6.300 Signal Processing

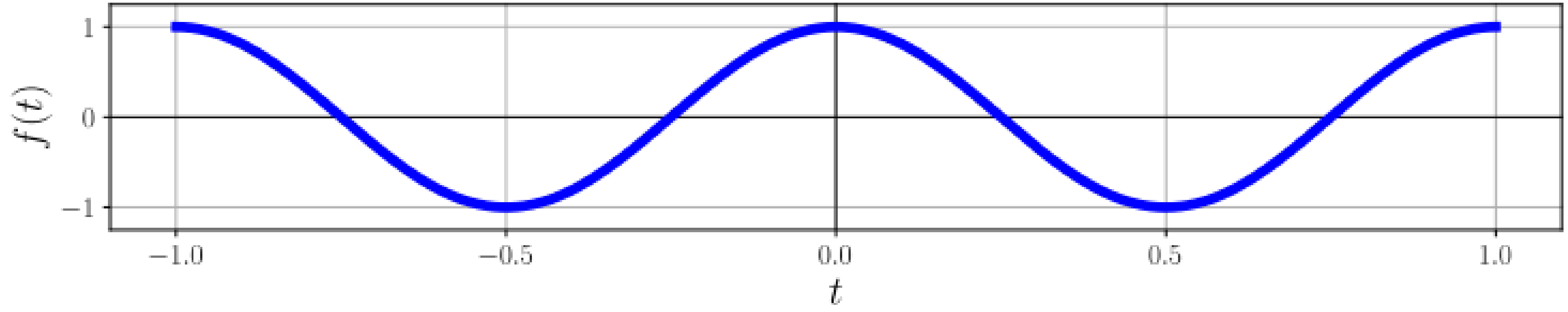
Week 2, Recitation A:
Continuous-Time Fourier Series and Symmetry

- Fourier Series
- Symmetry of Fourier Series

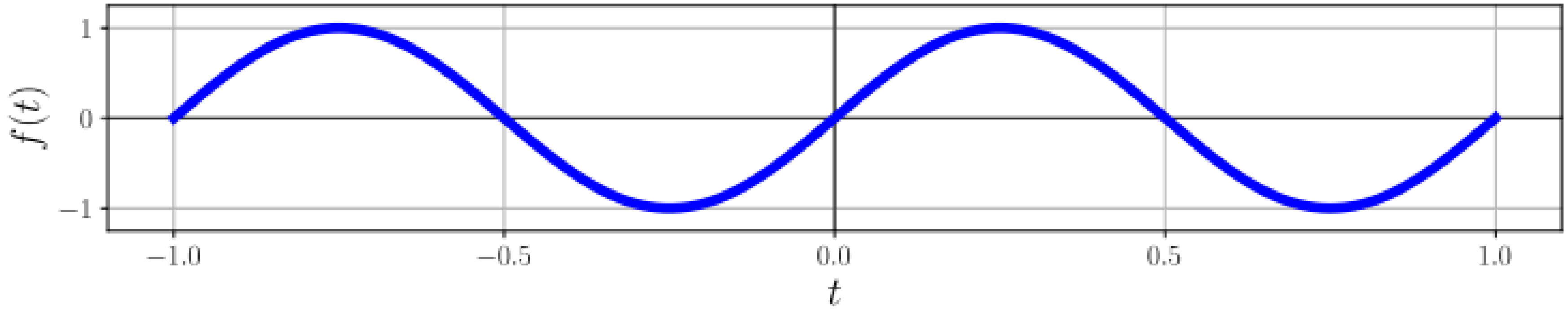
September 15, 2026

Symmetry, Anti-Symmetry, Asymmetry

A symmetric signal satisfies $f(t) = f(-t)$.



