

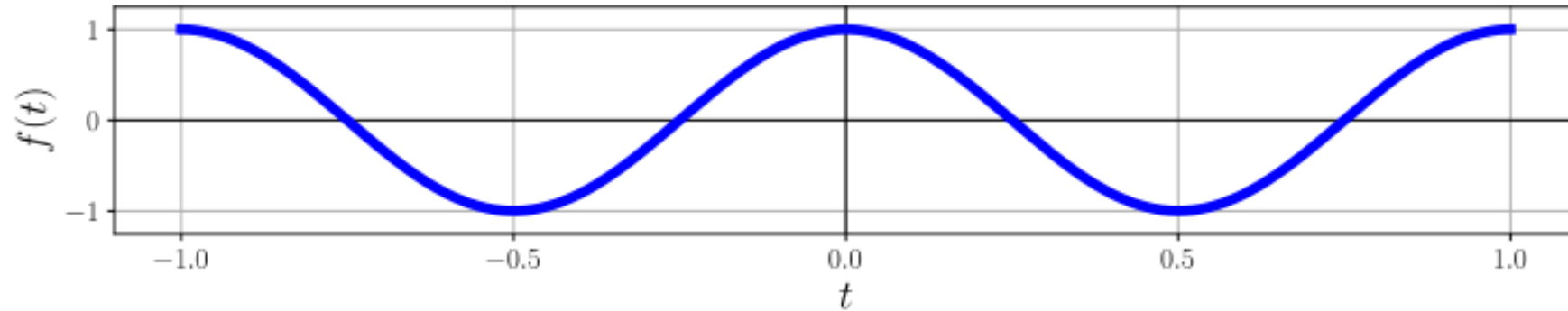
6.300 Signal Processing

Week 2, Recitation A: Continuous-Time Fourier Series and Symmetry

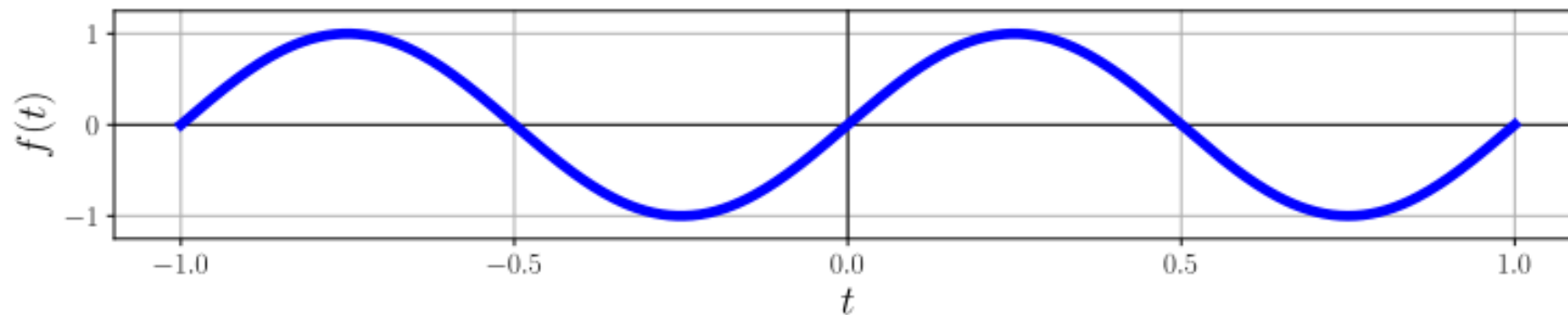
- Fourier Series
- Symmetry of Fourier Series

Symmetry, Anti-Symmetry, Asymmetry

A **symmetric** signal satisfies $f(t) = f(-t)$.



