Time-Varying Signals

Real-world signals (i.e., speech, music, ...) often have frequency content that varies with time.

Problem with DFT: events that are local in time are global in frequency (and vice versa). Sudden changes and local variations can be difficult to detect.

Example: 2 tunes
How to tell them apart?

Short-time Fourier Transform

STFT is a compromise between time- and frequency-domain representations, representing the frequency content of the signal at various points in time.

Formally, we define the STFT of a signal $x$ as:

$$\text{STFT}\{x\}[m, k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n + m \times s]w[n]e^{-j2\pi kn/N}$$

where:

- $N$ is the length of a window
- $s$ is a “step size”
- $w[\cdot]$ is a window function
- $m$ is a time index, and $k$ is a frequency index
Short-time Fourier Transform

Conceptually, we are taking the DFT of successive windowed regions of the original signal (and these regions may overlap).

STFT and Spectrograms

The STFT enhances our ability to reason about the frequency content of signals at various points in time. It is often visualized using a spectrogram, which is defined to be the magnitude squared of the STFT.

Today

The Rest of Today: Examples of spectrograms

Recitation: Circular convolution, windowing

PSet, Lab: Filtering, Spectrograms