# 6.3000 Quiz 2 Fall 2026

**Name:**

**Kerberos (Athena) username:**

## Please WAIT until we tell you to begin.

This quiz is closed book, but you may use two $8.5 \times 11$ sheets of **handwritten** notes (four sides).

**You may NOT use any electronic devices (such as calculators and phones).**

If you have questions, please **come to us** at the front of the room to ask.

### Please enter all answers in the boxes provided.

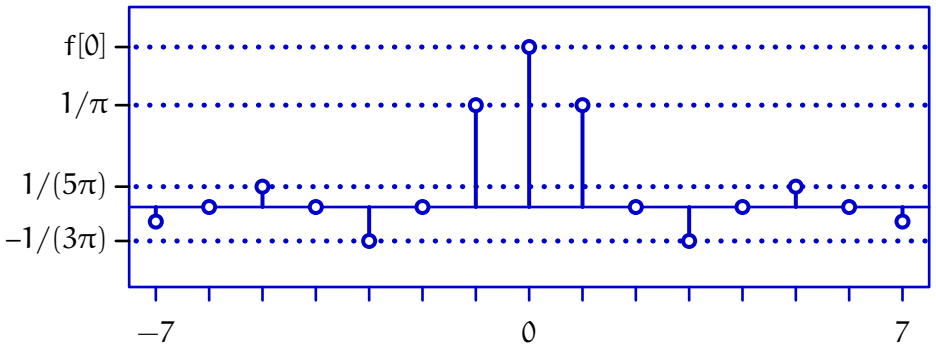
Work in other places may be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam.

### Trigonometric Identities Reference

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$
$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$$
$$\cos(a+b)-\cos(a-b) = -2\sin(a)\sin(b)$$
$$\sin(a+b)+\sin(a-b) = 2\sin(a)\cos(b)$$
$$\sin(a+b)-\sin(a-b) = 2\cos(a)\sin(b)$$
$$2\cos(a)\cos(b) = \cos(a-b)+\cos(a+b)$$
$$2\sin(a)\sin(b) = \cos(a-b)-\cos(a+b)$$
$$2\sin(a)\cos(b) = \sin(a+b)+\sin(a-b)$$
$$2\cos(a)\sin(b) = \sin(a+b)-\sin(a-b)$$

# 1 Mystery Filter

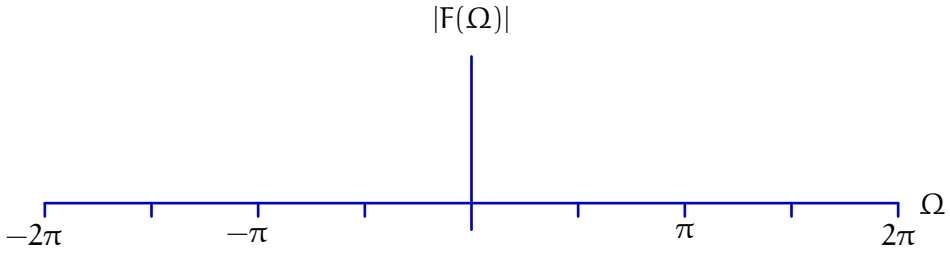
Let $f[n]$ represent the unit sample response of an ideal lowpass filter. The following figure shows the portion of $f[n]$ that falls in the range $-7 \leqslant n \leqslant 7$:




**Part a.** Determine the cutoff frequency $\Omega_c$ of this lowpass filter:

$\Omega_c =$ ____

and sketch the magnitude of the frequency response $|F(\Omega)|$ for frequencies $-2\pi \leqslant \Omega \leqslant 2\pi$ on the axes below. Label the important parameters of your sketch.




Find a closed-form expression for $f[n]$ and enter it in the following box:

$f[n] =$ ____

Determine the numerical value of $f[0]$ and enter it in the box below:

$f[0] =$ ____

**Part b.** Let $p[n]$ represent a discrete-time signal whose discrete-time Fourier Transform (DTFT) is given by

$$P(\Omega) = F(\Omega) - 1$$

where $F(\Omega)$ represents the low-pass frequency response found in part a. Determine an expression for $p[n]$ in terms of $f[n]$ and enter this expression in the box below.

