

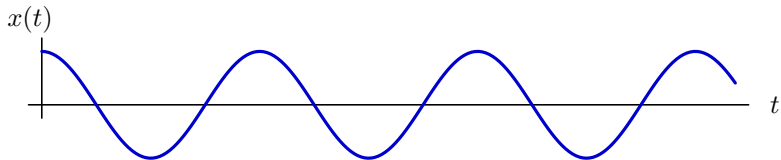
6.3000: Signal Processing

Sampling and Aliasing

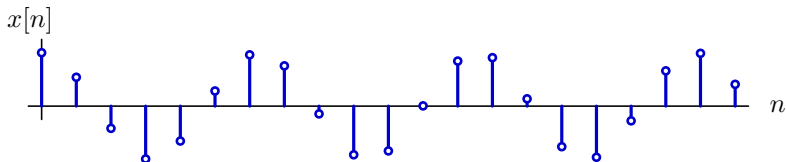
September 22, 2026

Tones and Sinusoids

A “tone” is a pressure that changes sinusoidally with time.



In 6.3000, we will think of this as a “continuous-time” (CT) signal. In contrast, a “discrete-time” (DT) signal is a sequence of numbers.



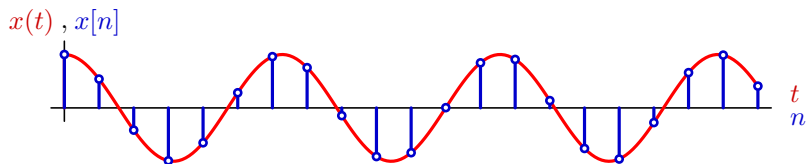
Mathematically:

$$x(t) = A \cos(\omega t)$$

$$x[n] = A \cos(\Omega n)$$

CT and DT Representations

Assume that $x[n]$ represents “samples” of $x(t)$:



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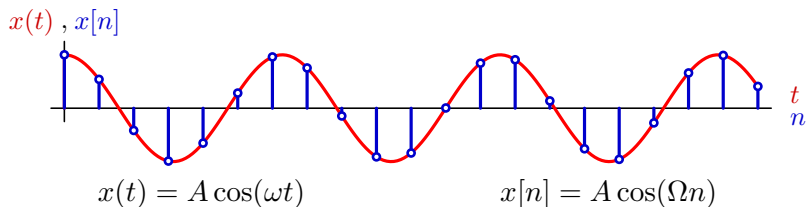
- What are the units of ω , t , Ω , and n ?

Let f represent the “frequency” of the tone in cycles/second.

- Determine ω in terms of f .
- Determine Ω in terms of ω .
- Determine Ω in terms of f .

CT and DT Representations

Assume that $x[n]$ represents “samples” of $x(t)$:



- What are the units of ω , t , Ω , and n ?

The product ωt is measured in units of **radians** (dimensionless ratio).

Time t is measured in units of **seconds**.

Therefore ω is measured in units of **radians/second**.

The product Ωn is measured in units of **radians** (domain of $\cos(\cdot)$).

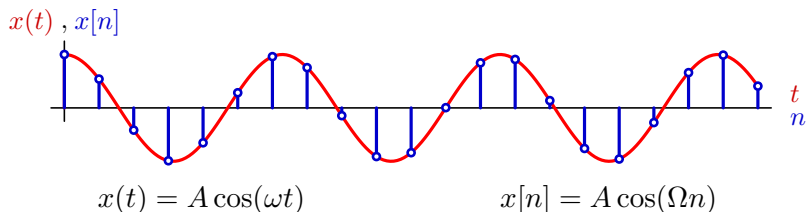
Discrete time n is a **dimensionless** integer.

Therefore Ω is measured in units of **radians**.

For convenience, we often think of n as measured in **number of samples** and Ω in **radians/sample**.

CT and DT Representations

Assume that $x[n]$ represents “samples” of $x(t)$:



Let f represent the “frequency” of the tone in cycles/second.

- Determine ω in terms of f .
- Determine Ω in terms of ω . [$\rightarrow f_s$]
- Determine Ω in terms of f .

$$\omega[\text{rad/sec}] = 2\pi[\text{rad/cycle}]f[\text{cycles/sec}]$$

$$\Omega[\text{rad/sample}] = \frac{\omega[\text{rad/sec}]}{f_s[\text{samples/sec}]} \quad \text{where } f_s = \text{sample frequency}$$

$$\Omega[\text{rad/sample}] = \frac{2\pi[\text{rad/cycle}]f[\text{cycles/sec}]}{f_s[\text{samples/sec}]}$$

Check Yourself

Compare two signals:

$$x_1[n] = \cos \frac{3\pi n}{4}$$

$$x_2[n] = \cos \frac{5\pi n}{4}$$

How many of the following statements are true?

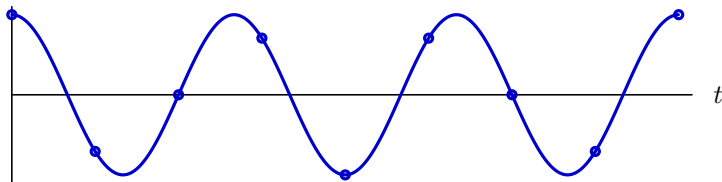
$x_1[n]$ has period $N=8$.