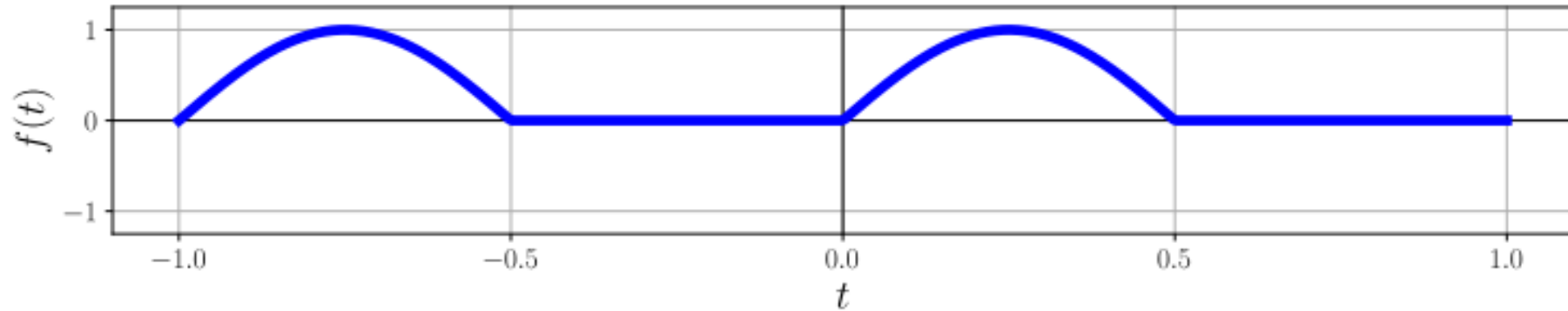
An **anti-symmetric** signal satisfies $f(t) = -f(-t)$.



What we've called **symmetry/anti-symmetry** is also called **even/odd symmetry**. Might as well know both.

Symmetry, Anti-Symmetry, Asymmetry

An **asymmetric** signal is neither symmetric nor anti-symmetric — it lacks any semblance of symmetry.



We can express an **asymmetric** signal as the **sum** of a **symmetric part** and an **anti-symmetric part**.

$$f(t) = f_{\text{symmetric}}(t) + f_{\text{anti-symmetric}}(t)$$

How do we get $f_{\text{symmetric}}(t)$ from $f(t)$?

How do we get $f_{\text{anti-symmetric}}(t)$ from $f(t)$?

Symmetry, Anti-Symmetry, Asymmetry

How do we get $f_{\text{symmetric}}(t)$ from $f(t)$?

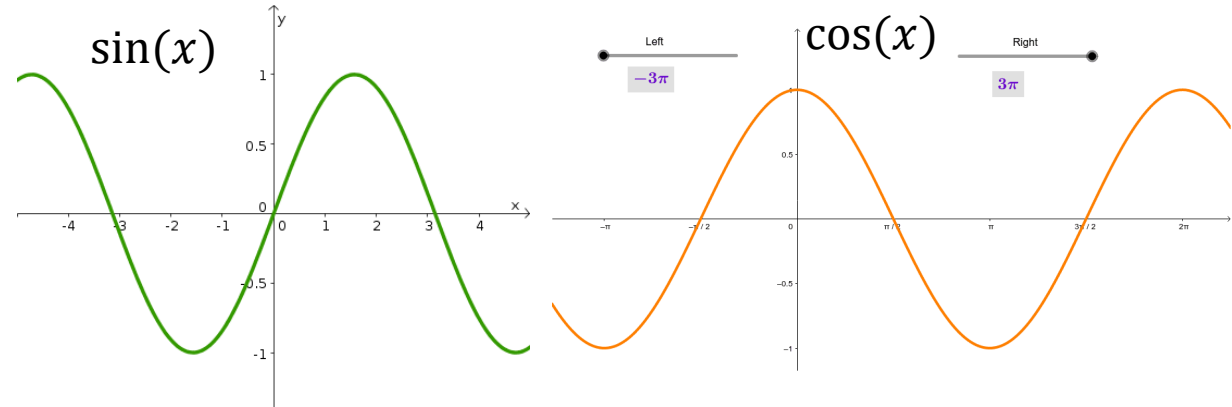
How do we get $f_{\text{anti-symmetric}}(t)$ from $f(t)$?