$p[n] =$

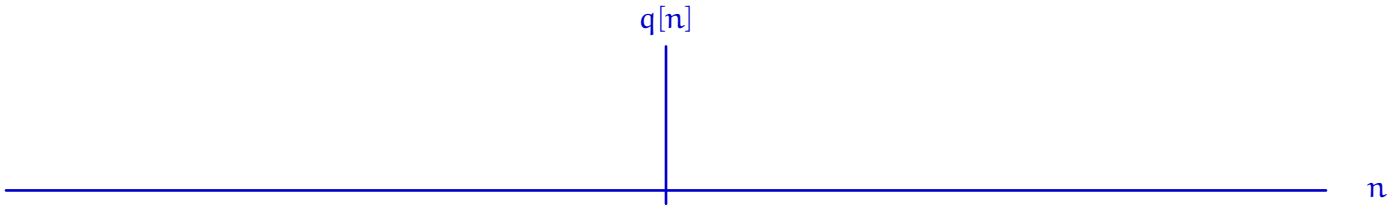
**Part c.** Let $q[n]$ represent a discrete-time signal whose discrete-time Fourier Transform (DTFT) is given by

$$Q(\Omega) = F(3\Omega) \times \Big(1+2\cos(\Omega)\Big)$$

where $F(\Omega)$ represents the low-pass frequency response found in part a. Determine a closed-form expression for $q[n]$ and enter it in the box below:

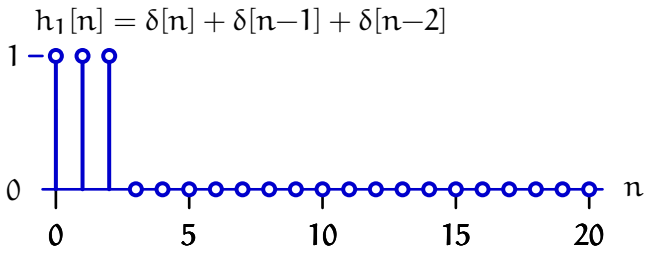
$q[n] =$

Sketch $q[n]$ on the axes below. Show a range of at least $-14 < n < 14$. Label the important parameters of your sketch.



# 2 Discrete-Time Signals and Systems

Let $S_1$ represent a linear, time-invariant system with unit-sample response $h_1[n]$.




**Part a.** The input/output relation for system $S_1$ can be represented by the following linear difference equation

$$y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

where $x[n]$ represents the input to system $S_1$ and $y[n]$ represents the resulting output of that system.

Determine the numerical values of $a_1$, $a_2$, $b_0$, $b_1$, $b_2$ and enter those values in the boxes below:

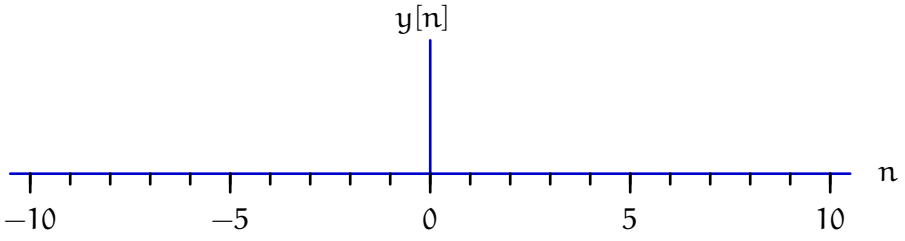
$a_1 =$ ____ $a_2 =$ ____

$b_0 =$ ____ $b_1 =$ ____ $b_2 =$ ____

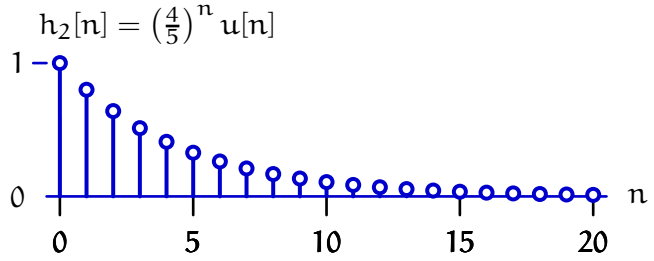
**Part b.** Determine the output $y[n]$ of system $S_1$ when the input to that system is

$$x[n] = \begin{cases} \delta[n \bmod 3] & n \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

Sketch the output $y[n]$ for $-10 \leqslant n \leqslant 10$ on the axes below.



Let $S_2$ represent a linear, time-invariant system with unit-sample response $h_2[n]$.

