Quiz 2 Review

Quiz 2: Tuesday, November 10, 2:00 PM, Walker Memorial (50-340)

Quiz day

When and where

Tuesday, November 10, regular class block (2:00 PM), in Walker Memorial (50-340). No lecture or recitation that day.

What you may bring

Two 8.5×11 sheets of **handwritten** notes (four sides). A pencil. No electronics.

What it covers

Weeks 4 through 8: CT/DT Fourier transforms, systems, convolution, frequency response, communication systems, and the DFT (lectures, recitations, homework).

Where answers go

In the boxes. Work on other quiz pages counts for partial credit; flag any worksheet you continue on.

Problem 1: short answer (24 points)

1 Short-Answer Questions (24 points)

Part a. The left column of the following table shows the input/output relations for four different systems, where x represents the input and y represents the output of the system. For each system, determine if that system is additive and/or homogeneous and/or time-invariant, and enter **Yes** or **No** in the boxes to the right.

	additive? (**Yes** or **No**)	homogeneous? (**Yes** or **No**)	time-invariant? (**Yes** or **No**)
$y[n] = \left(-\frac{1}{2}\right)^n \left(x[n]+1\right)$			
$y[n] = \sin\left(x[n]\right)$			
$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$			
$y(t) = tx(t)$			

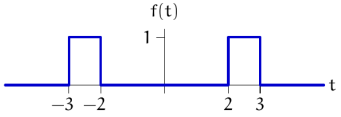
Before we solve it

- Which of these can you decide without writing any algebra?
- Can a system be time-invariant without being linear?
- Which system is the only one that is a convolution?

1b

Part b. Determine the frequency response $F(\omega)$ of a linear, time-invariant system with the following impulse response:

$$f(t) = \begin{cases} 1 & \text{if } 2 < |t| < 3 \\ 0 & \text{otherwise} \end{cases}$$



Enter a closed form expression[1] for $F(\omega)$ in the box below.

$F(\omega) =$

Before we solve it

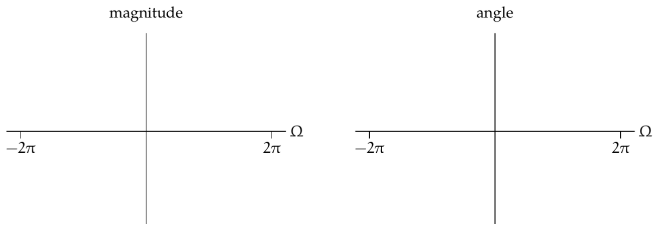
- What standard signals is $f(t)$ built from?
- Before any integral: what must $F(0)$ equal?

1c

Part c. Determine the frequency response of a linear, time-invariant system with the following unit-sample response:

$$g[n] = \delta[n] - \delta[n-2]$$

Sketch the magnitude and angle of the frequency response on the axes below. Label the key points.



Before we solve it

- Where must $|G(\Omega)|$ be zero?
- What does a pure delay do to the phase?

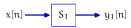
Problem 2: cascaded systems (22 points)

2a

2 Cascaded Systems (22 points)

Part a.

Let S_1 represent a linear, time-invariant (LTI) system whose unit-sample response $h_1[n]$ is a unit step:



$$h_1[n] = \begin{cases} 1 & \text{if } n \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

Determine the response $y_1[n]$ of this system when the input $x[n]$ is the following geometric sequence:

$$x[n] = \begin{cases} (0.9)^n & \text{if } n \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

Enter the first 5 values of $y_1[n]$ in the boxes below.

$y_1[0] =$

$y_1[1] =$

$y_1[2] =$

$y_1[3] =$

$y_1[4] =$

Enter a closed-form expression[2] for $y_1[n]$ in the box below.

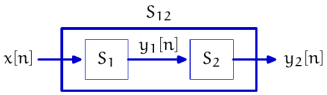
$y_1[n] =$

Before we solve it

- What does convolving with a unit step do to *any* signal?
- What sets the limits on the convolution sum?

Part b.

Let S_{12} represent the linear, time-invariant (LTI) system that results when two LTI systems (S_1 and S_2) are connected in cascade (so that the output of S_1 is the input to S_2) as shown in the figure to the right.



Determine the unit-sample response $h_{12}[n]$ of S_{12} when the unit-sample response $h_1[n]$ of system S_1 is a unit-step function $u[n]$ and the unit-sample response $h_2[n]$ of system S_2 is also a unit-step function $u[n]$:

$h_1[n] = u[n]$

$h_2[n] = u[n]$

Enter a closed-form expression[3] for $h_{12}[n]$ in the box below.

$h_{12}[n] =$

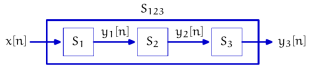
Before we solve it

- What shape do you expect for $u * u$?
- If you take a first difference of $h_{12}[n]$, what should come back?

2c

Part c.

Let S_{123} represent the LTI system that results when three LTI systems (S_1, S_2, and S_3) are connected in cascade, as shown in the figure to the right.



