

6.3000: Signal Processing

Fourier Series Complex Form

Synthesis Equation (making a signal from components):

$$f(t) = f(t + T) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

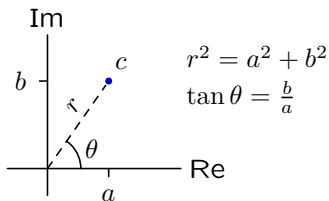
Analysis Equation (finding the components)

$$a_k = \frac{1}{T} \int_T f(t) e^{-jk\omega_o t} dt$$

where $\omega_o = \frac{2\pi}{T}$

Representations of Complex Numbers

Let c represent a complex number.



rectangular form: $c = a + jb$

polar (phasor) form: $r\angle\theta$

Euler form: $r e^{j\theta}$

Find

$$\angle(jc) - \angle(c)$$

which can also be written as

$$\arg(jc) - \arg(c)$$

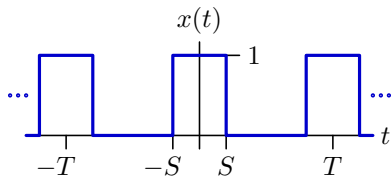
Complex Numbers

How many of the following are true?

- $\frac{1}{\cos \theta + j \sin \theta} = \cos \theta - j \sin \theta$
- $(\cos \theta + j \sin \theta)^n = \cos(n\theta) + j \sin(n\theta)$
- $|2 + j2 + e^{\frac{j\pi}{4}}| = |2 + j2| + |e^{\frac{j\pi}{4}}|$
- $\operatorname{Im}(j^j) > \operatorname{Re}(j^j)$
- $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1} 1$

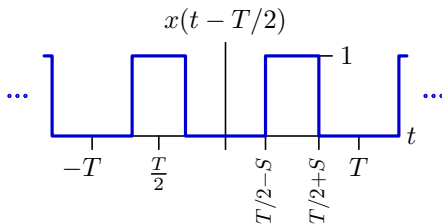
Pulse Train

Find the Fourier series coefficients a_k for $x(t)$:



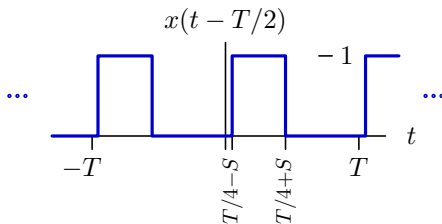
Pulse Train

What would happen to Fourier series if you delayed $x(t)$ by $T/2$?



Pulse Train

What would happen if you delayed $x(t)$ by $T/4$?



Parseval's Theorem

Determine an expression for

$$\int_T (f(t))^2 dt$$

in terms of the Fourier series coefficients a_k of $f(t)$.

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Fourier Series Matching

Match the signals (left column) to Fourier series coefficients (right).

