Quiz 2 Review

Quiz 2: Tuesday, November 10, 2:00 PM, Walker Memorial (50-340)

Quiz day

When and where

Tuesday, November 10, regular class block (2:00 PM), in Walker Memorial (50-340). No lecture or recitation that day.

What you may bring

Two 8.5×11 sheets of **handwritten** notes (four sides). A pencil. No electronics.

What it covers

Weeks 4 through 8: CT/DT Fourier transforms, systems, convolution, frequency response, communication systems, and the DFT (lectures, recitations, homework).

Where answers go

In the boxes. Work on other quiz pages counts for partial credit; flag any worksheet you continue on.

Problem 1: short answer (24 points)

1 Short-Answer Questions (24 points)

Part a. The left column of the following table shows the input/output relations for four different systems, where x represents the input and y represents the output of the system. For each system, determine if that system is additive and/or homogeneous and/or time-invariant, and enter **Yes** or **No** in the boxes to the right.

	additive? (**Yes** or **No**)	homogeneous? (**Yes** or **No**)	time-invariant? (**Yes** or **No**)
$y[n] = \left(-\frac{1}{2}\right)^n \left(x[n]+1\right)$			
$y[n] = \sin\left(x[n]\right)$			
$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$			
$y(t) = tx(t)$			

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Before we solve it

- Which of these can you decide without writing any algebra?

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Before we solve it

- Which of these can you decide without writing any algebra?
- Can a system be time-invariant without being linear?
- Which system is the only one that is a convolution?

1a Answer and checks

System	Additive	Homog.	Time-inv.
$y[n] = \left(-\frac{1}{2}\right)^n \left(x[n] + 1\right)$	**No**	**No**	**No**
$y[n] = \sin(x[n])$	**No**	**No**	**Yes**
$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$	**Yes**	**Yes**	**Yes**
$y(t) = t\, x(t)$	**Yes**	**Yes**	**No**

Method: feed $x_1 + x_2$, then αx, then $x[n - m]$ (replacing *every* n). One counterexample proves No; Yes needs the algebra for all inputs.

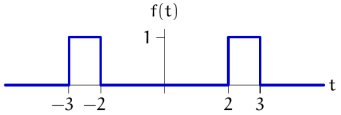
Verify

- Zero in, zero out? The first system gives $(-\frac{1}{2})^n$ for $x = 0$ because of the $+1$: not linear.
- Gain that depends on n or t ($(-\frac{1}{2})^n$, t)? Possibly linear, but not TI.
- Nonlinear function of x alone ($\sin$)? TI, but not linear.
- A convolution with a fixed h (the integrator, $h = u$)? LTI.

1b

Part b. Determine the frequency response $F(\omega)$ of a linear, time-invariant system with the following impulse response:

$$f(t) = \begin{cases} 1 & \text{if } 2 < |t| < 3 \\ 0 & \text{otherwise} \end{cases}$$



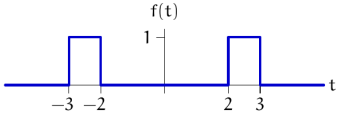
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$F(\omega) =$

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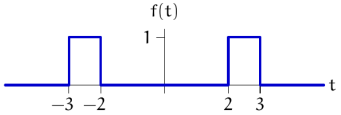
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- What standard signals is $f(t)$ built from?

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$F(\omega) =$

Before we solve it

- What standard signals is $f(t)$ built from?
- Before any integral: what must $F(0)$ equal?

1b Answer and checks

$$f = p_3 - p_2, \quad P_S(\omega) = \frac{2\sin S\omega}{\omega} \quad \Rightarrow \quad F(\omega) = \frac{2\sin 3\omega}{\omega} - \frac{2\sin 2\omega}{\omega}$$

Approach

Decompose into pieces whose transforms you already know; use linearity.

