

6.300: Signal Processing

Discrete Fourier Transform 1

- Discrete Fourier Transform (DFT)
- Relations to Discrete-Time Fourier Series (DTFS)
- Relations to Discrete-Time Fourier Transform (DTFT)

Yet Another Fourier Representation

Do we really need another Fourier Representation?

Fourier series represent signals as sums of sinusoids.

$$x[n] = x[n+N] = \sum_{k=\langle N \rangle} X[k] e^{j \frac{2\pi k}{N} n}$$

Only sinusoids that are periodic in time (n) with period N are useful for constructing signals that are periodic in N . To be periodic in N , the frequency of a sinusoid must be an integer (k) multiple of the fundamental frequency ($2\pi/N$). Because of aliasing, there are only N such unique frequencies.

Fourier transforms have no periodicity constraints.

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega$$

Signals may include components at any real-valued frequency. Therefore the synthesis formula must include all possible real-valued frequencies.

Discrete Fourier Transform: discrete frequencies for aperiodic signals.

Yet Another Fourier Representation

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$$x[n] = x[n+N] = \sum_{k=\langle N \rangle} X[k] e^{j \frac{2\pi k}{N} n} \leftarrow \text{limited to periodic signals}$$

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Fourier transforms have no periodicity constraints.

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega \leftarrow \text{inconvenient for numerical methods}$$

Signals may include components at any real-valued frequency. Therefore the synthesis formula must include all possible real-valued frequencies.

Discrete Fourier Transform: discrete frequencies for aperiodic signals.

Discrete Fourier Transform

Definition and comparison to other Fourier representations.

analysis

synthesis

DTFS:
$$X[k] = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-j \frac{2\pi k}{N} n}$$

$$x[n] = \sum_{k=\langle N \rangle} X[k] e^{j \frac{2\pi k}{N} n}$$

DTFT:
$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j \Omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j \Omega n} d\Omega$$

DFT:
$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi k}{N} n}$$

$$x[n] = \sum_{k=\langle N \rangle} X[k] e^{j \frac{2\pi k}{N} n}$$

Major differences:

DTFS: $x[n]$ must be periodic in n ; N is the period of $x[n]$

DTFT: $x[n]$ is arbitrary; there is no N

DFT: only a portion of $x[n]$ is considered; N is length of that portion

Relation Between DFT and DTFS

If we compute the DFT with a window length N that is equal to the period of a periodic signal, then the DFT and DTFS coefficients are equal.

Let $x_1[n] = \cos \frac{2\pi n}{64}$. Then the DFT coefficients computed with $N=64$ are

$$X_1[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn} = \frac{1}{2} \delta[k-1] + \frac{1}{2} \delta[k-63]$$

as plotted below.



These DFT coefficients are the same as the Fourier series coefficients.

Check Yourself

If a signal is not periodic in the DFT window length $N=64$, then there are no Fourier series coefficients to compare.

Let $x_2[n] = \cos \frac{3\pi n}{64}$.

Is $x_2[n]$ periodic in $N=64$?

What is the period of $x_2[n]$?

1. $N = 32$
2. $N = 64$
3. $N = 128$
4. $N = 64/3$
5. $N = 64 \times 3$
0. none of the above

Check Yourself

If a signal is not periodic in the DFT window length $N=64$, then there are no Fourier series coefficients to compare.

Let $x_2[n] = \cos \frac{3\pi n}{64}$. Is $x_2[n]$ periodic in $N=64$? What is the period of $x_2[n]$?

If $x_2[n]$ is periodic with period N , then

$$x_2[n] = x_2[n+N] \quad \forall n$$

$$\cos\left(\frac{3\pi n}{64}\right) = \cos\left(\frac{3\pi(n+N)}{64}\right)$$

$$\frac{3\pi N}{64} = m2\pi$$

where m is an integer.

$$N = m \frac{128}{3}$$

The smallest m that makes sense is $m=3$ for which $N = 128$.

Therefore, $x_2[n]$ is **not** periodic in $N = 64$.

Check Yourself

If a signal is not periodic in the DFT window length $N=64$, then there are no Fourier series coefficients to compare.

Let $x_2[n] = \cos \frac{3\pi n}{64}$.