Symmetric and Antisymmetric Parts in CTFS

$$f_S(t) = \frac{f(t) + f(-t)}{2} \quad f_A(t) = \frac{f(t) - f(-t)}{2}$$

$$f(t) = f_S(t) + f_A(t)$$

$$f(t) = \sum_{k=0}^{\infty} (c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t))$$



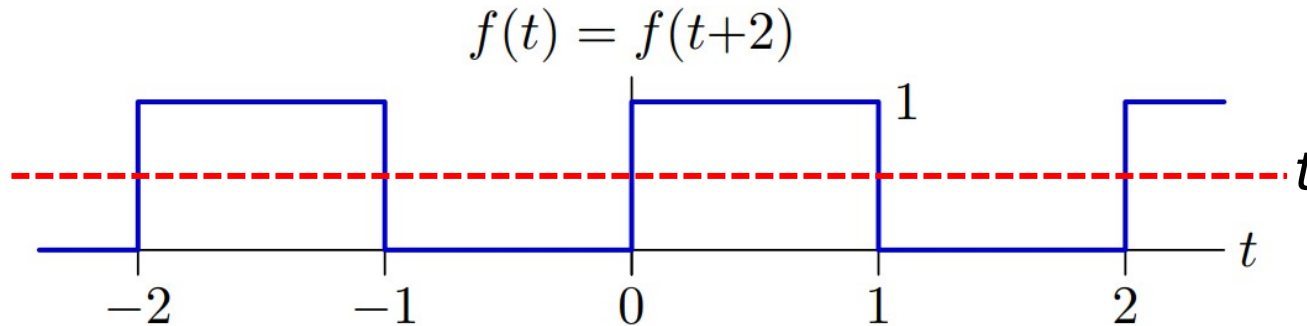
- c_k 's (cosines) alone only represent the symmetric part of the signal.
- d_k 's (sines) alone only represent the antisymmetric part of the signal.

$$f(-t) = \sum_{k=0}^{\infty} (c_k \cos(k\omega_0 t) - d_k \sin(k\omega_0 t))$$

The symmetric part shows up in the c_k coefficients, and the antisymmetric part shows up in the d_k coefficients.

Examples in Lecture

- Find the Fourier series coefficients for the following square wave:



Why are the C_k coefficients zero (except c_0)?

$$T = 2$$
$$\omega_o = \frac{2\pi}{T} = \pi$$

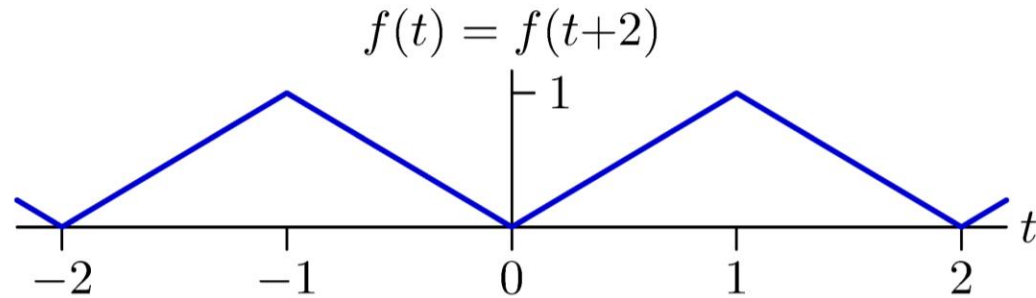
$$c_0 = \frac{1}{T} \int_0^T f(t) dt = \frac{1}{2} \int_0^2 f(t) dt = \frac{1}{2}$$

$$c_k = \frac{2}{T} \int_0^T f(t) \cos(k\omega_o t) dt = \int_0^1 \cos(k\pi t) dt = \left. \frac{\sin(k\pi t)}{k\pi} \right|_0^1 = 0 \text{ for } k = 1, 2, 3, \dots$$

$$d_k = \frac{2}{T} \int_0^T f(t) \sin(k\omega_o t) dt = \int_0^1 \sin(k\pi t) dt = - \left. \frac{\cos(k\pi t)}{k\pi} \right|_0^1 = \begin{cases} \frac{2}{k\pi} & k = 1, 3, 5, \dots \\ 0 & \text{otherwise} \end{cases}$$

The other example

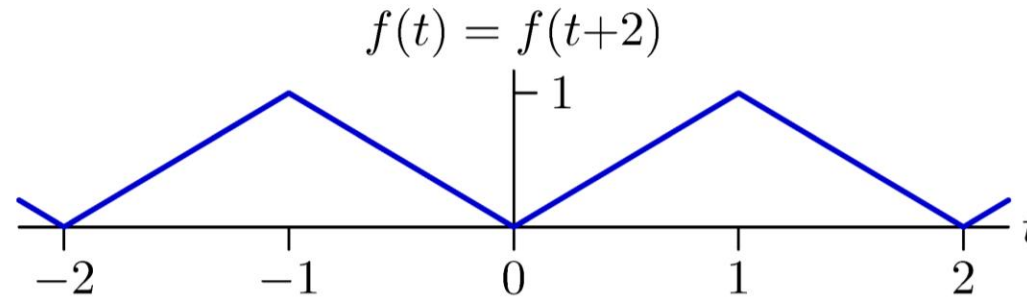
Find the Fourier series coefficients for the following triangle wave:



Which coefficients are zero?
Which are non-zero?