Verify

- $F(0)$ = area of $f = 2$; limits give $6 - 4$ ✓
- f real and symmetric $\Rightarrow$ F real and symmetric; a stray j or $e^{-j\omega}$ means a shift crept in ✓
- Wider pulse, narrower sinc: first zeros at $\pi/3$, $\pi/2$ ✓

1b Alternative 1: integrate directly

Two disjoint intervals, so two integrals:

$$F(\omega) = \int_{-3}^{-2} e^{-j\omega t}\,dt + \int_{2}^{3} e^{-j\omega t}\,dt = \frac{e^{-j\omega t}}{-j\omega}\bigg|_{-3}^{-2} + \frac{e^{-j\omega t}}{-j\omega}\bigg|_{2}^{3}$$
$$= \frac{\left(e^{j2\omega} - e^{j3\omega}\right) + \left(e^{-j3\omega} - e^{-j2\omega}\right)}{-j\omega} = \frac{\left(e^{j2\omega} - e^{-j2\omega}\right) - \left(e^{j3\omega} - e^{-j3\omega}\right)}{-j\omega}$$
$$= \frac{2j\sin 2\omega - 2j\sin 3\omega}{-j\omega} = \frac{2\sin 3\omega}{\omega} - \frac{2\sin 2\omega}{\omega}$$

Verify

The regrouping step is the check: pair $e^{ja\omega}$ with $e^{-ja\omega}$. If a term has no partner, an integration limit is wrong.

1b Alternative 2: two shifted narrow pulses

Each piece is a pulse of width 1 (half-width $\frac{1}{2}$) centered at $t = \pm\frac{5}{2}$:

$$f(t) = p_{1/2}\big(t - \tfrac{5}{2}\big) + p_{1/2}\big(t + \tfrac{5}{2}\big), \qquad P_{1/2}(\omega) = \frac{2\sin(\omega/2)}{\omega}$$

Shift by $\pm\frac{5}{2}$ multiplies by $e^{\mp j\frac{5}{2}\omega}$; the two phases add to a cosine:

$$F(\omega) = \Big(e^{-j\frac{5}{2}\omega} + e^{j\frac{5}{2}\omega}\Big)\frac{2\sin(\omega/2)}{\omega} = \frac{4\cos\big(\frac{5\omega}{2}\big)\sin\big(\frac{\omega}{2}\big)}{\omega}$$

Verify

Same answer as $p_3 - p_2$: the sum-to-product identity gives $2\sin 3\omega - 2\sin 2\omega = 4\cos\frac{5\omega}{2}\sin\frac{\omega}{2}$

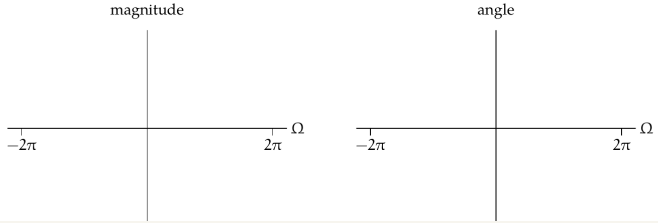
✓ $F(0) = 4 \cdot 1 \cdot \frac{\omega/2}{\omega} \to 2 =$ area ✓

1c

Part c. Determine the frequency response of a linear, time-invariant system with the following unit-sample response:

$$g[n] = \delta[n] - \delta[n-2]$$

Sketch the magnitude and angle of the frequency response on the axes below. Label the key points.

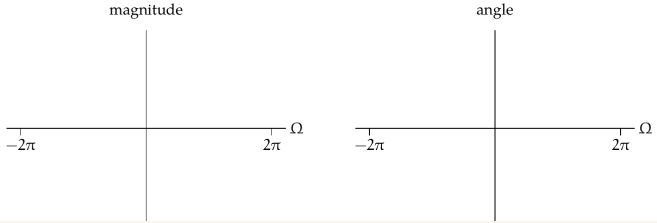


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Before we solve it

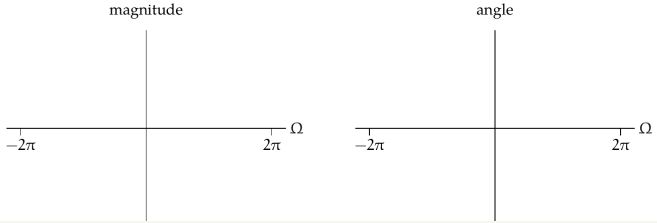
- Where must $|G(\Omega)|$ be zero?

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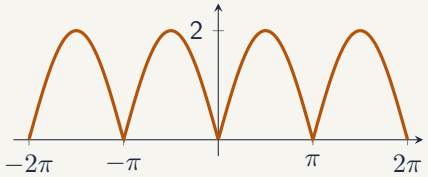
- Where must $|G(\Omega)|$ be zero?
- What does a pure delay do to the phase?