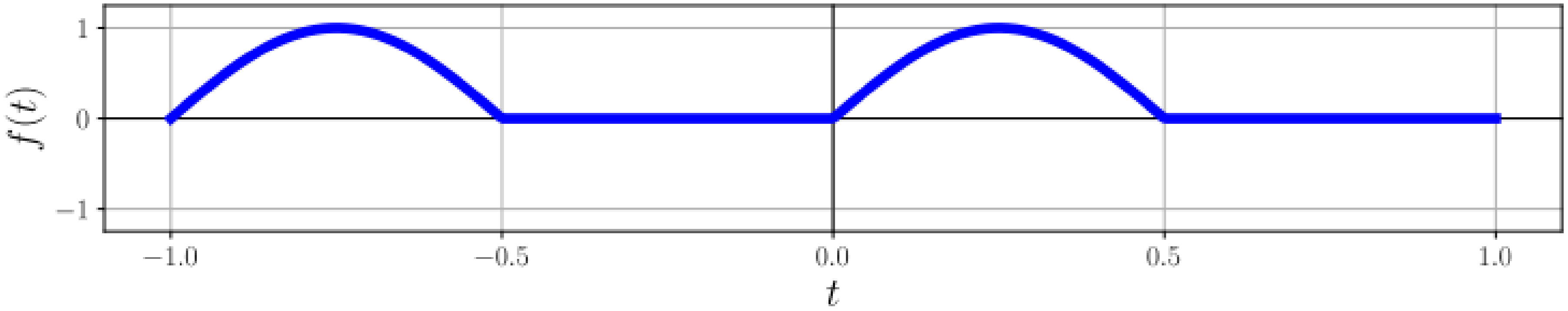
An anti-symmetric signal satisfies $f(t) = -f(-t)$.



What we've called symmetry/anti-symmetry is also called even/odd symmetry. Might as well know both.

Symmetry, Anti-Symmetry, Asymmetry

An **asymmetric** signal is neither symmetric nor anti-symmetric — it lacks any semblance of symmetry.



We can express an **asymmetric** signal as the **sum** of a **symmetric part** and an **anti-symmetric part**.

$$f(t) = f_{\text{symmetric}}(t) + f_{\text{anti-symmetric}}(t)$$

How do we get $f_{\text{symmetric}}(t)$ from $f(t)$?

How do we get $f_{\text{anti-symmetric}}(t)$ from $f(t)$?

Symmetry, Anti-Symmetry, Asymmetry

How do we get $f_{\text{symmetric}}(t)$ from $f(t)$?
How do we get $f_{\text{anti-symmetric}}(t)$ from $f(t)$?

Add $f(t)$ and $f(-t)$ to cancel out the anti-symmetric part.
This leaves the symmetric part.

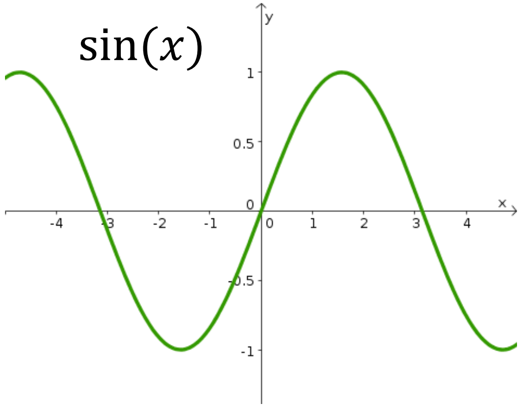
$$f_{\text{symmetric}}(t) = \frac{f(t) + f(-t)}{2}$$

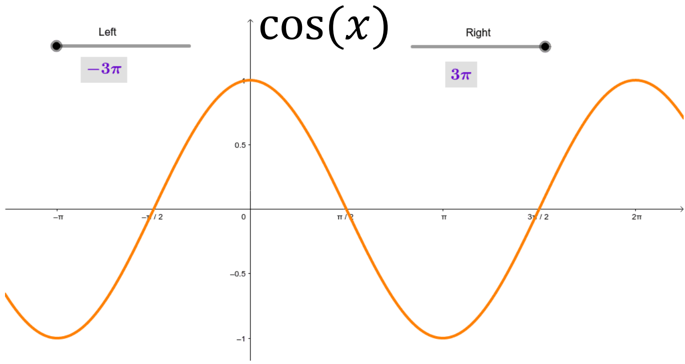
Add $f(t)$ and $-f(-t)$ to cancel out the symmetric part.
This leaves the anti-symmetric part.

