFT that results when each element of $F_b[k]$ is squared:

$$F_c[k] = F_b^2[k].$$

Use an inverse Discrete Fourier Transform with length $N = 9$ to find $f_c[n]$:

$$f_c[n] \xleftarrow{\text{DFT}} N=9 F_c[k]$$

Enter your answer in the boxes below.

- $f_c[0]$: 45
- $f_c[1]$: 36
- $f_c[2]$: 27
- $f_c[3]$: 18
- $f_c[4]$: 9
- $f_c[5]$: 9
- $f_c[6]$: 18
- $f_c[7]$: 27
- $f_c[8]$: 36
3. Discrete-Time Convolutions  

Consider the convolution of two of the following signals, which are equal to 0 outside the indicated ranges.

Determine if each of the following signals can be constructed by convolving \((a \text{ or } b \text{ or } c)\) with \((a \text{ or } b \text{ or } c)\). If it can, indicate which signals should be convolved. If it cannot, put an \(\times\) in both boxes.

Notice that there are ten possible answers:
\((a * a), (a * b), (a * c), (b * a), (b * b), (b * c), (c * a), (c * b), (c * c), \text{ or } (\times, \times)\).

Notice also that the answer may not be unique.
4. Phase Matching [20 points]

The following periodic signal $x(t)$ has period $T = 8$.

$$x(t) = x(t - 8)$$

The magnitude and phase of the Fourier series coefficients $a_k$ of $x(t)$ are given below.

| $k$ | $|a_k|$ | $\angle a_k$ |
|-----|-------|------------|
| 0   |       | $\pi$      |
| 1   |       | $-\pi$     |

Determine which angle function from the next page corresponds to each of these signals:

- A, B, ... F or none: E
- A, B, ... F or none: B
- A, B, ... F or none: F
- A, B, ... F or none: none
Note: $\times$ means the angle is undefined because the magnitude of that coefficient is zero.
5. Combinations [20 points]
Consider the following 2-D signals (labeled A through H). In each:

- black represents a value of 0, and white represents a value of 1
- the origin \((r = 0, c = 0)\) is in the upper-left corner
- the image is 63 pixels wide and 63 pixels tall
- all lines are white and one pixel wide

On the second and third pages of the separate image handout are several plots of DFT coefficient magnitudes. For each of the combinations of signals below (where \(\oplus\) denotes element-wise addition, \(\times\) denotes element-wise multiplication, and \(\ast\) denotes circular convolution), indicate which of the numbered images best matches the DFT coefficient magnitudes of that combination. Enter a single number in each box.

\[
\begin{array}{cccccc}
A + B & 11 & A \times B & 9 & A \oplus B & 8 & E \times F & 14 \\
C + B & 17 & C \times B & 18 & C \oplus B & 8 & E \ast F & 13 \\
A + G & 10 & A \times G & 7 & A \oplus G & 8 & (A + B) + E & 1 \\
F + G & 2 & F \times G & 5 & F \oplus G & 8 & (A + B) \times E & 3 \\
C + D & 4 & C \times D & 6 & C \oplus D & 8 & (A + B) \ast E & 16 \\
\end{array}
\]
Each plot below shows the magnitude of the DFT coefficients associated with some 2-D signal. In each, \((k_r = 0, k_c = 0)\) is in the center of the image. Black represents 0, and pure white represents the highest value in the image (not necessarily 1).
Worksheet (intentionally blank)
Worksheet (intentionally blank)