1c Answer and checks

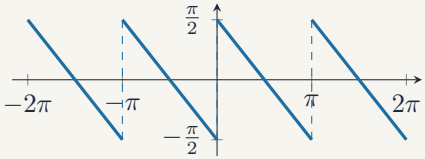
$$G(\Omega) = 1 - e^{-j2\Omega} = e^{-j\Omega}\big(e^{j\Omega} - e^{-j\Omega}\big) = j\,2\sin\Omega\, e^{-j\Omega}$$

$$|G| = 2|\sin\Omega|, \qquad \angle G = \tfrac{\pi}{2} - \Omega \;\; (+\pi \text{ where } \sin\Omega < 0)$$

magnitude



angle



Verify

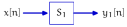
$G(0) = \sum g[n] = 0$ ✓ $G(\pi) = \sum g[n](-1)^n = 0$ ✓ 2π-periodic ✓ phase slope -1 = one-sample delay ✓

Problem 2: cascaded systems (22 points)

2 Cascaded Systems (22 points)

Part a.

Let S_1 represent a linear, time-invariant (LTI) system whose unit-sample response $h_1[n]$ is a unit step:



$$h_1[n] = \begin{cases} 1 & \text{if } n \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

Determine the response $y_1[n]$ of this system when the input $x[n]$ is the following geometric sequence:

$$x[n] = \begin{cases} (0.9)^n & \text{if } n \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

Enter the first 5 values of $y_1[n]$ in the boxes below.

$y_1[0] =$

$y_1[1] =$

$y_1[2] =$

$y_1[3] =$

$y_1[4] =$

Enter a closed-form expression[2] for $y_1[n]$ in the box below.

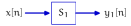
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2a

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Before we solve it

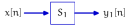
- What does convolving with a unit step do to *any* signal?

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Enter a closed-form expression[2] for $y_1[n]$ in the box below.

$y_1[n] =$

Before we solve it

- What does convolving with a unit step do to *any* signal?
- What sets the limits on the convolution sum?

2a Answer and checks

$$y_1[n] = \sum_m (0.9)^m u[m]\, u[n-m] = \sum_{m=0}^{n} (0.9)^m = \frac{1 - 0.9^{\,n+1}}{1 - 0.9} = 10 - 9\,(0.9)^n$$

Approach

$u[m]$ forces $m \geq 0$ and $u[n-m]$ forces $m \leq n$: convolution with a step is a running sum. So $y_1[0], \ldots, y_1[4]$ are the partial sums $1,\ 1+0.9,\ 1+0.9+0.9^2,\ \ldots$

Common mistake

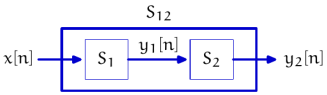
$n+1$ terms, so $r^{\,n+1}$ in the numerator.

Verify

- $n = 0$: $10 - 9 = 1$ ✓
- $n \to \infty$: $10 = 1/(1-0.9)$ ✓
- Difference it: $y_1[n] - y_1[n-1] = 9(0.9)^{n-1}(0.1) = (0.9)^n = x[n]$ ✓

Part b.

Let S_{12} represent the linear, time-invariant (LTI) system that results when two LTI systems (S_1 and S_2) are connected in cascade (so that the output of S_1 is the input to S_2) as shown in the figure to the right.



Determine the unit-sample response $h_{12}[n]$ of S_{12} when the unit-sample response $h_1[n]$ of system S_1 is a unit-step function $u[n]$ and the unit-sample response $h_2[n]$ of system S_2 is also a unit-step function $u[n]$:

$h_1[n] = u[n]$

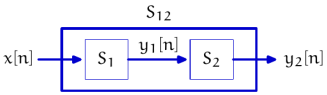
$h_2[n] = u[n]$

Enter a closed-form expression[3] for $h_{12}[n]$ in the box below.

$h_{12}[n] =$

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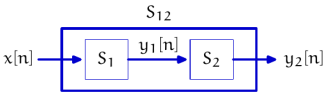
Before we solve it

- What shape do you expect for $u * u$?

2b

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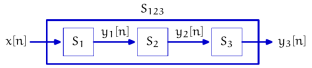
Before we solve it

- What shape do you expect for $u * u$?
- If you take a first difference of $h_{12}[n]$, what should come back?