$$f_{\text{anti-symmetric}}(t) = \frac{f(t) - f(-t)}{2}$$

Symmetric and Antisymmetric Parts in CTFS

$$f_S(t) = \frac{f(t) + f(-t)}{2} \qquad f_A(t) = \frac{f(t) - f(-t)}{2}$$





$$f(t) = f_S(t) + f_A(t)$$

$$f(t) = \sum_{k=0}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t))$$

- c_k's (cosines) alone only represent the symmetric part of the signal.
- d_k 's (sines) alone only represent the antisymmetric part of the signal.

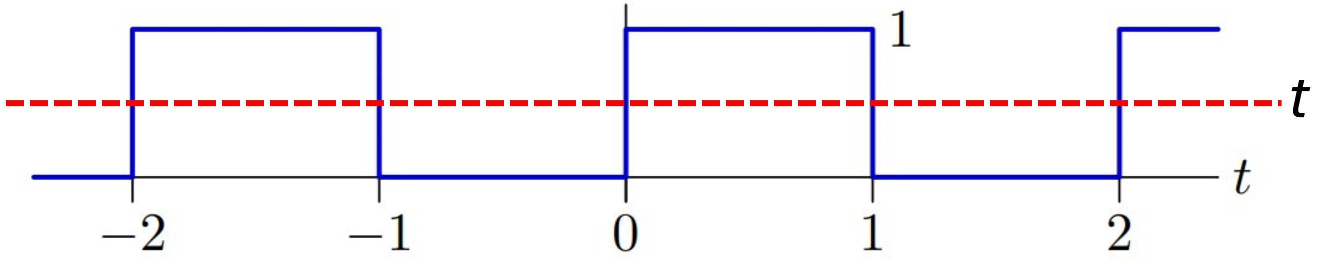
$$f(-t) = \sum_{k=0}^{\infty} (c_k \cos(k\omega_0 t) - d_k \sin(k\omega_0 t))$$

The symmetric part shows up in the c_k coefficients, and the antisymmetric part shows up in the d_k coefficients.

Examples in Lecture

- Find the Fourier series coefficients for the following square wave:

$$f(t) = f(t+2)$$



Why are the C_k coefficients zero (except c_0)?

$$T = 2$$

$$\omega_o = \frac{2\pi}{T} = \pi$$

$$c_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2}\int_0^2 f(t)dt = \frac{1}{2}$$

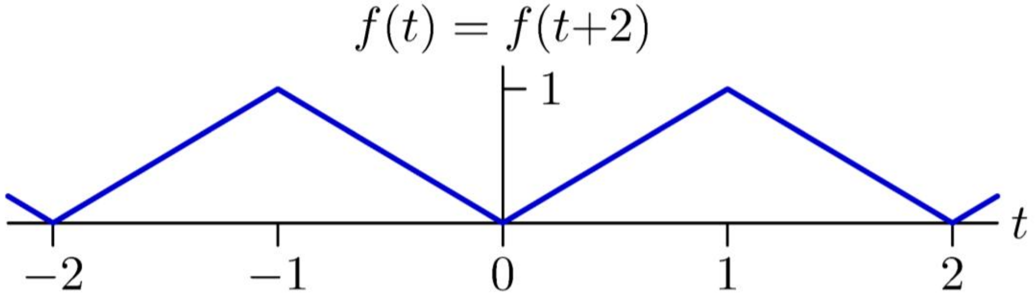
$$c_k = \frac{2}{T}\int_0^T f(t)\cos(k\omega_o t)\,dt = \int_0^1 \cos(k\pi t)dt = \frac{\sin(k\pi t)}{k\pi}\Big|_0^1 = 0 \text{ for } k = 1, 2, 3, \ldots$$

$$d_k = \frac{2}{T}\int_0^T f(t)\sin(k\omega_o t)\,dt = \int_0^1 \sin(k\pi t)dt = -\frac{\cos(k\pi t)}{k\pi}\Big|_0^1 = \begin{cases} \frac{2}{k\pi} & k = 1, 3, 5, \ldots \\ 0 & \text{otherwise} \end{cases}$$

