

6.300 Signal Processing

Week 3, Recitation B: Discrete-Time Fourier Series

- DTFS
- Fourier Series properties

Discrete-Time Fourier Series

Synthesis:

$$x[n] = x[n + N] = \sum_{k=\langle N \rangle} X[k] e^{j\frac{2\pi}{N}kn}$$

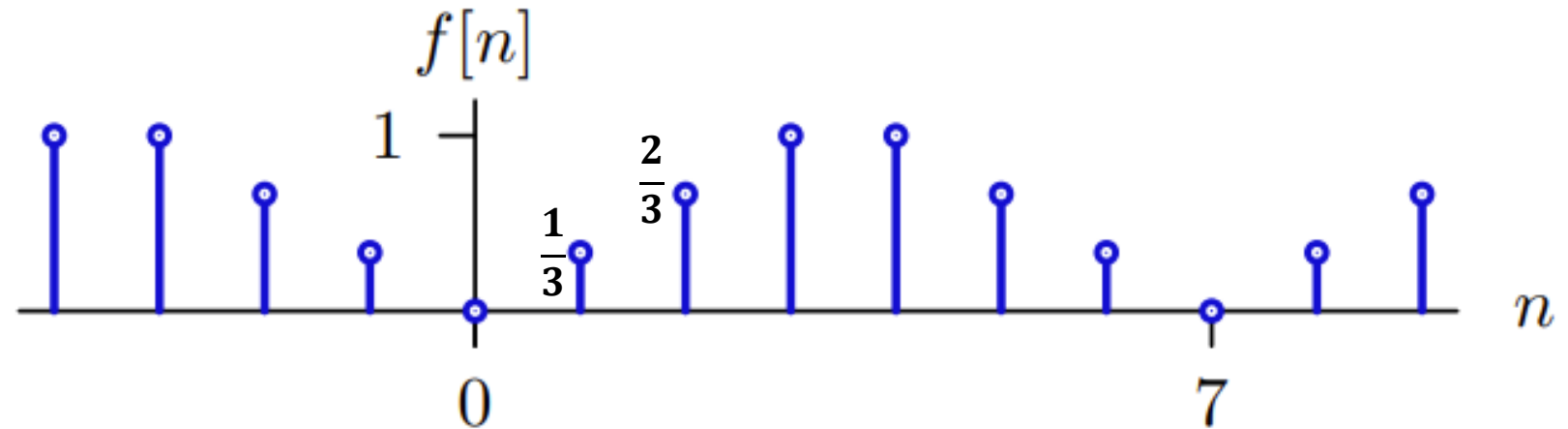
Analysis:

$$X[k] = X[k + N] = \frac{1}{N} \sum_{n=n_0}^{n_0+N-1} x[n] e^{-j\Omega_0 kn}$$

discrete in time $\Rightarrow$ periodic in frequency

Find the DT Fourier Series Coefficients

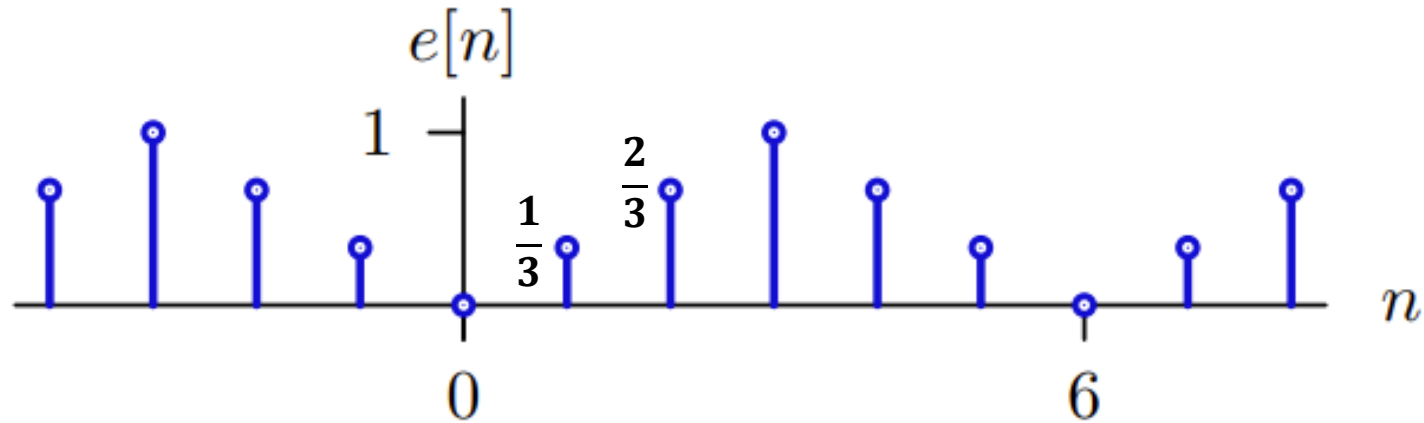
Let $f[n]$ represent a periodic DT signal with period $N = 7$:



Determine the Fourier series coefficients $F[k]$ for $f[n]$.