The other example

Find the Fourier series coefficients for the following triangle wave:



$$T = 2$$

$$\omega_o = \frac{2\pi}{T} = \pi$$

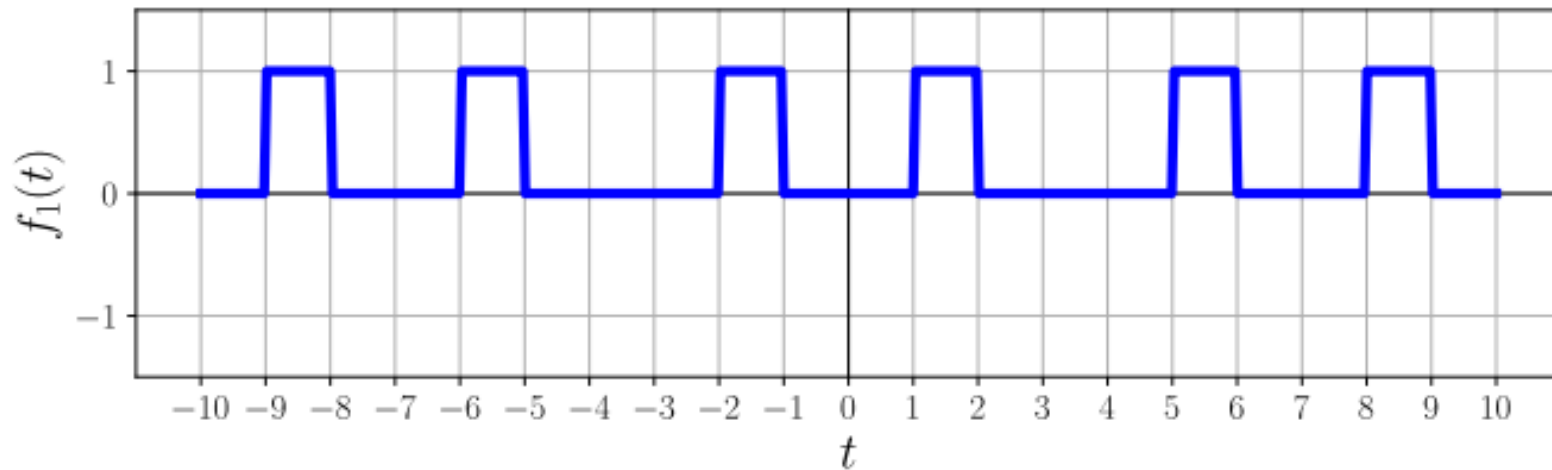
$$c_0 = \frac{1}{T} \int_0^T f(t) dt = \frac{1}{2} \int_0^2 f(t) dt = \frac{1}{2}$$

$$c_k = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos \frac{2\pi kt}{T} dt = 2 \int_0^1 t \cos(\pi kt) dt = \begin{cases} -\frac{4}{\pi^2 k^2} & k \text{ odd} \\ 0 & k = 2, 4, 6, \dots \end{cases}$$

$$d_k = 0 \quad (\text{by symmetry})$$

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_1(t)$, shown below.