2c

Part c.

Let S_{123} represent the LTI system that results when three LTI systems (S_1, S_2, and S_3) are connected in cascade, as shown in the figure to the right.



Determine the unit-sample response $h_{123}[n]$ of S_{123} when the unit-sample response $h_1[n]$ of system S_1 is a unit-step function $u[n]$, the unit-sample response $h_2[n]$ of system S_2 is a unit-step function $u[n]$, and the unit-sample response $h_3[n]$ of system S_3 is also a unit-step function $u[n]$:

$h_1[n] = u[n]$
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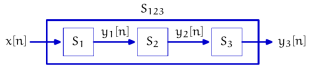
Enter the first 5 values of $h_{123}[n]$ in the boxes below.

$h_{123}[0] =$
$h_{123}[1] =$
$h_{123}[2] =$
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2c

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Determine the unit-sample response $h_{123}[n]$ of S_{123} when the unit-sample response $h_1[n]$ of system S_1 is a unit-step function $u[n]$, the unit-sample response $h_2[n]$ of system S_2 is a unit-step function $u[n]$, and the unit-sample response $h_3[n]$ of system S_3 is also a unit-step function $u[n]$:

$h_1[n] = u[n]$
$h_2[n] = u[n]$
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Enter the first 5 values of $h_{123}[n]$ in the boxes below.

$h_{123}[0] =$
$h_{123}[1] =$
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$h_{123}[4] =$

Before we solve it

- $h_{12}[n] = (n+1)u[n]$ is linear in n. After one more accumulator, what degree polynomial in n do you expect?

2b, 2c Answers and checks

$$h_{12}[n] = (u * u)[n] = \sum_{m=0}^{n} 1 = (n+1)\,u[n], \qquad h_{123}[n] = \sum_{m=0}^{n} (m+1) = \frac{(n+1)(n+2)}{2}\,u[n]$$

n	h_1	h_{12}	h_{123}
0	1	1	**1**
1	1	2	**3**
2	1	3	**6**
3	1	4	**10**
4	1	5	**15**

Verify

- Each accumulator is undone by a first difference: $\Delta h_{123} = h_{12}$, $\Delta h_{12} = u$ ✓
- $(4+1)(4+2)/2 = 15$ ✓
- Don't drop the $u[n]$: the system is causal.

2b, 2c What would happen if we tried the frequency perspective?

The appeal

Convolution in time means multiplication in frequency: $H_{12} = H_1^2$, $H_{123} = H_1^3$. No sums to evaluate.

The catch!!

$H_1(\Omega) = \sum_{n \geq 0} e^{-j\Omega n}$ does not converge: the unit step is not a signal the DTFT, as we defined it, can handle.

Problem 3: pulsed relations (27 points)

Problem 3

3 Pulsed Relations (27 points)

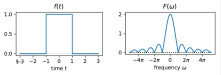
Let $f(t)$ represent the following signal

$$f(t) = \begin{cases} 1 & \text{if } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

and let $F(\omega)$ represent its Fourier transform

$$F(\omega) = 2\frac{\sin(\omega)}{\omega}$$

as shown on the right.



Identify which plot (A-I) shows the magnitude of each of the following derived signals.

$f(t/2)$: ☐ $f(t)\sin(2\pi t)$: ☐

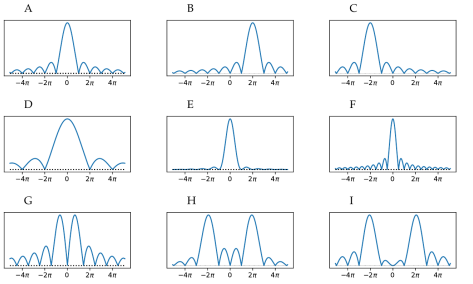
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Problem 3

3 Pulsed Relations (27 points)

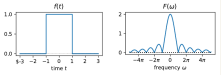
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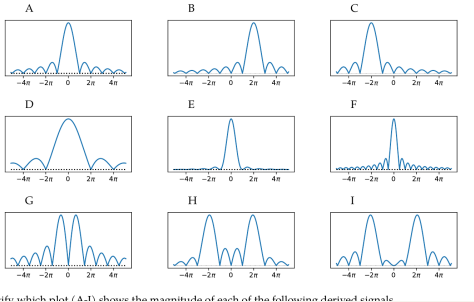
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Before we solve it