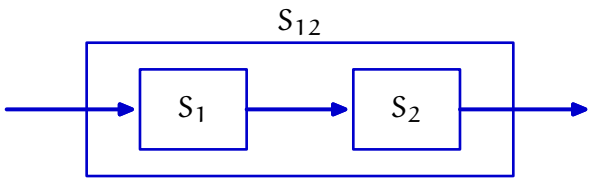



Notice that $h_2[n] = 0$ for all $n < 0$.

**Part c.** Find the frequency response $H_2(\Omega)$ of system $S_2$ and enter an expression for $H_2(\Omega)$ in the box below:

$H_2(\Omega) =$

**Part d.** Let $S_{12}$ represent a system in which $S_1$ and $S_2$ are connected in cascade as shown below:




Determine the frequency response $H_{12}$ of the system $S_{12}$ and enter an expression for $H_{12}$ in the box below.

$H_{12}(\Omega) =$

Let $h_{12}[n]$ represent the unit-sample response of the system $S_{12}$. Enter expressions for the first four values of $h_{12}[n]$ in the boxes below.

$h_{12}[0] =$ $h_{12}[1] =$

$h_{12}[2] =$ $h_{12}[3] =$

**Part e.** If the input to system $S_{12}$ is

$$x[n] = (-1)^n$$

for all time $(-\infty < n < \infty)$ then the output can be expressed as follows:

$$y[n] = \lambda(-1)^n$$

Determine $\lambda$ and enter its numerical value in the box below:

$\lambda =$ [ ]

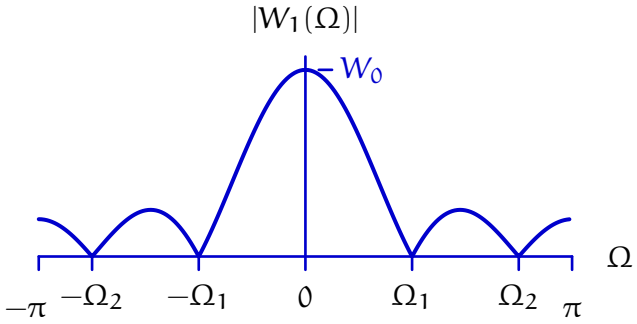
# 3 From DTFT to DFT

An aperiodic discrete-time signal $w_1[n]$ is described as follows:

$$w_1[n] = \begin{cases} 1 & 0 \leqslant n < M \\ 0 & \text{otherwise} \end{cases}$$

where $M$ is a positive integer.

**Part a.** The magnitude of the discrete-time Fourier transform (DTFT) of $w_1[n]$ is shown below for the frequency interval $-\pi \leqslant \Omega \leqslant \pi$:




Determine the value of $M$ and enter that value in the box below:

$M =$ ☐

Determine the value of $W_0$ and enter that value in the box below:

$W_0 =$ ☐

Determine the value of $\Omega_1$ and enter it in the box below:

$\Omega_1 =$ ☐

Determine the value of $\Omega_2$ and enter it in the box below:

$\Omega_2 =$ ☐

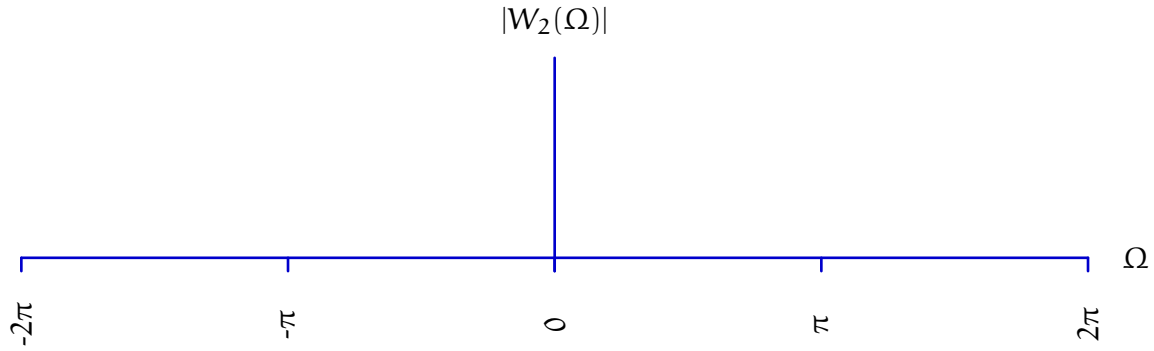
**Part b.** A new signal $w_2[n]$ is derived from $w_1[n]$ as follows:

$$w_2[n] = \begin{cases} w_1[n] & \text{if } n \text{ is even} \\ -w_1[n] & \text{if } n \text{ is odd} \end{cases}$$