If without $c_0 = \frac{1}{2}$ "DC" part, $f(t)$ is antisymmetric around t=0, thus only having non-zero d_k 's

The other example

Find the Fourier series coefficients for the following triangle wave:



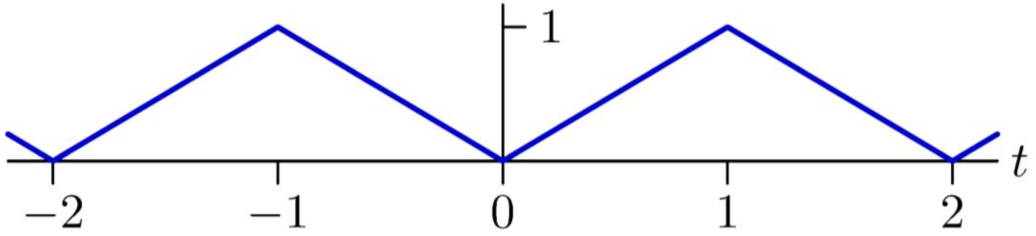
Which coefficients are zero?

Which are non-zero?

The other example

Find the Fourier series coefficients for the following triangle wave:

$$f(t) = f(t+2)$$



$$T = 2$$

$$\omega_o = \frac{2\pi}{T} = \pi$$

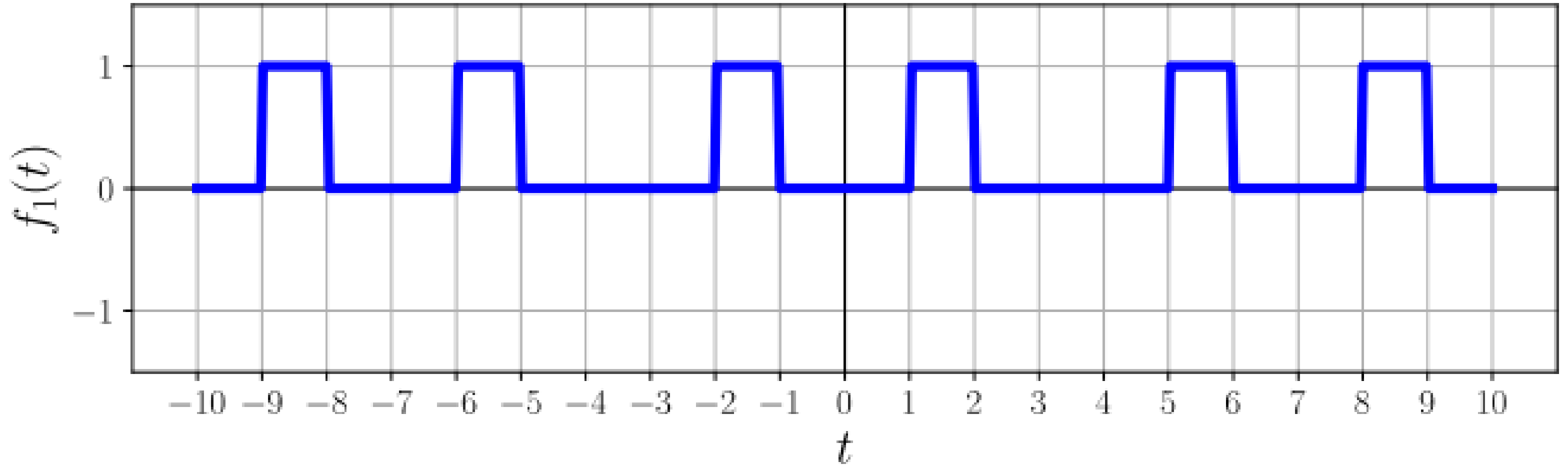
$$c_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2}\int_0^2 f(t)dt = \frac{1}{2}$$

$$c_k = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\frac{2\pi k t}{T}\,dt = 2\int_0^1 t\cos(\pi k t)\,dt = \begin{cases} -\frac{4}{\pi^2 k^2} & k \text{ odd} \\ 0 & k = 2, 4, 6, \ldots \end{cases}$$

$d_k = 0$ (by symmetry)

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_1(t)$, shown below.



