Final Examination

Name:
Kerberos Username:

Enter all answers in the boxes provided.
Other work on pages with QR codes may be considered when assigning partial credit.

You have three hours.
This quiz is closed book, but you may use three 8.5 × 11 sheets of paper (six sides).
No calculators, computers, cell phones, music players, or other electronic devices.
1. Discrete-Time Transforms [24 points]

Part 1.

Let $X_1(\Omega)$ represent the Discrete-Time Fourier Transform (DTFT) of the signal

$$x_1[n] = \begin{cases} 
-2 & n = -2 \\
1 & n = -1, 0, 1 \\
2 & n = 2 \\
0 & \text{otherwise}
\end{cases}$$

Determine a closed form expression for $X_1(\Omega)$ in terms of sin and cos functions (no complex exponentials), and enter your expression in the box below.

Let $X_2(\Omega)$ represent the DTFT of the signal

$$x_2[n] = (-j)^n + e^{j\pi n/4}$$

Write a closed form expression for $X_2(\Omega)$ in the box below.
**Part 2.**

Let $X_3[k]$ represent the Discrete Fourier Transform (DFT) of $x_3[n]$ computed with $N = 6$. Define a new signal $y_3[n]$ in terms of $x_3[n]$:

$$y_3[n] = 9 - 2x_3[n]$$

Let $Y_3[k]$ represent the DFT of $y_3[n]$. Determine expressions for $Y_3[k]$ in terms of $X_3[k]$, and enter those expressions in the boxes below.

$$Y_3[0] = \quad Y_3[1] = \quad Y_3[2] =$$


Let $X_4[k]$ represent the DFT of $x_4[n]$ computed with $N = 6$. Define a new signal

$$y_4[n] = 5(-1)^nx_4[n]$$

Let $Y_4[k]$ represent the DFT of $y_4[n]$. Determine expressions for $Y_4[k]$ in terms of $X_4[k]$, and enter those expressions in the boxes below.

$$Y_4[0] = \quad Y_4[1] = \quad Y_4[2] =$$

**Part 3.**

Let $F_i[k_r, k_c]$ represent the Discrete Fourier Transforms (DFTs) of the following two-dimensional signals $f_i[r, c]$, where $0 \leq r < 64$ and $0 \leq c < 64$. For each signal, list all of the indices of the form $[k_r, k_c]$, where $-31 \leq k_r \leq 32$ and $-31 \leq k_c \leq 32$, for which $F_i[k_r, k_c] \neq 0$ in the box provided.

Notice that $f_i[r, c]$ may have positive, negative, or even complex values.

<table>
<thead>
<tr>
<th>$f_i[r, c]$</th>
<th>list of 2D indices $([k_r, k_c])$ for which $F_i[k_r, k_c] \neq 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\cos \frac{6\pi r}{64})(\cos \frac{8\pi c}{64})$</td>
<td></td>
</tr>
<tr>
<td>$(\cos \frac{6\pi r}{64}) + (\cos \frac{8\pi c}{64})$</td>
<td></td>
</tr>
<tr>
<td>$\cos \left(\frac{6\pi r}{64} + \frac{8\pi c}{64}\right)$</td>
<td></td>
</tr>
<tr>
<td>$\cos \left(\frac{6\pi r}{64} - \frac{8\pi c}{64}\right)$</td>
<td></td>
</tr>
<tr>
<td>$\sin \left(\frac{6\pi r}{64} + \frac{8\pi c}{64}\right)$</td>
<td></td>
</tr>
<tr>
<td>$(1 + \cos \frac{6\pi r}{64})(1 + \cos \frac{8\pi c}{64})$</td>
<td></td>
</tr>
<tr>
<td>$e^{j2\pi \frac{3r}{64}}e^{j2\pi \frac{4c}{64}}$</td>
<td></td>
</tr>
<tr>
<td>$e^{j2\pi \frac{3r}{64}} + e^{j2\pi \frac{4c}{64}}$</td>
<td></td>
</tr>
<tr>
<td>$(-1)^r + (-1)^c$</td>
<td></td>
</tr>
<tr>
<td>$(\cos \frac{32\pi r}{64})^2$</td>
<td></td>
</tr>
</tbody>
</table>
Worksheet (intentionally blank)
2. **Surveying** [20 points]

Farmland in the midwestern United States is often divided into “sections” of uniform size. When viewed from above, this land often looks much like a grid, as can be seen in the photograph below:

![Farmland from above](image)

The following shows the magnitudes of a portion of the DFT coefficients of a different satellite photograph of farmland, taken from directly overhead.