Find an expression for $W_2(\Omega)$ in terms of $W_1(\Omega)$, and enter your expression in the box below:

$W_2(\Omega) =$

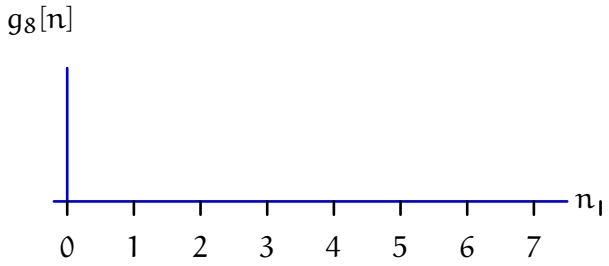
Plot the magnitude of $W_2(\Omega)$ over the range $-2\pi \leqslant \Omega \leqslant 2\pi$ on the axes below. Label all of the important features of your sketch.



**Part c.** Let $F_8[k]$ denote the DFT of $w_1[n]$ computed with an analysis window length $N = 8$. Determine the values of $F_8[k]$ for $k = 0, 2, 4$ and enter those values in the boxes below:

$F_8[0] =$ $F_8[2] =$ $F_8[4] =$

**Part d.** Let $g_8[n]$ denote the signal with DFT $G_8[k] = F_8[k] \times F_8[k]$. The analysis window has length $N = 8$. Sketch $g_8[n]$ for $0 \leqslant n < 8$ on the axes provided below.



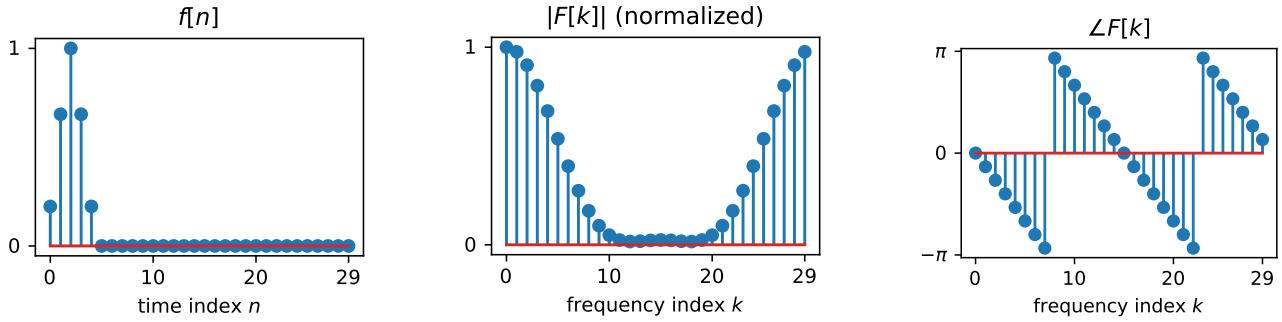

Enter the numerical values of $g_8[n]$ for $0 \leqslant n < 8$ in the boxes below:

$g_8[0] =$ $g_8[1] =$ $g_8[2] =$ $g_8[3] =$

$g_8[4] =$ $g_8[5] =$ $g_8[6] =$ $g_8[7] =$

# 4 Related DFTs

Let $f[n]$ represent a periodic discrete-time signal with period $N = 30$. The following plots show $f[n]$ along with the magnitude and angle of its Discrete Fourier Transform (DFT) computed with an analysis window of length N=30. The magnitude plot has been scaled so that the maximum magnitude is 1.




Six signals ($g_1[n]$ through $g_6[n]$) are derived from $f[n]$ as shown below.