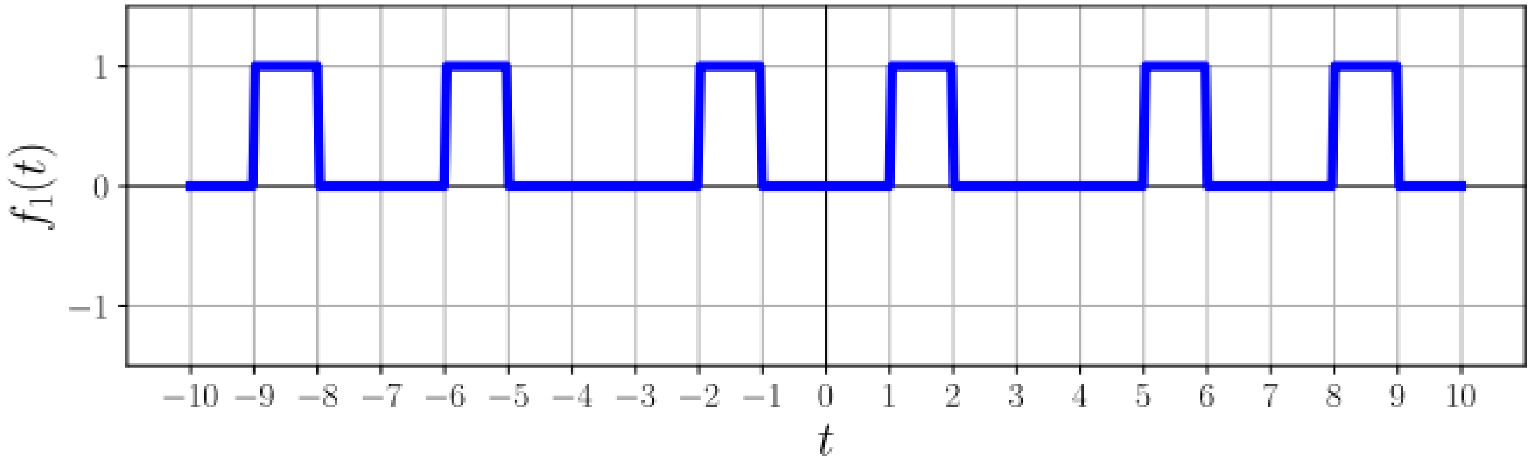
What is the fundamental period (T) of $f_1(t)$?

Does it matter which interval you integrate over? (0 to T? $-\frac{1}{2}T$ to $\frac{1}{2}T$? $\frac{5}{16}T$ to $\frac{21}{16}T$?)

Do you notice anything about the c_k and/or d_k terms?

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_1(t)$, shown below.



The fundamental period is $T = 7$.

Integrating over any length-T interval yields the same result. (The calculations are on the next slide.)

Because $f_1(t)$ is symmetric, $d_k = 0$ for all k.

Fourier Series and Symmetry

c_0 is the average value of $f_1(t)$ over one period.

$$c_0 = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_1(t)\,dt = \frac{2}{7}$$