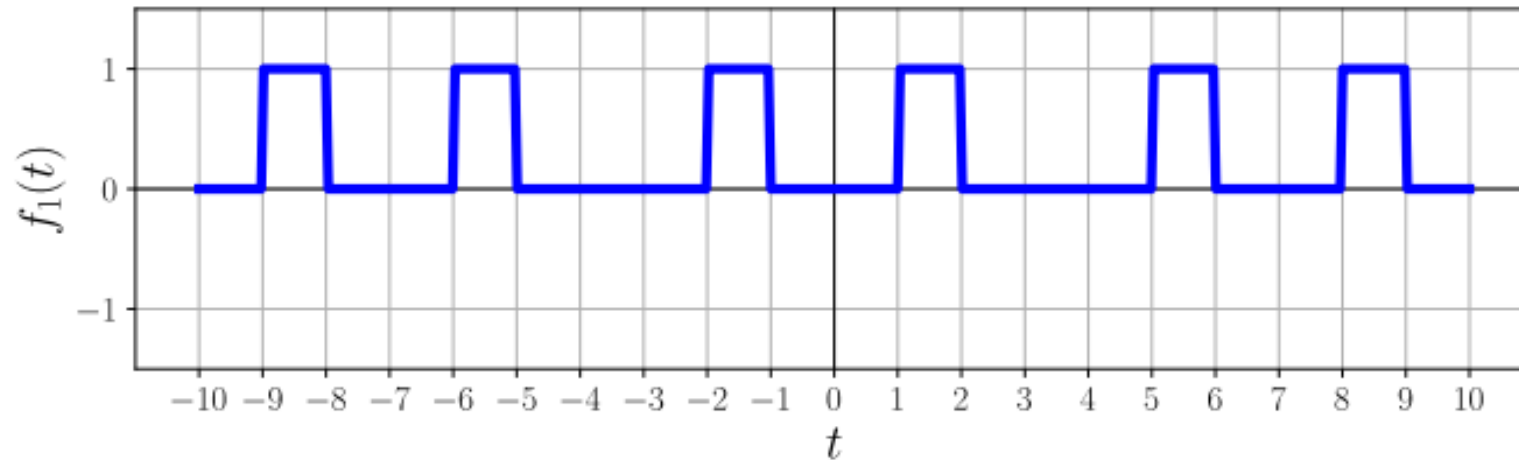
What is the fundamental period (T) of $f_1(t)$?

Does it matter which interval you integrate over?
(0 to T ? $-\frac{1}{2}T$ to $\frac{1}{2}T$? $\frac{5}{16}T$ to $\frac{21}{16}T$?)

Do you notice anything about the c_k and/or d_k terms?

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_1(t)$, shown below.



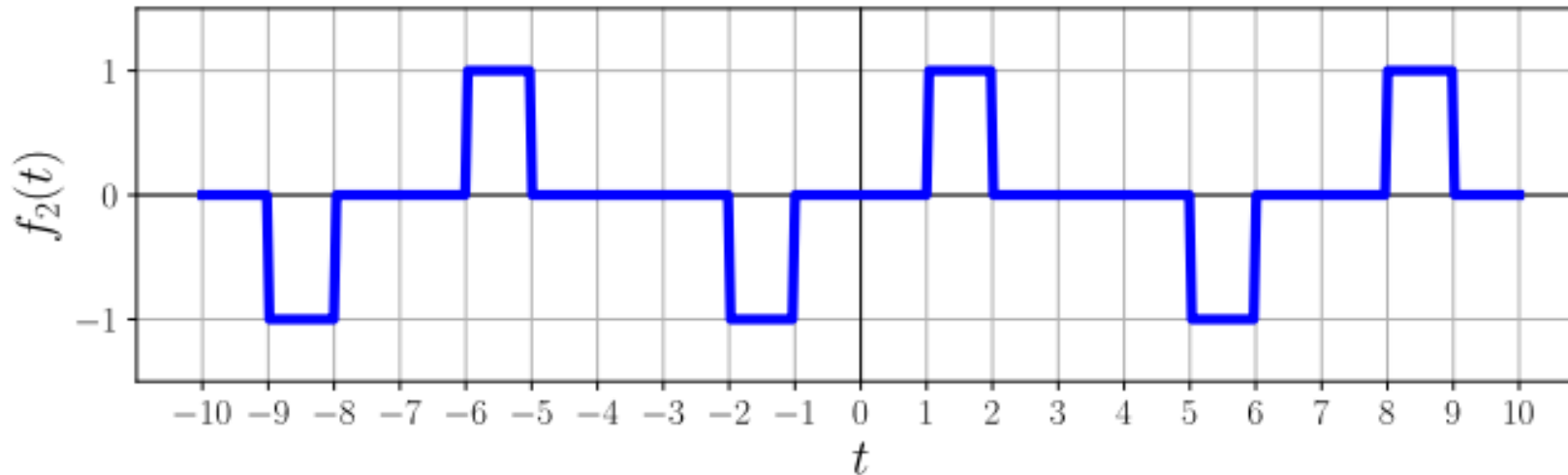
The fundamental period is $T = 7$.