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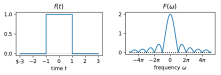
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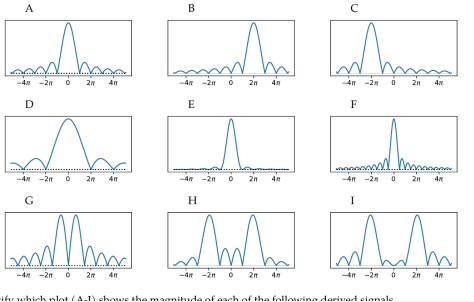
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Before we solve it

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- Which ones change the width of the pulse, and which way does the sinc go?

Problem 3

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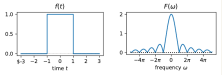
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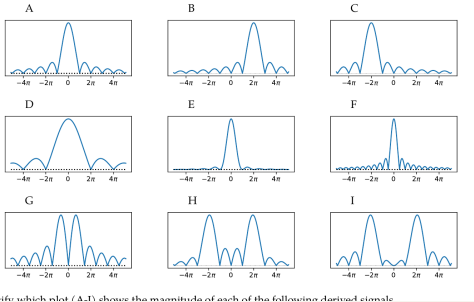
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Before we solve it

- Which of the nine operations leave the *magnitude* of the transform unchanged?
- Which ones change the width of the pulse, and which way does the sinc go?
- Which ones move the spectrum, and where?

Problem 3 Answers

Signal	Transform	Evidence in the plot	Plot
$f(t/2)$	$2F(2\omega)$	main lobe half as wide: first zero at $\pi/2$	**F**
$f(2t)$	$\frac{1}{2}F(\omega/2)$	main lobe twice as wide: first zero at 2π	**D**
$(f * f)(t)$	$F(\omega)^2$	same zeros, sidelobes crushed	**E**
$f(t-1)$	$e^{-j\omega}F(\omega)$	magnitude identical to $\lvert F\rvert$	**A**
$tf(t)$	$j\,dF/d\omega$	anti-symmetric signal $\Rightarrow$ zero at $\omega = 0$; lobes still $\approx \pi$ apart	**G**
$f(t)e^{\pm j2\pi t}$	$F(\omega \mp 2\pi)$	one sinc moved to $\pm 2\pi$	**B**, **C**
$f(t)\cos 2\pi t$	$\frac{1}{2}[F(\omega-2\pi)+F(\omega+2\pi)]$	two peaks; tails near 0 cancel: small center bump	**I**
$f(t)\sin 2\pi t$	$\frac{1}{2j}[F(\omega-2\pi)-F(\omega+2\pi)]$	two peaks; tails near 0 add: larger center bump	**H**

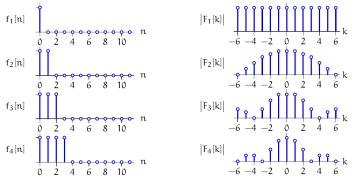
Verify

- Each of A–I is used exactly once.
- $F(0)$ is the area under the signal. Scaling, shifting and self-convolution keep the area positive, so A, D, E, F peak at $\omega = 0$; $tf(t)$ is anti-symmetric and the modulated signals have zero average, so G, H, I, B, C do not.
- A pulse of half-width S has sinc zeros spaced π/S apart: $\pi/2$ for the wide pulse (F), π for the unit pulse and everything made from it by shift, modulation or convolution, 2π for the narrow pulse (D).

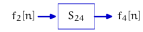
Problem 4: pulses in, pulses out (27 points)

4 Pulses In Pulses Out (27 points)

The first 12 samples of four **periodic** signals that are each periodic in N=12 are shown in the left column below, and the magnitudes of their Fourier series coefficients are shown in the right column.



Part a. Consider a system S_{24} that produces $f_4[n]$ as output when $f_2[n]$ is its input.



Determine the unit-sample response $h_{24}[n]$ and frequency response $H_{24}[k]$ of system S_{24} and enter expressions for $h_{24}[n]$ and $H_{24}[k]$ in the boxes below.