![DFT coefficients](image)

In order to answer the questions on the following page, you may assume that the sections in the image are nearly identical, nearly rectangular (but not necessarily square), and laid out on a nearly perfect grid. Given this information, answer the questions on the following page about the image whose DFT coefficients are shown above.
Approximately how many sections of land are there in the image whose DFT coefficients are shown on the previous page? 

Explain your method.

If the image represented an area that was 10.5 miles wide and 4.0 miles tall, what are the approximate dimensions of each of the sections of land, in miles?

Height: 

Width: 

Explain your method.
3. Filtering  [22 points]

Part 1.

For this problem, we will start with a DT filter whose frequency response is:

\[ H(\Omega) = H(\Omega + 2\pi) = \begin{cases} 
1, & \text{if } 0 \leq |\Omega| < \frac{\pi}{3} \\
0, & \text{otherwise}
\end{cases} \]

Now, consider constructing a new filter of the following form based on \( H \):

\[ x[n] \rightarrow w[n] \rightarrow H(\Omega) \rightarrow z[n] \rightarrow y[n] \]

\[ (\neg 1)^n \rightarrow \neg 1^n \]

Mathematically, the relationship between the signals \( x[n] \) and \( y[n] \) is given by:

\[ w[n] = (-1)^n \times x[n] \]

\[ z[n] = (w * h)[n] \]

\[ y[n] = (-1)^n \times z[n] \]

Consider applying this new filter to a signal \( x_1 \), whose DTFT magnitudes are shown below:

On the following page, sketch a graph of the DTFT magnitudes of the resulting signal. Label all key points on your graph.
\[ |Y_1(\Omega)| \]

\[ \begin{array}{c}
\Omega \\
-2\pi & -\pi & \pi & 2\pi
\end{array} \]

\[ \begin{array}{c}
|Y_1(\Omega)| \\
-1 & 1
\end{array} \]
Part 2.
Consider the following image, which we’ll refer to as $x[r,c]$. This image has height $R$ and width $C$.

In addition, consider several other signals:

$$H_1[k_r, k_c] = j \sin \left( \frac{10\pi k_r}{R} \right)$$

$$h_2[r, c] = \sin \left( \frac{10\pi r}{R} \right)$$

For each of the expressions below, indicate which of the images on the following page is represented by that expression. In each of those images, black represents the lowest value (not necessarily 0), and white represents the highest value (not necessarily 1).

$$(x \times h_1)[r, c]$$

$$(x \times h_2)[r, c]$$

$$(x \odot h_1)[r, c]$$

$$(x \odot h_2)[r, c]$$
4. Time and Frequency Patterns  [16 points]

Eight time waveforms are shown in the left panels below. The corresponding Fourier transform magnitudes are shown in the right panels, however, the order has been shuffled. For each panel on the right, find the corresponding time waveform from the left panels, and enter its number in the box provided.

Each time function is plotted on the same time scale, and each frequency function is plotted on the same frequency scale. Time functions are zero outside the range shown.
Worksheet (intentionally blank)
5. Echos  [18 points]

Assume that a single echo interferes with a speaker’s voice that is being recorded by a microphone as illustrated in the following figure.

We can represent this recording situation as a linear, time-invariant system, with the speaker’s voice as the input and the recorded microphone signal as the output. Assume that the impulse response of this system is

\[ h(t) = \delta(t - T_1) + \epsilon \delta(t - T_2) \]

where \( T_1 \) represents that delay of the direct path from speaker to microphone, \( T_2 \) represents that delay through the echo path, and \( \epsilon \) represents the amplitude of the echo.
Part 1.

On the axes below, sketch the magnitude and angle of the frequency response of this system in the absence of an echo (i.e., when $\epsilon = 0$). Label all important values.
Part 2.

Now we’ll consider the case where there is an echo. The following plots show that magnitude and angle of the frequency response of this system for $|\omega| < 1500\text{rad/s}$, for some values of $T_1$, $T_2$, and $\epsilon$.

Determine values of $T_1$, $T_2$, and $\epsilon$ that are consistent with the graphs above.

\[ T_1 = \]

\[ T_2 = \]

\[ \epsilon = \]
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