Determine the unit-sample response $h_{123}[n]$ of S_{123} when the unit-sample response $h_1[n]$ of system S_1 is a unit-step function $u[n]$, the unit-sample response $h_2[n]$ of system S_2 is a unit-step function $u[n]$, and the unit-sample response $h_3[n]$ of system S_3 is also a unit-step function $u[n]$:

$h_1[n] = u[n]$
$h_2[n] = u[n]$
$h_3[n] = u[n]$

Enter the first 5 values of $h_{123}[n]$ in the boxes below.

$h_{123}[0] =$
$h_{123}[1] =$
$h_{123}[2] =$
$h_{123}[3] =$
$h_{123}[4] =$

Before we solve it

- $h_{12}[n] = (n+1)u[n]$ is linear in n. After one more accumulator, what degree polynomial in n do you expect?

Problem 3: pulsed relations (27 points)

Problem 3

3 Pulsed Relations (27 points)

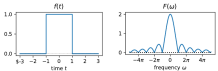
Let $f(t)$ represent the following signal

$$f(t) = \begin{cases} 1 & \text{if } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

and let $F(\omega)$ represent its Fourier transform

$$F(\omega) = 2\frac{\sin(\omega)}{\omega}$$

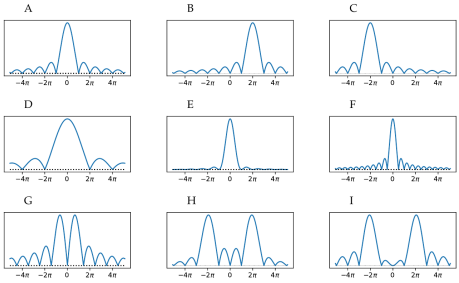
as shown on the right.



Identify which plot (A-I) shows the magnitude of each of the following derived signals.

$f(t/2)$: ☐
$(f * f)(t)$: ☐
$f(2t)$: ☐
$f(t-1)$: ☐
$tf(t)$: ☐
$f(t)\sin(2\pi t)$: ☐
$f(t)\cos(2\pi t)$: ☐
$f(t)e^{j2\pi t}$: ☐
$f(t)e^{-j2\pi t}$: ☐

Each of the following plots shows the magnitude of the Fourier transform of a signal derived from $f(t)$. Note that the magnitude scales for these plots are not specified and may differ from one another.



Identify which plot (A-I) shows the magnitude of each of the following derived signals.

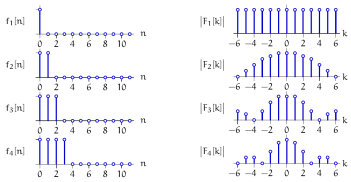
Before we solve it

- Which of the nine operations leave the *magnitude* of the transform unchanged?
- Which ones change the width of the pulse, and which way does the sinc go?
- Which ones move the spectrum, and where?

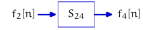
Problem 4: pulses in, pulses out (27 points)

4 Pulses In Pulses Out (27 points)

The first 12 samples of four **periodic** signals that are each periodic in N=12 are shown in the left column below, and the magnitudes of their Fourier series coefficients are shown in the right column.



Part a. Consider a system S_{24} that produces $f_4[n]$ as output when $f_2[n]$ is its input.

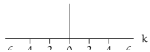


Determine the unit-sample response $h_{24}[n]$ and frequency response $H_{24}[k]$ of system S_{24} and enter expressions for $h_{24}[n]$ and $H_{24}[k]$ in the boxes below.

$h_{24}[n] =$

$H_{24}[k] =$

Sketch the magnitude of the frequency response $H_{24}[k]$ on the axes below.



Before we solve it

- How do you build $f_4[n]$ out of copies of $f_2[n]$?
- Where must $|H_{24}[k]|$ be zero, just from the two spectra?

4b

Part b. Consider 16 possible systems that are each defined by its input signal $f_i[n]$ and output signal $f_o[n]$:



Some of these systems could be linear and time invariant (LTI); others cannot.

Determine which of the 16 systems cannot possibly be LTI, and enter **X** in the corresponding box below.

i \ o	1	2	3	4
1				
2				
3				
4				

Before we solve it

- What can an LTI system do to a harmonic whose input coefficient is already zero?
- Which row of the grid must be all blank?

Summary

Quiz problem	Lecture	Recitation
1a LTI properties	Systems (Oct 8)	Systems (Oct 8)
1b, 3 CT Fourier transform	CT Fourier Transform (Sep 29)	CT Fourier Transform (Sep 29)
1c DT frequency response	DT Fourier Transform (Oct 1); Frequency Response and Filtering (Oct 20)	Convolution and Filtering (Oct 20)
2 Cascades, convolution	Unit-Sample Response and Convolution (Oct 15)	Superposition and Convolution (Oct 15)
3 Modulation	Communication Systems (Oct 22)	Communication Systems (Oct 22)
4 Filtering periodic signals	DT Fourier Series (Sep 24); Frequency Response and Filtering (Oct 20)	DFT and Circular Convolution (Oct 29)

Practice quizzes

On the Quiz 2 info page: Spring 2022, Fall 2022, Fall 2023, Spring 2025 (this one), with solutions, plus review slides and story sheet.

TA-led review session

Sunday, November 8, 1–3 PM, in 34-101.