Integrating over any length- T interval yields the same result. (The calculations are on the next slide.)

Because $f_1(t)$ is symmetric, $d_k = 0$ for all k .

Fourier Series and Symmetry

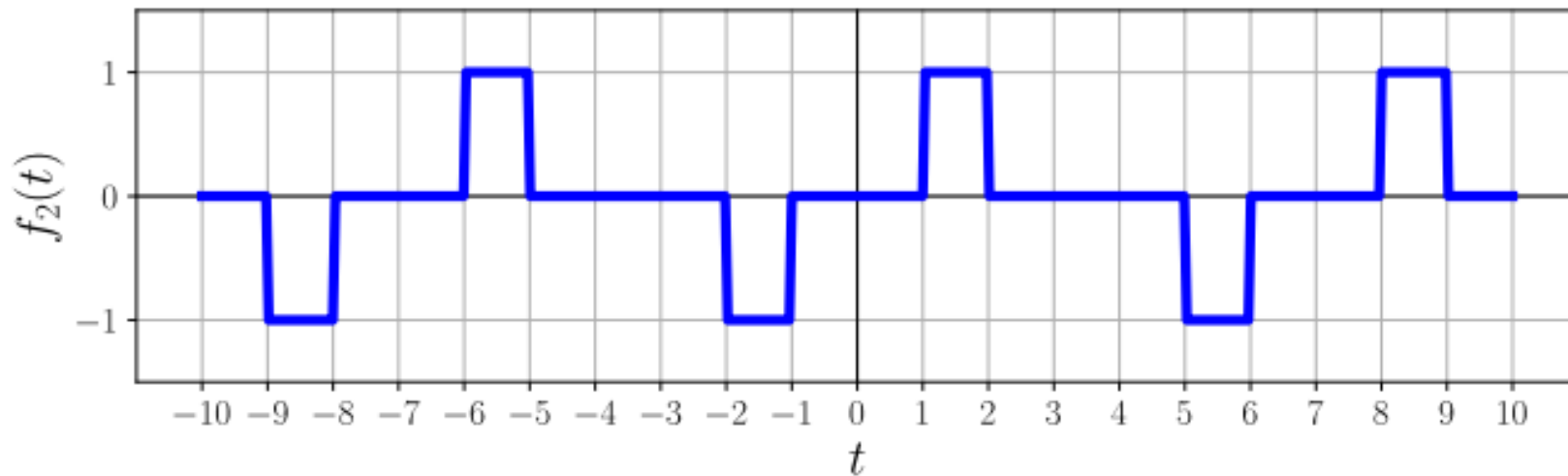
Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_2(t)$, shown below.



Do you notice anything about the c_k and/or d_k terms?

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_2(t)$, shown below.