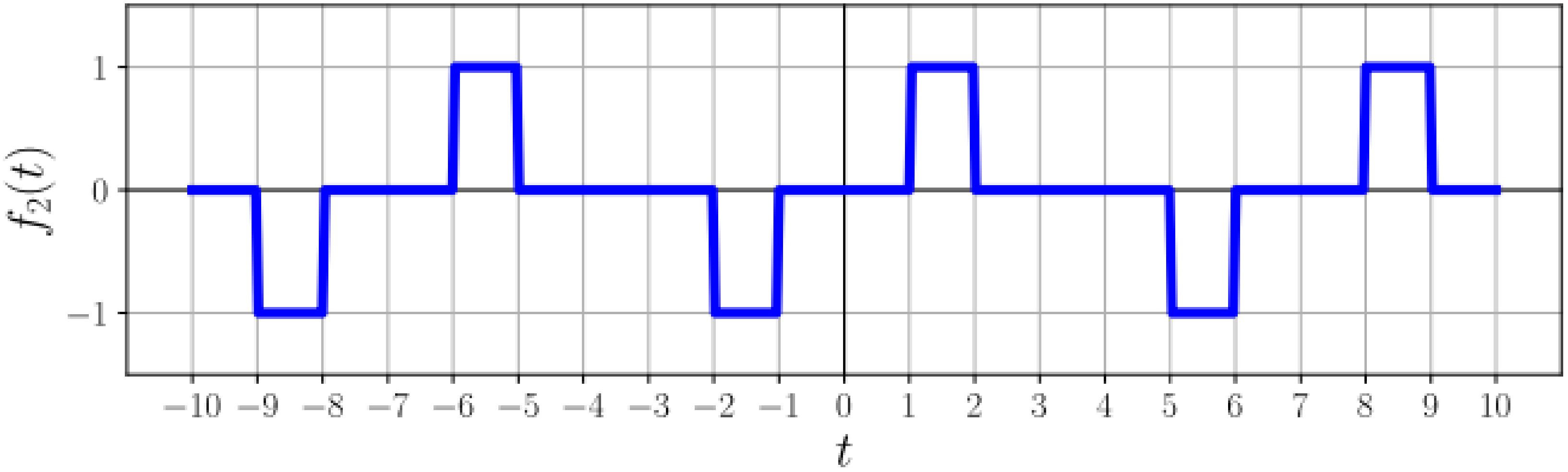
Compute the remaining coefficients via integration.

$$c_k = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_1(t)\cos\left(k\tfrac{2\pi}{7}t\right)dt = \frac{2}{\pi k}\left(\sin\left(\frac{4\pi}{7}k\right) - \sin\left(\frac{2\pi}{7}k\right)\right)$$

$$d_k = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_1(t)\sin\left(k\tfrac{2\pi}{7}t\right)dt = 0$$

Fourier Series and Symmetry

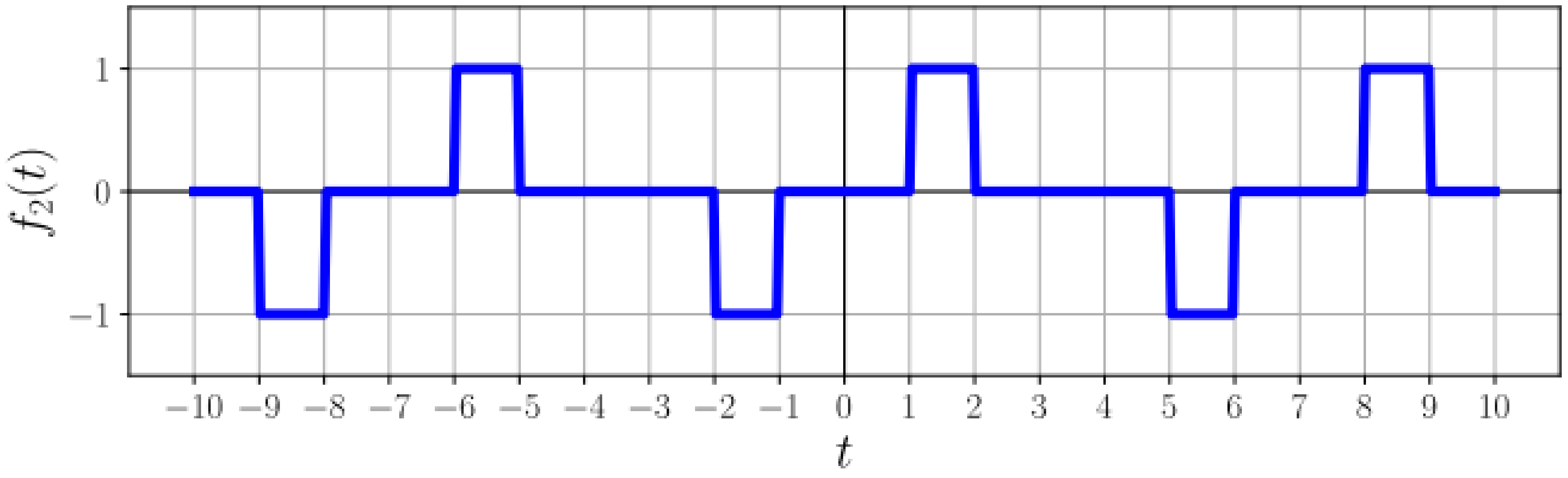
Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_2(t)$, shown below.



