

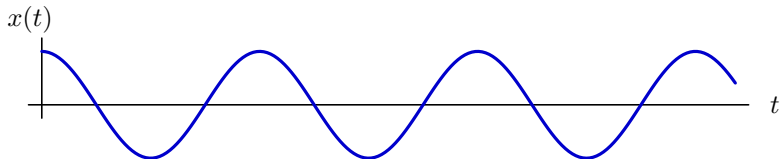
6.3000: Signal Processing

Sampling and Aliasing

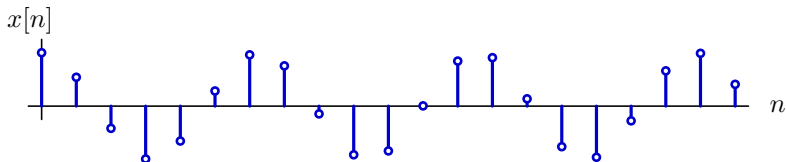
September 22, 2026

Tones and Sinusoids

A “tone” is a pressure that changes sinusoidally with time.



In 6.3000, we will think of this as a “continuous-time” (CT) signal. In contrast, a “discrete-time” (DT) signal is a sequence of numbers.



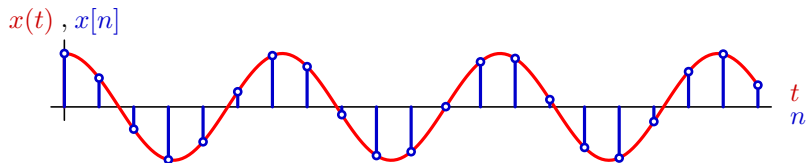
Mathematically:

$$x(t) = A \cos(\omega t)$$

$$x[n] = A \cos(\Omega n)$$

CT and DT Representations

Assume that $x[n]$ represents “samples” of $x(t)$:



$$x(t) = A \cos(\omega t)$$

$$x[n] = A \cos(\Omega n)$$

- What are the units of ω , t , Ω , and n ?

Let f represent the “frequency” of the tone in cycles/second.

- Determine ω in terms of f .
- Determine Ω in terms of ω .
- Determine Ω in terms of f .

Check Yourself

Compare two signals:

$$x_1[n] = \cos \frac{3\pi n}{4}$$

$$x_2[n] = \cos \frac{5\pi n}{4}$$

How many of the following statements are true?

$x_1[n]$ has period $N=8$.

$x_2[n]$ has period $N=8$.

$x_1[n] = x_2[n]$.

Frequencies

Consider the following CT signal:

$$f(t) = 6 \cos(42\pi t) + 4 \cos(18\pi t - 0.5\pi)$$

What is the fundamental period of this signal?

Frequencies

Now imagine that this same signal

$$f(t) = 6 \cos(42\pi t) + 4 \cos(18\pi t - 0.5\pi)$$

is sampled with a sampling rate of $f_s = 60$ Hz to obtain a discrete-time signal $f[n]$, which is periodic in n with fundamental period N . What is the fundamental period N ?

Aliasing

Consider 3 CT signals:

$$f_1(t) = \cos(4000 \pi t) \quad ; \quad f_2(t) = \cos(5000 \pi t) \quad ; \quad f_3(t) = \cos(6000 \pi t)$$

Each of these is sampled so that

$$f_1[n] = f_1(n\Delta) \quad ; \quad f_2[n] = f_2(n\Delta) \quad ; \quad f_3[n] = f_3(n\Delta)$$

where $\Delta = 1/3600$

Which list goes from lowest to highest (baseband) frequency?

0. $f_1[n]$ $f_2[n]$ $f_3[n]$

2. $f_2[n]$ $f_1[n]$ $f_3[n]$

4. $f_3[n]$ $f_1[n]$ $f_2[n]$

1. $f_1[n]$ $f_3[n]$ $f_2[n]$

3. $f_2[n]$ $f_3[n]$ $f_1[n]$

5. $f_3[n]$ $f_2[n]$ $f_1[n]$

Tones in Python

Determine `EXPR1` and `EXPR2` below to generate a 1000 Hz cosine tone using a sampling rate of 44,100 samples/second. The tone should last 2.5 seconds.

```
import math

from lib6300.audio import wav_write

f = [cos(EXPR1 * n) for n in range(0, EXPR2)]

wav_write(f, 44100, 'output.wav')
```