Find the DT Fourier Series Coefficients

How would the answer change if the period were $N = 6$?



Determine the Fourier series coefficients $E[k]$ for $e[n]$.

Find the DT Fourier Series Coefficients

- What are the Fourier Series coefficients of the following signal?

$$x[n] = 1 + \cos\left(\frac{2\pi}{5}n\right)$$

Find the DT Fourier Series Coefficients

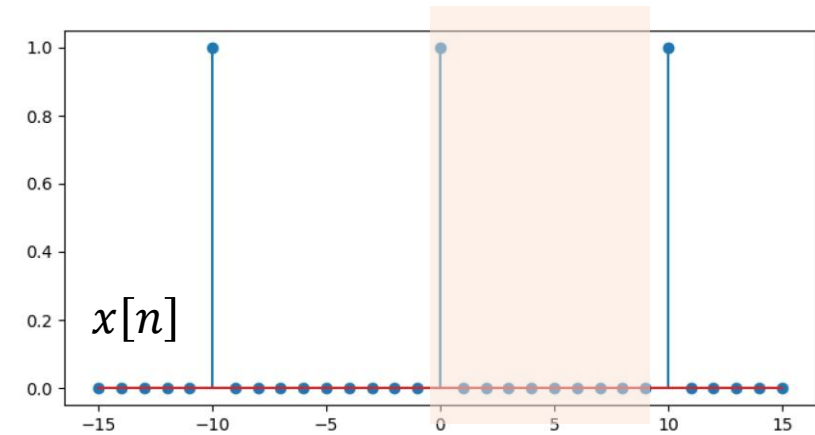
- What are the Fourier Series coefficients of the following signal?

$$x[n] = 1 + \sin\left(\frac{\pi}{4}n\right)$$

Find the DT Fourier Series Coefficients

- What are the Fourier Series coefficients of the following signal?

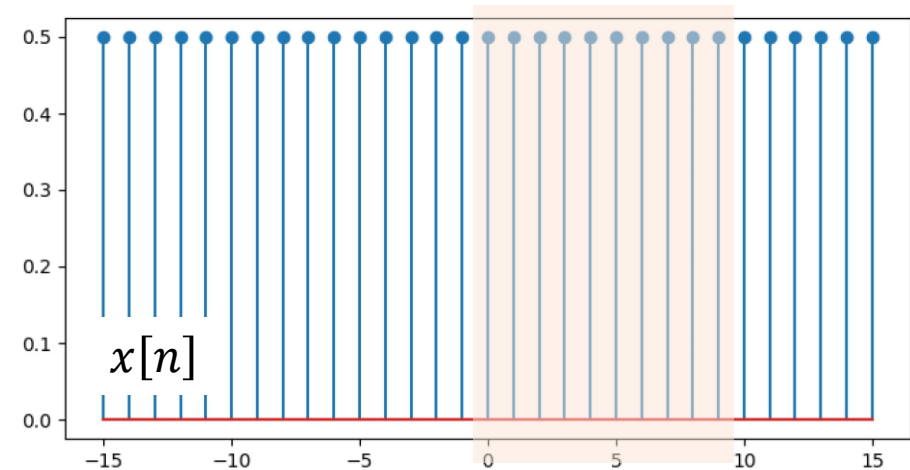
$$x[n] = \begin{cases} 1 & \text{if } n \bmod 10 \equiv 0 \\ 0 & \text{otherwise} \end{cases}$$



Find the DT Fourier Series Coefficients

- What are the Fourier Series coefficients of the following signal with a period of $N=10$?