Do you notice anything about the c_k and/or d_k terms?

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_2(t)$, shown below.



Do you notice anything about the c_k and/or d_k terms?

Because $f_2(t)$ is anti-symmetric, $c_k = 0$ for all k.

Fourier Series and Symmetry

c_0 is the average value of $f_2(t)$ over one period.

$$c_0 = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_2(t)dt = 0$$

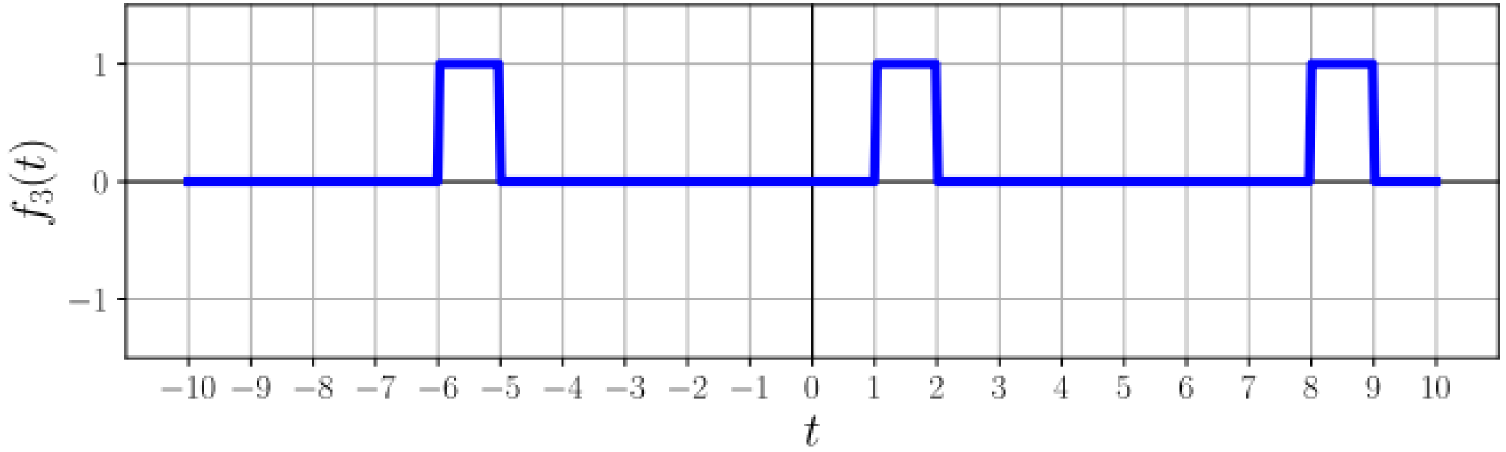
Compute the remaining coefficients via integration.

$$c_k = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_2(t)\cos\left(k\tfrac{2\pi}{7}t\right)dt = 0$$

$$d_k = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_2(t)\sin\left(k\tfrac{2\pi}{7}t\right)dt = \frac{2}{\pi k}\left(\cos\left(\frac{2\pi}{7}k\right) - \cos\left(\frac{4\pi}{7}k\right)\right)$$

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_3(t)$, shown below.