$x_2[n]$ has period $N=8$.

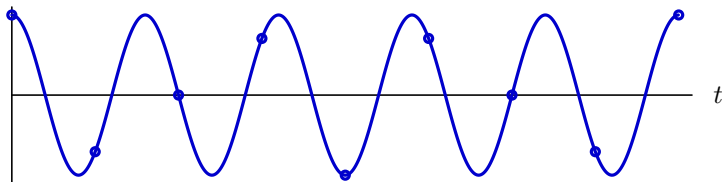
$x_1[n] = x_2[n]$.

Check Yourself

$$\cos \frac{3\pi t}{4} = \cos \omega_1 t$$



$$\cos \frac{5\pi t}{4} = \cos \omega_2 t$$



$$\omega_1 \neq \omega_2$$

but $x_1[n] = x_2[n]!$

$$\omega_1 + \omega_2 = 2\pi$$

$$\cos \omega_1 n = \cos(2\pi - \omega_2)n = \cos(2\pi n - \omega_2 n) = \cos(-\omega_2 n) = \cos \omega_2 n$$

Check Yourself

Compare two signals:

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How many of the following statements are true? 3

$x_1[n]$ has period $N=8$. ✓

$x_2[n]$ has period $N=8$. ✓

$x_1[n] = x_2[n]$. ✓

Frequencies

Consider the following CT signal:

$$f(t) = 6 \cos(42\pi t) + 4 \cos(18\pi t - 0.5\pi)$$

What is the fundamental period of this signal?

Frequencies

Consider the following CT signal:

$$f(t) = 6 \cos(42\pi t) + 4 \cos(18\pi t - 0.5\pi)$$

What is the fundamental period of this signal?

We need to find the smallest time T for which both $\cos(42\pi t)$ and $\cos(18\pi t - 0.5\pi)$ go through an integer number of cycles.

$\cos(42\pi t)$ goes through one cycle every $\frac{1}{21}$ seconds, and $\cos(18\pi t - 0.5\pi)$ goes through one cycle every $\frac{1}{9}$ seconds. So we want the smallest integers m and n such that $T = \frac{m}{21} = \frac{n}{9}$. Solving we find that $m = 7$ and $n = 3$, which gives us $T = \frac{1}{3}$ seconds.

Frequencies

Now imagine that this same signal

$$f(t) = 6 \cos(42\pi t) + 4 \cos(18\pi t - 0.5\pi)$$

is sampled with a sampling rate of $f_s = 60$ Hz to obtain a discrete-time signal $f[n]$, which is periodic in n with fundamental period N . What is the fundamental period N ?

Aliasing

Consider 3 CT signals:

$$f_1(t) = \cos(4000 \pi t) \quad ; \quad f_2(t) = \cos(5000 \pi t) \quad ; \quad f_3(t) = \cos(6000 \pi t)$$

Each of these is sampled so that

$$f_1[n] = f_1(n\Delta) \quad ; \quad f_2[n] = f_2(n\Delta) \quad ; \quad f_3[n] = f_3(n\Delta)$$

where $\Delta = 1/3600$

Which list goes from lowest to highest (baseband) frequency?

0. $f_1[n]$ $f_2[n]$ $f_3[n]$

2. $f_2[n]$ $f_1[n]$ $f_3[n]$

4. $f_3[n]$ $f_1[n]$ $f_2[n]$

1. $f_1[n]$ $f_3[n]$ $f_2[n]$

3. $f_2[n]$ $f_3[n]$ $f_1[n]$

5. $f_3[n]$ $f_2[n]$ $f_1[n]$

Aliasing

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where $\Delta = 1/3600$

Which list goes from lowest to highest DT frequency? **5**

0. $f_1[n] \quad f_2[n] \quad f_3[n]$

1. $f_1[n] \quad f_3[n] \quad f_2[n]$

2. $f_2[n] \quad f_1[n] \quad f_3[n]$

3. $f_2[n] \quad f_3[n] \quad f_1[n]$

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Tones in Python

Determine `EXPR1` and `EXPR2` below to generate a 1000 Hz cosine tone using a sampling rate of 44,100 samples/second. The tone should last 2.5 seconds.

```
import math

from lib6300.audio import wav_write

f = [cos(EXPR1 * n) for n in range(0, EXPR2)]

wav_write(f, 44100, 'output.wav')
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EXPR1 is the DT frequency, which we can calculate as follows:

$$\Omega \left[\frac{\text{radians}}{\text{sample}} \right] = 2\pi \left[\frac{\text{radians}}{\text{cycle}} \right] \times f \left[\frac{\text{cycles}}{\text{second}} \right] / f_s \left[\frac{\text{sample}}{\text{second}} \right]$$

Substituting the constants above yields

```
EXPR1=2*math.pi*1000/44100
```

EXPR2 corresponds to the total number of samples needed for 2.5 seconds of audio, which is

```
EXPR2=int(2.5*44100)
```