$h_{24}[n] =$

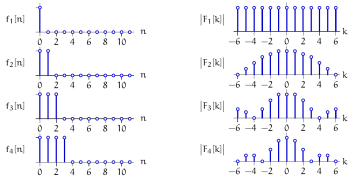
$H_{24}[k] =$

Sketch the magnitude of the frequency response $H_{24}[k]$ on the axes below.

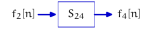


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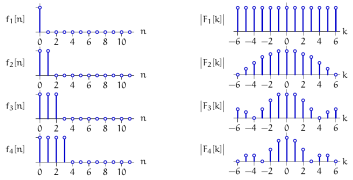


Before we solve it

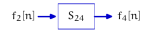
- How do you build $f_4[n]$ out of copies of $f_2[n]$?

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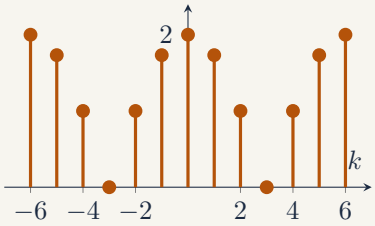
Sketch the magnitude of the frequency response $H_{24}[k]$ on the axes below.



Before we solve it

- How do you build $f_4[n]$ out of copies of $f_2[n]$?
- Where must $|H_{24}[k]|$ be zero, just from the two spectra?

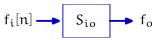
$$h_{24}[n] = \delta[n] + \delta[n-2], \qquad H_{24}[k] = 1 + e^{-j\pi k/3} = e^{-j\pi k/6}\, 2\cos\!\left(\tfrac{\pi k}{6}\right)$$



Verify

- $H[0] = \sum h = 2$; averages $4/12$ vs $2/12$ ✓
- Zeros at $k = \pm 3$: exactly where $F_4 = 0$ and $F_2 \neq 0$ ✓
- Where $F_2 = 0$ already ($k = \pm 6$), H is unconstrained: the answer is not unique, and that is fine.

Part b. Consider 16 possible systems that are each defined by its input signal $f_i[n]$ and output signal $f_o[n]$:



Some of these systems could be linear and time invariant (LTI); others cannot.

Determine which of the 16 systems cannot possibly be LTI, and enter **X** in the corresponding box below.

i \ o	1	2	3	4
1				
2				
3				
4				

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i \ o	1	2	3	4
1				
2				
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Before we solve it

- What can an LTI system do to a harmonic whose input coefficient is already zero?

4b

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i \ o	1	2	3	4
1				
2				
3				
4				

Before we solve it

- What can an LTI system do to a harmonic whose input coefficient is already zero?
- Which row of the grid must be all blank?

4b Answer and checks

$i \backslash o$	1	2	3	4
1				
2	X		X	
3	X	X		X
4	X	X	X	

Zeros: f_1 none; f_2 ± 6; f_3 ± 4; f_4 $\pm 3, \pm 6$.

Rule

$F_o[k] = H[k]F_i[k]$: an LTI system cannot create a harmonic the input lacks. S_{io} can be LTI only if $\text{Zeros}(F_i) \subseteq \text{Zeros}(F_o)$. Don't compute H.

Verify

- Diagonal blank ($h = \delta$) ✓
- More input zeros ⇒ more X's: 0, 2, 3, 3 ✓
- The only off-diagonal blank below row 1 is $(2, 4)$, the system from 4a ✓

Summary

Quiz problem	Lecture	Recitation
1a LTI properties	Systems (Oct 8)	Systems (Oct 8)
1b, 3 CT Fourier transform	CT Fourier Transform (Sep 29)	CT Fourier Transform (Sep 29)
1c DT frequency response	DT Fourier Transform (Oct 1); Frequency Response and Filtering (Oct 20)	Convolution and Filtering (Oct 20)
2 Cascades, convolution	Unit-Sample Response and Convolution (Oct 15)	Superposition and Convolution (Oct 15)
3 Modulation	Communication Systems (Oct 22)	Communication Systems (Oct 22)
4 Filtering periodic signals	DT Fourier Series (Sep 24); Frequency Response and Filtering (Oct 20)	DFT and Circular Convolution (Oct 29)

Practice quizzes

On the Quiz 2 info page: Spring 2022, Fall 2022, Fall 2023, Spring 2025 (this one), with solutions, plus review slides and story sheet.

TA-led review session

Sunday, November 8, 1–3 PM, in 34-101.