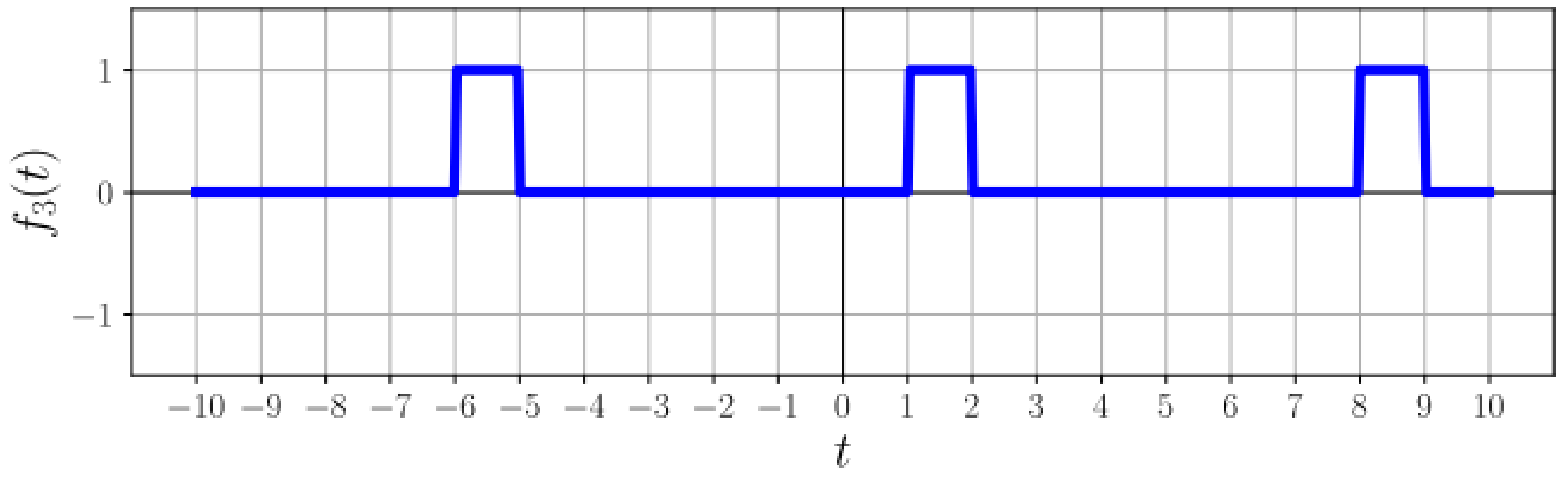
Do you notice anything about the c_k and/or d_k terms?

How does $f_3(t)$ relate to $f_1(t)$ and $f_2(t)$?

How do the Fourier series coefficients for $f_3(t)$ relate to the coefficients for $f_1(t)$ and $f_2(t)$?

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_3(t)$, shown below.



$f_3(t)$ is asymmetric. Both c_k and d_k are non-zero.

Notice that $f_3(t) = \frac{1}{2}f_1(t) + \frac{1}{2}f_2(t)$. So, the coefficients for $f_3(t)$ are half those for $f_1(t)$ plus half those for $f_2(t)$.

Fourier Series and Symmetry

c_0 is the average value of $f_3(t)$ over one period.

$$c_0 = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_3(t)\,dt = \frac{1}{7}$$

Compute the remaining coefficients via integration.

$$c_k = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_3(t)\cos\left(k\tfrac{2\pi}{7}t\right)dt = \frac{1}{\pi k}\left(\sin\left(\frac{4\pi}{7}k\right) - \sin\left(\frac{2\pi}{7}k\right)\right)$$

$$d_k = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_3(t)\sin\left(k\tfrac{2\pi}{7}t\right)dt = \frac{1}{\pi k}\left(\cos\left(\frac{2\pi}{7}k\right) - \cos\left(\frac{4\pi}{7}k\right)\right)$$