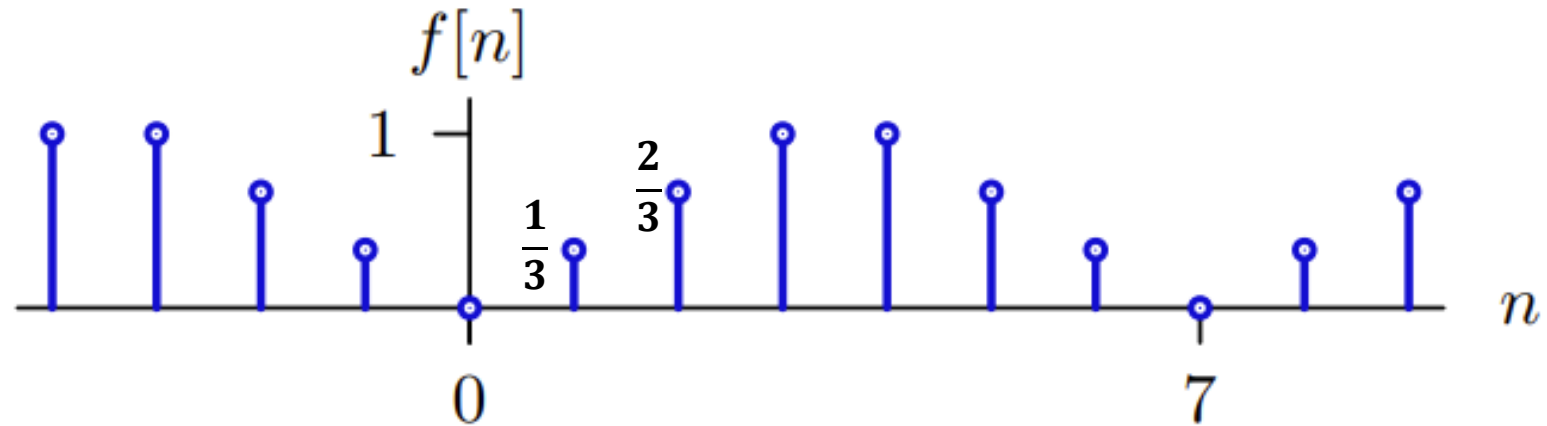
$$x[n] = 0.5$$



Find the DT Fourier Series Coefficients

Consider a new signal $g[n]$ derived from $f[n]$ as follows:

$$g[n] = 9 - 3f[n - 1]$$



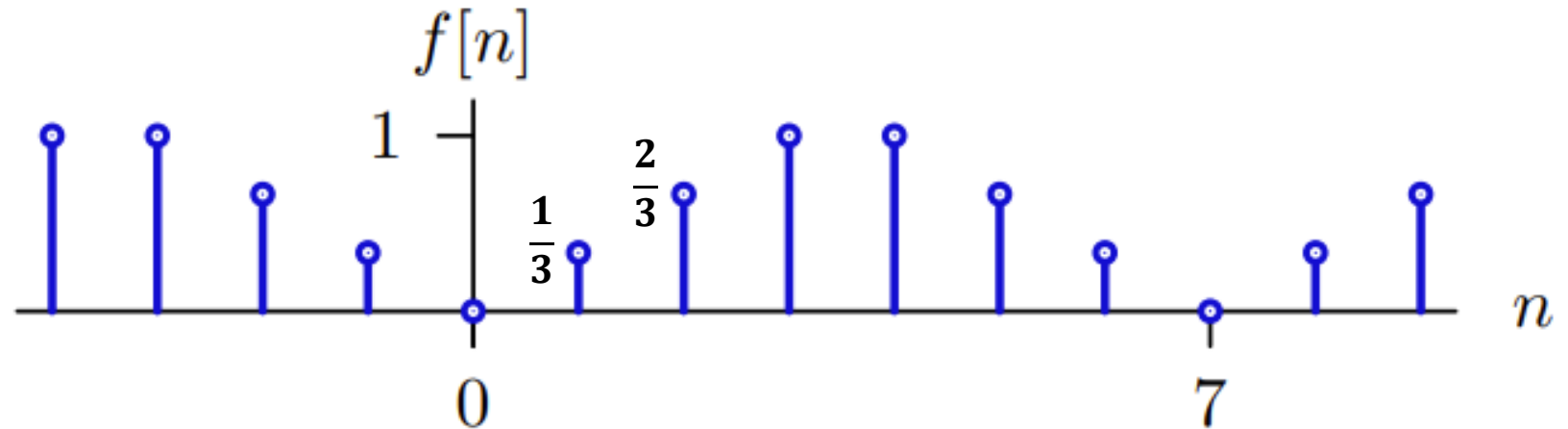
Find the DTFS coefficients of $g[n]$.

Find the DT Fourier Series Coefficients

Consider another new signal

$$h[n] = (-1)^n f[n]$$

where



Find the DTFS coefficients of $h[n]$.