

6.300 Quiz 1

Fall 2026

Name:

MIT ID:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use one hand-written 8.5×11 sheet of notes.

You may NOT use any electronic devices (such as calculators and phones).

Show your work in the space below each part, and enter your final answer in the box provided.

Work on the worksheet pages at the end of this booklet will be considered for partial credit if you point to it from the answer box. Do not remove any pages and do not hand in loose sheets: only this booklet is scanned.

Trigonometric Identities Reference

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a) + \cos(b) = 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

$$\sin(a) + \sin(b) = 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$$

$$\cos(a+b) + \cos(a-b) = 2 \cos(a) \cos(b)$$

$$\sin(a+b) + \sin(a-b) = 2 \sin(a) \cos(b)$$

$$2 \cos(a) \cos(b) = \cos(a-b) + \cos(a+b)$$

$$2 \sin(a) \cos(b) = \sin(a+b) + \sin(a-b)$$

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

$$\cos(a) - \cos(b) = -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)$$

$$\sin(a) - \sin(b) = 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)$$

$$\cos(a+b) - \cos(a-b) = -2 \sin(a) \sin(b)$$

$$\sin(a+b) - \sin(a-b) = 2 \cos(a) \sin(b)$$

$$2 \sin(a) \sin(b) = \cos(a-b) - \cos(a+b)$$

$$2 \cos(a) \sin(b) = \sin(a+b) - \sin(a-b)$$

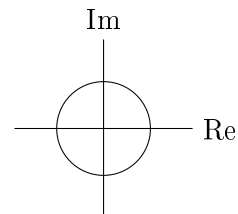
1. Simplify & Locate (20 points)

In each of the following, a complex number is given. Write the simplified form, either in polar or rectangular format, and mark its value on the complex plane with a dot. If an expression has more than one possible value, giving any one of them is fine.

(1)

$$\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)^j$$

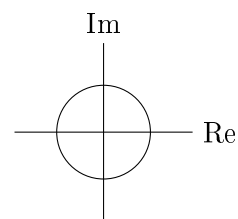
Simplified expression:



(2)

$$\sum_{n=0}^8 e^{j\frac{\pi}{5}n}$$

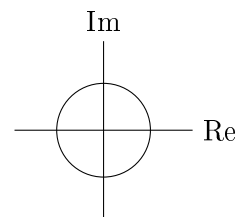
Simplified expression:



(3)

$$\left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{6}\right)\right)^4$$

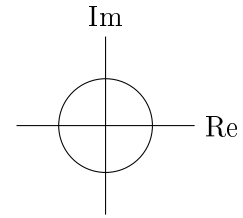
Simplified expression:



(4)

$$\sqrt[5]{\left(\frac{\sqrt{2}}{2}(1+j)\right)^8}$$

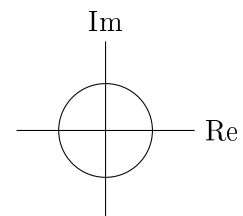
Simplified expression:



(5)

$$\text{Im} \left(\frac{1}{2e^{j(\tan^{-1}(2)+\tan^{-1}(3))}} \right)$$

Simplified expression:



- $y =$

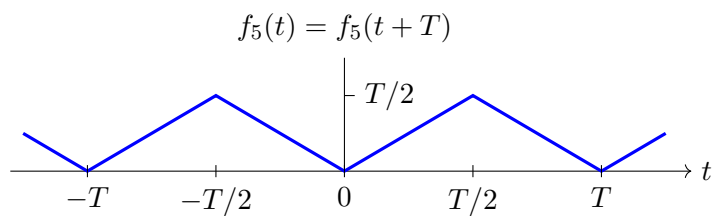
$$F_3[k] =$$

-

$$F_4[k] =$$

k	-3	-2	-1	0	1	2	3
$F_4[k]$							

- (5) For the following signal $f_5(t)$, find its relationship with $f_1(t)$, express its Fourier series coefficients $F_5[k]$ in terms of $F_1[k]$, and enter the values of $F_5[k]$ for k from -3 to 3 below.



Expression describing the relationship of $f_5(t)$ with $f_1(t)$:

$F_5[k] =$

k	-3	-2	-1	0	1	2	3
$F_5[k]$							

4. Discrete Varieties (30 points)

- (1) Signal $x[n] = 1 + \sin\left(\frac{2\pi n}{5}\right) + 2\cos\left(\frac{2\pi n}{5}\right) + 3\cos\left(\frac{4\pi n}{5} + \frac{\pi}{2}\right)$. Find its DT Fourier series coefficients when computed with period $N = 5$, and fill in the $X[k]$ values for the corresponding k in the table below.

k	0	1	2	3	4	5	6	7	8	9
$X[k]$										

- (2) If the same signal $x[n]$ above is computed with period $N = 10$, find the corresponding Fourier series coefficients $X_{10}[k]$ and fill in the table below.

k	0	1	2	3	4	5	6	7	8	9
$X_{10}[k]$										

- (3) Consider a time-stretched version of the signal, $y[n] = \begin{cases} x\left[\frac{n}{2}\right], & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$, computed with period $N = 10$. Find the corresponding Fourier series coefficients $Y_{10}[k]$ and fill in the table below.

k	0	1	2	3	4	5	6	7	8	9
$Y_{10}[k]$										

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Worksheet (intentionally blank)