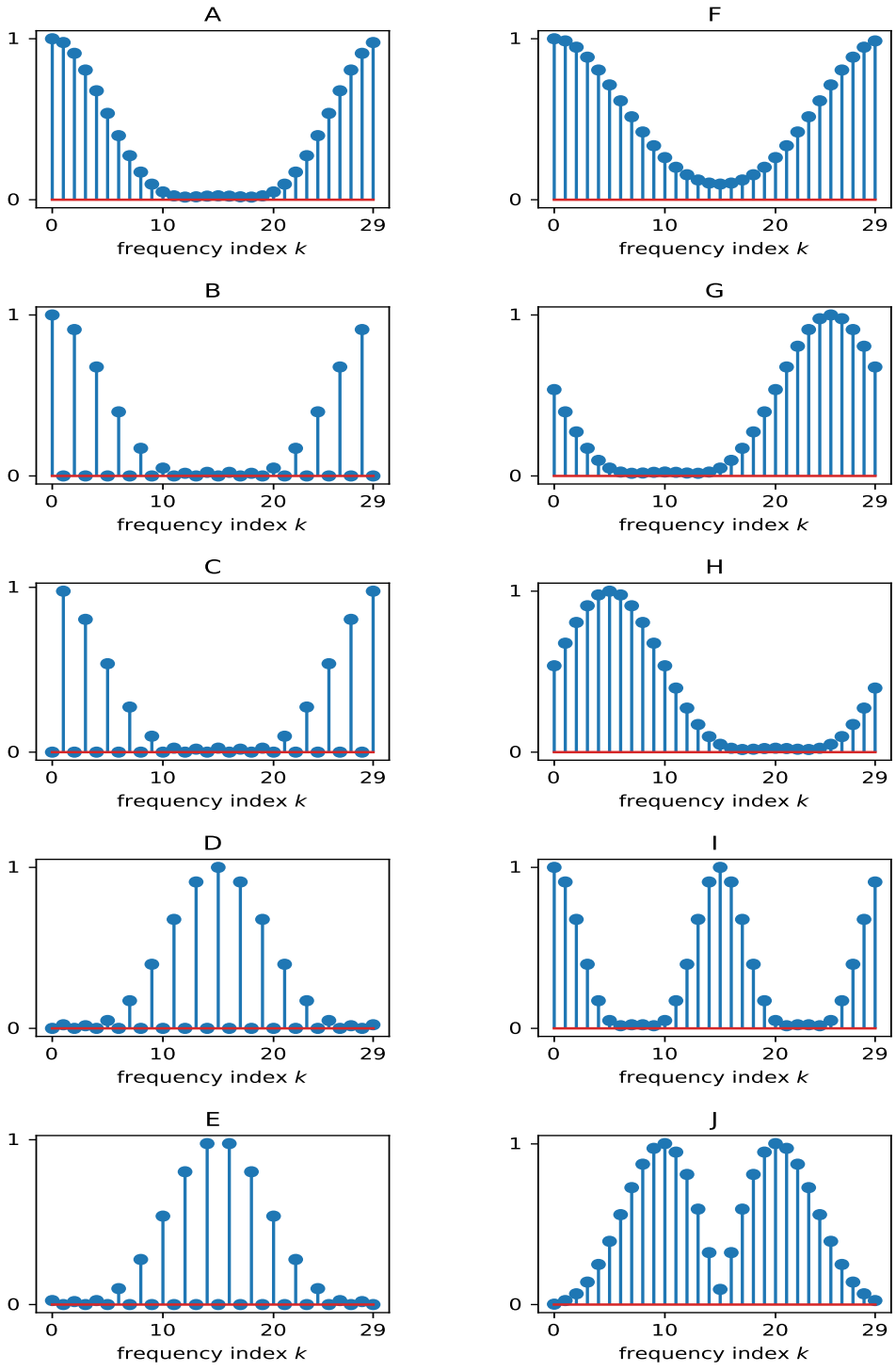
| | normalized magnitude | angle |
|---|---|---|
| $g_1[n] = f[-n]$ | | |
| $g_2[n] = f[n-1]$ | | |
| $g_3[n] = f^2[n]$ | | |
| $g_4[n] = e^{j\pi n/3} f[n]$ | | |
| $g_5[n] = \big(f[n] + f[(n-15)]\big)/2$ | | |
| $g_6[n] = \begin{cases} f[n/2] & \text{if } n \text{ is even} \\ 0 & \text{otherwise} \end{cases}$ | | |

Determine which of the panels (A-J) on the following page shows the magnitude of the DFT of each of the derived signals and write the letter for that panel in the corresponding box in the magnitude column above. If none of panels A-J shows the correct magnitude plot, enter **X**.

