

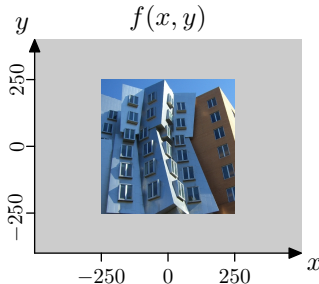
6.3000: Signal Processing

Fourier Series Trig Form

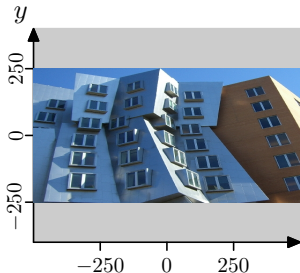
Representing Signals as Fourier Series

- Synthesis: making a signal from components
- Analysis: finding the components

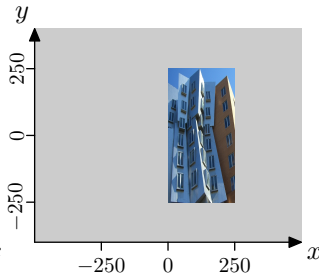
Start With Some Basic Transformations



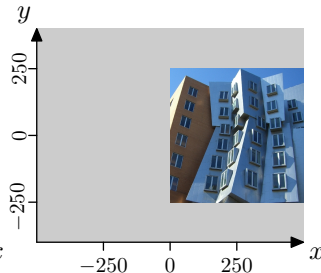
How many images match the expressions beneath them?



$$f_1(x, y) = f(2x, y) ?$$



$$f_2(x, y) = f(2x - 250, y) ?$$



$$f_3(x, y) = f(-x - 250, y) ?$$

Fourier Series

Fourier representations are a major theme of this subject.

The basic ideas were described in lecture:

Synthesis Equation (making a signal from components):

$$f(t) = f(t + T) = c_0 + \sum_{k=1}^{\infty} c_k \cos\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{\infty} d_k \sin\left(\frac{2\pi kt}{T}\right)$$

Analysis Equations (finding the components):

$$c_0 = \frac{1}{T} \int_T f(t) dt$$

$$c_k = \frac{2}{T} \int_T f(t) \cos\left(\frac{2\pi kt}{T}\right) dt ; \quad k \geq 1$$

$$d_k = \frac{2}{T} \int_T f(t) \sin\left(\frac{2\pi kt}{T}\right) dt ; \quad k \geq 1$$

Warm Up

Find the Fourier series coefficients (c_k and d_k) for

$$f(t) = \cos(t)$$

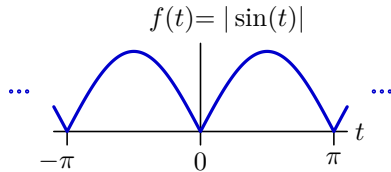
Fourier Series Coefficients

How many of the following functions have **exactly one** non-zero Fourier series coefficient?

- $f_1(t) = \cos^2 t$
- $f_2(t) = \sin t \cos t$
- $f_3(t) = 4 \cos^3 t - 3 \cos t$
- $f_4(t) = \cos(12t) \cos(4t) \cos(2t)$

Rectified Sine Wave

Consider a Fourier series representation of the following function.



- What is the approximate value of c_0 ?
- Which non-DC Fourier coefficient has the largest absolute value? What's the sign of that coefficient?
- Determine an expression for the Fourier coefficients of $f(t)$.
- Compute the sum of the first 100 terms in the Fourier series of $f(t)$.

Verify Fourier Series of Rectified Sine Wave Numerically

Compute the sum of the first 100 terms in the Fourier series of $f(t)$.

Trig Table

$$\sin(a+b) = \sin(a) \cos(b) + \cos(a) \sin(b)$$

$$\sin(a-b) = \sin(a) \cos(b) - \cos(a) \sin(b)$$

$$\cos(a+b) = \cos(a) \cos(b) - \sin(a) \sin(b)$$

$$\cos(a-b) = \cos(a) \cos(b) + \sin(a) \sin(b)$$

$$\tan(a+b) = (\tan(a)+\tan(b))/(1-\tan(a) \tan(b))$$

$$\tan(a-b) = (\tan(a)-\tan(b))/(1+\tan(a) \tan(b))$$

$$\sin(A) + \sin(B) = 2 \sin((A+B)/2) \cos((A-B)/2)$$

$$\sin(A) - \sin(B) = 2 \cos((A+B)/2) \sin((A-B)/2)$$

$$\cos(A) + \cos(B) = 2 \cos((A+B)/2) \cos((A-B)/2)$$

$$\cos(A) - \cos(B) = -2 \sin((A+B)/2) \sin((A-B)/2)$$

$$\sin(a+b) + \sin(a-b) = 2 \sin(a) \cos(b)$$

$$\sin(a+b) - \sin(a-b) = 2 \cos(a) \sin(b)$$

$$\cos(a+b) + \cos(a-b) = 2 \cos(a) \cos(b)$$

$$\cos(a+b) - \cos(a-b) = -2 \sin(a) \sin(b)$$

$$2 \cos(A) \cos(B) = \cos(A-B) + \cos(A+B)$$

$$2 \sin(A) \sin(B) = \cos(A-B) - \cos(A+B)$$

$$2 \sin(A) \cos(B) = \sin(A+B) + \sin(A-B)$$

$$2 \cos(A) \sin(B) = \sin(A+B) - \sin(A-B)$$