Name:
Kerberos Username:

Enter all answers in the boxes provided.
Other work on pages with QR codes may be considered when assigning partial credit.

You have one hour and fifty minutes.
This quiz is closed book, but you may use one 8.5 \times 11 sheet of paper (two sides).
No calculators, computers, cell phones, music players, or other electronic devices.
1. Series [20 points]

Part 1.

Consider a signal \( x_1[n] = 3(-j)^n + 7e^{j\pi n/4} \)

What are the Fourier series coefficients of \( x_1[\cdot] \), when analyzed with \( N \) equal to the fundamental period of \( x_1 \)? Simplify your answer completely so that it does not contain any expressions involving \( e \), \( \sin \), \( \cos \).

\[ X_1[k] = \]
Part 2.
Consider a function $x_2(\cdot)$ that is periodic in $T = 2$ seconds. Its CTFS coefficients $X_2[k]$ are shown below:

$$X_2[k] = \begin{cases} 
1/2 & \text{if } k \in \{0, 2\} \\
1 & \text{if } k = 1 \\
0 & \text{otherwise}
\end{cases}$$

You may assume that any coefficients that are not pictured above are 0.

On the axes below, plot the magnitude and phase of $x_2(t)$ as functions of $t$, and label all key values:
2. Steps [26 points]

Part 1. Let $x[n]$ represent the following discrete-time signal

$$x[n] = \begin{cases} 
0 & \text{for } n < 0 \\
a^0 & \text{for } n = 0, 1, 2 \\
a^1 & \text{for } n = 3, 4, 5 \\
a^2 & \text{for } n = 6, 7, 8 \\
\ldots 
\end{cases}$$

where $a$ is a real number between 0 and 1, as shown in the plot below.

Determine a closed form expression for $X(\Omega)$, which is the discrete-time Fourier transform of $x[n]$. 

$$X(\Omega) = \boxed{\text{[expression for } X(\Omega)\text{]}}$$
Part 2. Let \( x(t) \) represent the following continuous-time signal

\[
x(t) = \begin{cases} 
0 & \text{for } t < 0 \\
a^0 & \text{for } 0 \leq t < 3 \\
a^1 & \text{for } 3 \leq t < 6 \\
a^2 & \text{for } 6 \leq t < 9 \\
\vdots
\end{cases}
\]

where \( a \) is a real number between 0 and 1, as shown in the plot below.

Determine a closed-form expression for \( X(\omega) \), which is the continuous-time Fourier transform of \( x(t) \).

\[
X(\omega) = \quad \text{[Blank]}
\]
3. Related Signals [20 points]

For this problem, let \( x[\cdot] \) be a DT signal that is periodic in \( N = 6 \). One period of the Fourier series coefficients \( X[\cdot] \) is \( \{1, 0, j, 0, -j, 0\} \), i.e.,

\[
X[k] = X[k + 6] = \begin{cases} 
1 & \text{if } k = 0 \\
 j & \text{if } k = 2 \\
 -j & \text{if } k = 4 \\
0 & \text{otherwise}
\end{cases}
\]

**Part 1.**

Consider a new signal \( y_1[\cdot] \), described by: \( y_1[n] = 7 - 3x[n - 1] \). What are the DTFS coefficients of \( y_1[\cdot] \)?

\[
Y_1[0] = \boxed{} \quad Y_1[1] = \boxed{} \quad Y_1[2] = \boxed{} \\
Y_1[3] = \boxed{} \quad Y_1[4] = \boxed{} \quad Y_1[5] = \boxed{}
\]
Part 2. Consider another new signal \( y_2[n] = 5(-1)^n x[n] \). What are the DTFS coefficients of \( y_2[n] \)?

\[
\begin{align*}
Y_2[0] &= \phantom{\text{ }} \\
Y_2[1] &= \phantom{\text{ }} \\
Y_2[2] &= \phantom{\text{ }} \\
Y_2[3] &= \phantom{\text{ }} \\
Y_2[4] &= \phantom{\text{ }} \\
Y_2[5] &= \phantom{\text{ }}
\end{align*}
\]
4. Angular Trends  [14 points]

Each of the expressions on the left corresponds to a plot on the right. Enter the letter for that plot in the corresponding box.

\[ \angle e^{-jx} : \]

\[ \angle (1 + 0.8e^{jx}) : \]

\[ \angle \left( \frac{1 + 0.4e^{jx}}{2 + 0.8e^{jx}} \right) : \]

\[ \angle (1 + e^{jx}) : \]

\[ \angle (1 + 0.8e^{2jx}) : \]

\[ \angle (0.9e^{jx} + 0.8e^{-jx}) : \]

\[ \angle \left( \frac{1}{1 + 0.8e^{jx}} \right) : \]
5. **CT and DT [20 points]**

**Part 1.**

Consider taking a periodic signal $x(\cdot)$ (with a period of $T = 1$ second) and sampling at a sampling rate of 8 samples per second to obtain DT signal $x[\cdot]$ that is also periodic. Analyzing this signal, you find that:

- $x[\cdot]$ is an even function of $n$.
- $x[n]$ is positive for all values of $n$.
- the sum of all $x[n]$ in one period is 2.
- if you sum and subtract alternating samples in one period, you get 1; that is, $x[0] - x[1] + x[2] - \ldots + x[N-2] - x[N-1] = 1$.
- most of the Fourier series coefficients $X[\cdot]$ are 0; only two (per period) are nonzero.

What are two distinct CT functions $x(\cdot)$ that could have produced the results shown above?

\[ x(t) = \] 

or

\[ x(t) = \]
Part 2.

When computing the DTFS coefficients of a purely real signal with a fundamental period $N$, where $N$ is an even number, two coefficients in particular ($k = 0$ and $k = N/2$) are always real-valued (whereas other values can be complex). Why is this the case? Explain briefly (1-3 sentences).

When computing the DTFS coefficients of a purely real signal with a fundamental period $N$ where $N$ is odd, are there any coefficients that are similarly guaranteed to be real? If so, specify the values of $k$. Explain briefly (1-3 sentences).
When computing the CTFS coefficients of a purely real CT signal, are there any coefficients that are similarly guaranteed to be real? If so, specify the values of $k$. Does your answer depend on the value of $T$? Explain briefly (1-3 sentences).
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