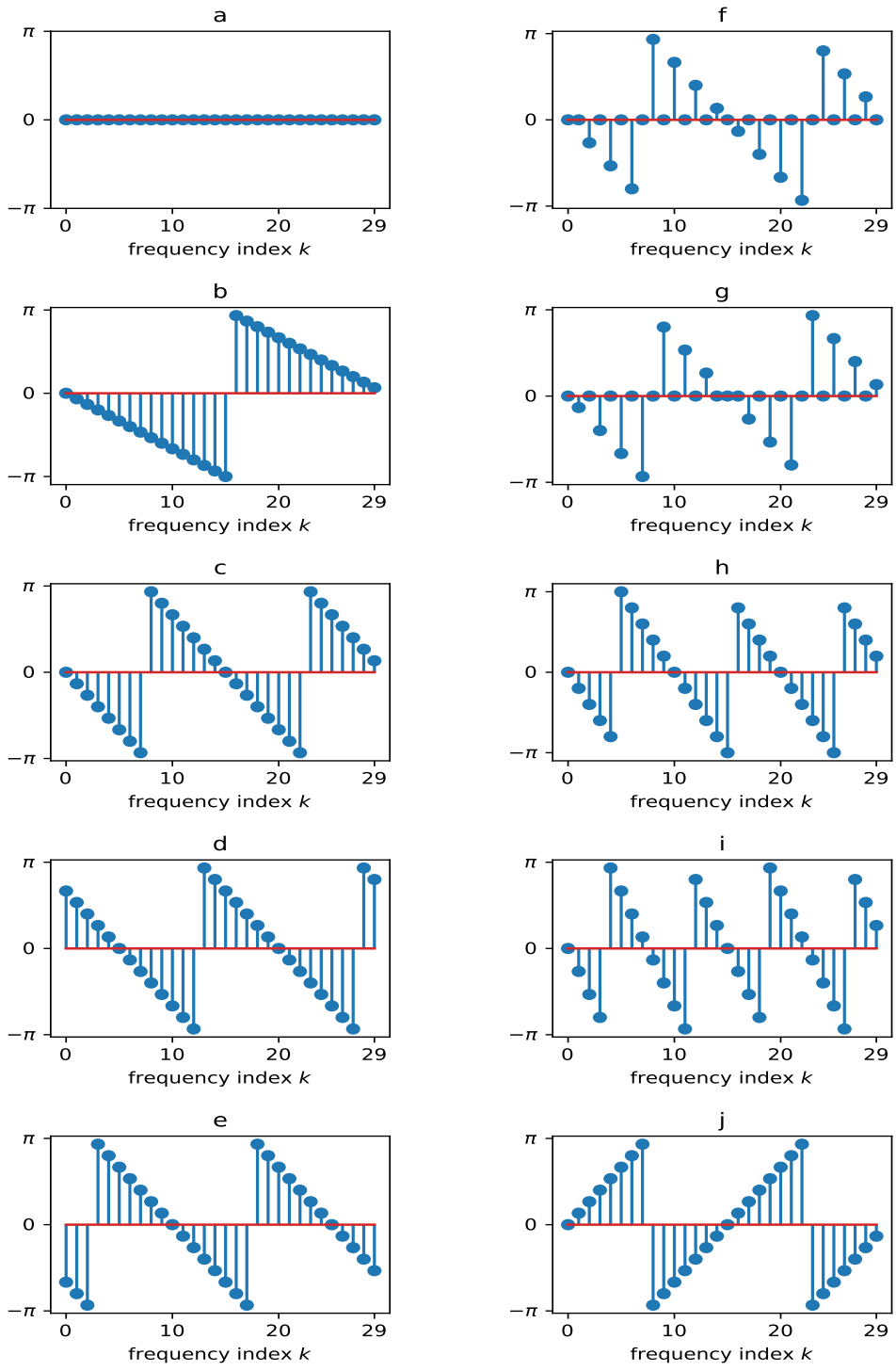
Similarly, determine which of the panels (a-j) on the subsequent page shows the angle of the DFT of each of the derived signals and write the letter for that panel in the corresponding box in the angle column above. If none of panels a-j shows the correct angle plot, enter **X**.

The same panel may be used more than once.

Magnitude Plots for Problem 4: the magnitudes in each plot have been scaled so that the largest magnitude in each plot is 1.



Angle Plots for Problem 4.



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