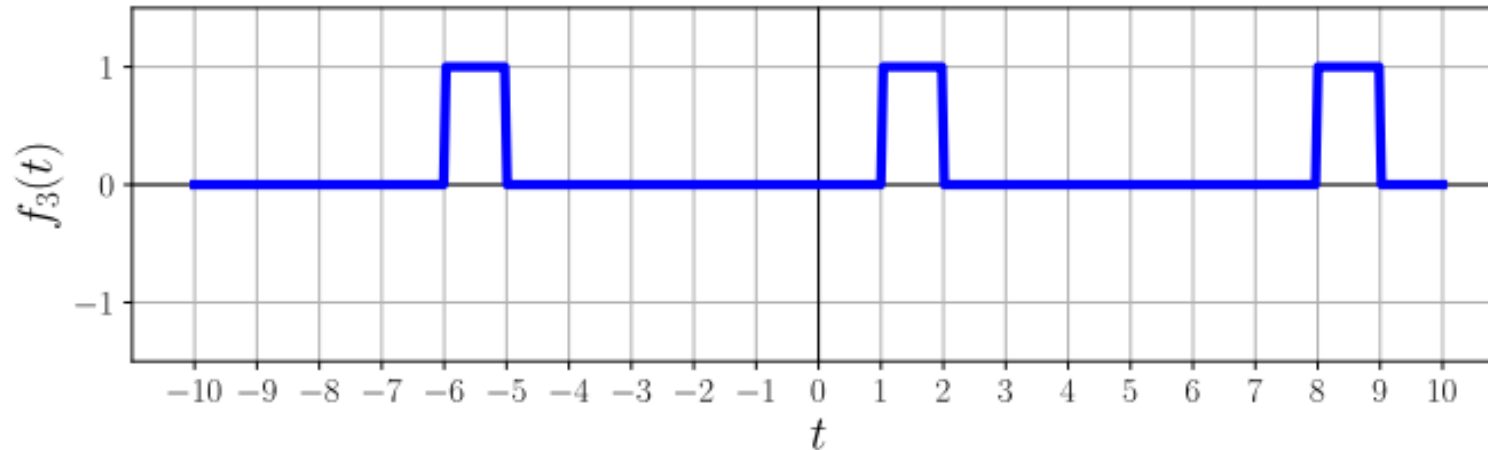


Do you notice anything about the c_k and/or d_k terms?

Because $f_2(t)$ is anti-symmetric, $c_k = 0$ for all k .

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_3(t)$, shown below.



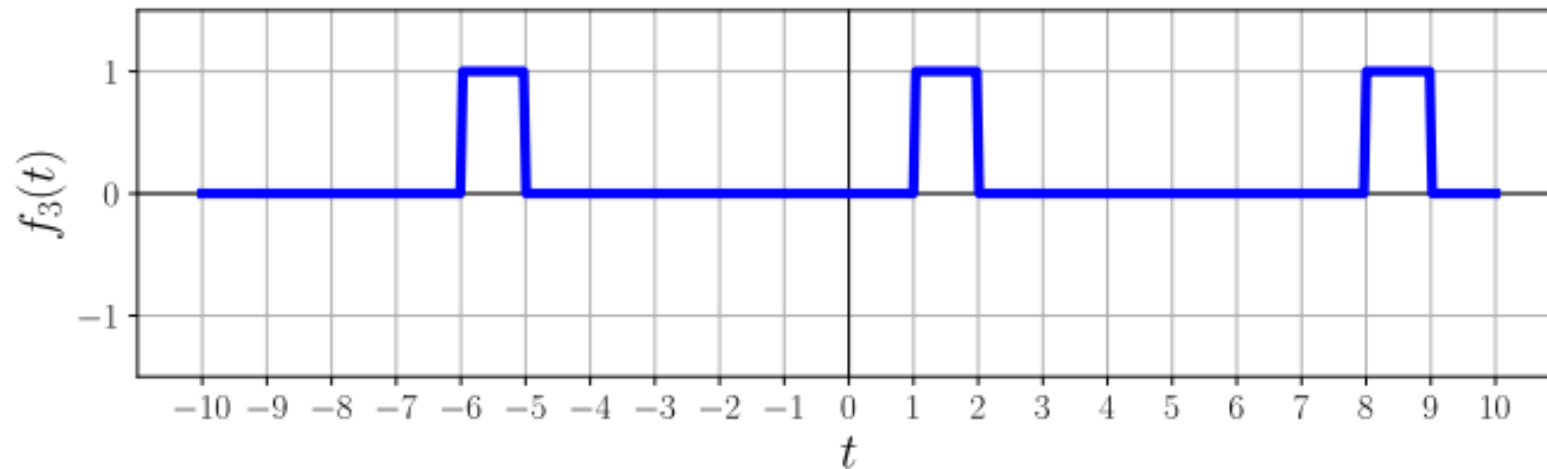
Do you notice anything about the c_k and/or d_k terms?

How does $f_3(t)$ relate to $f_1(t)$ and $f_2(t)$?

How do the Fourier series coefficients for $f_3(t)$ relate to the coefficients for $f_1(t)$ and $f_2(t)$?

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_3(t)$, shown below.



$f_3(t)$ is asymmetric. Both c_k and d_k are non-zero.

Notice that $f_3(t) = \frac{1}{2}f_1(t) + \frac{1}{2}f_2(t)$. So, the coefficients for $f_3(t)$ are half those for $f_1(t)$ plus half those for $f_2(t)$.