Is $x_2[n]$ periodic in $N=64$? **no**

What is the period of $x_2[n]$? **3. $N=128$**

1. $N = 32$
2. $N = 64$
3. **$N = 128$**
4. $N = 64/3$
5. $N = 64 \times 3$
0. none of the above

Relation Between DFT and DTFS

If a signal is not periodic in the DFT window length $N=64$, then there are no Fourier series coefficients to compare.

Let $x_2[n] = \cos \frac{3\pi n}{64}$. Then if $N=64$, the DFT coefficients are

$$X_2[k] = \frac{1}{N} \sum_{n=0}^{N-1} x_2[n] e^{-j\frac{2\pi}{N}kn}$$

are plotted below.

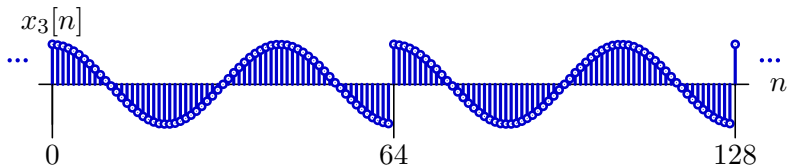


Even though $x_2[n]$ contains a single frequency $\Omega = 3\pi/64$, there are large coefficients at many different frequencies k .

The reason is that $x_2[n]$ is not periodic in n with period $N = 64$.

Relation Between DFT and DTFS

Although $x_2[n] = \cos \frac{3\pi n}{64}$ is not periodic in $N=64$, we can define a signal $x_3[n]$ that is equal to $x_2[n]$ for $0 \leq n < 64$ and that is periodic in $N=64$.



The DTFS coefficients for this signal are the same as the DFT coefficients for $x_2[n]$:



The large number of non-zero coefficients are necessary to produce the step discontinuity at $n = 64$.

Two Ways to Think About the DFT

We just compared the DFT to the DTFS:

1. The DFT of a signal $x[n]$ is equal to the DTFS of a version of $x[n]$ that is **periodically extended** so that it is periodic in N .

→ emphasizes the importance of **periodicity** in time.

2. We can also gain intuition for the DFT by comparing it to the DTFT.

Relation Between DFT and DTFT

The DFT can also be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $\frac{1}{N}$.

Let $x_w[n] = x[n] \times w[n]$ represent a **windowed** version of $x[n]$ where

$$w[n] = \begin{cases} 1 & 0 \leq n < N \\ 0 & \text{otherwise} \end{cases}$$

Then the **Fourier transform** of $x_w[n]$ is

$$X_w(\Omega) = \sum_{n=-\infty}^{\infty} x_w[n] e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} x[n] w[n] e^{-j\Omega n} = \sum_{n=0}^{N-1} x[n] e^{-j\Omega n}$$

Sample the resulting function of Ω at $\Omega = \frac{2\pi k}{N}$:

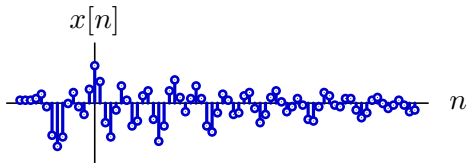
$$X_w\left(\frac{2\pi k}{N}\right) = \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi k}{N}\right)n}$$

Divide both sides by N :

$$\frac{1}{N} X_w\left(\frac{2\pi k}{N}\right) = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi k}{N}\right)n} = X[k], \quad \text{which is the DFT of } x[n]$$

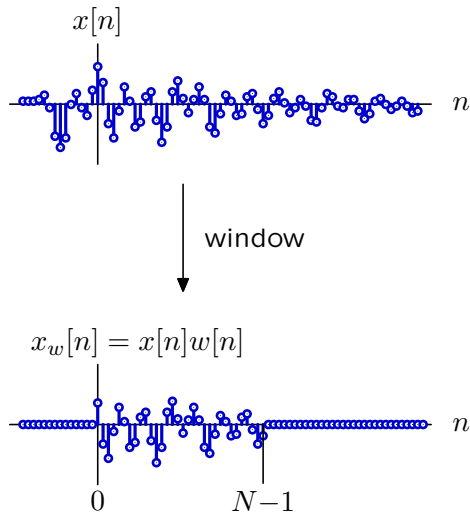
Relation Between DFT and DTFT

Graphical depiction of relation between DFT and DTFT.



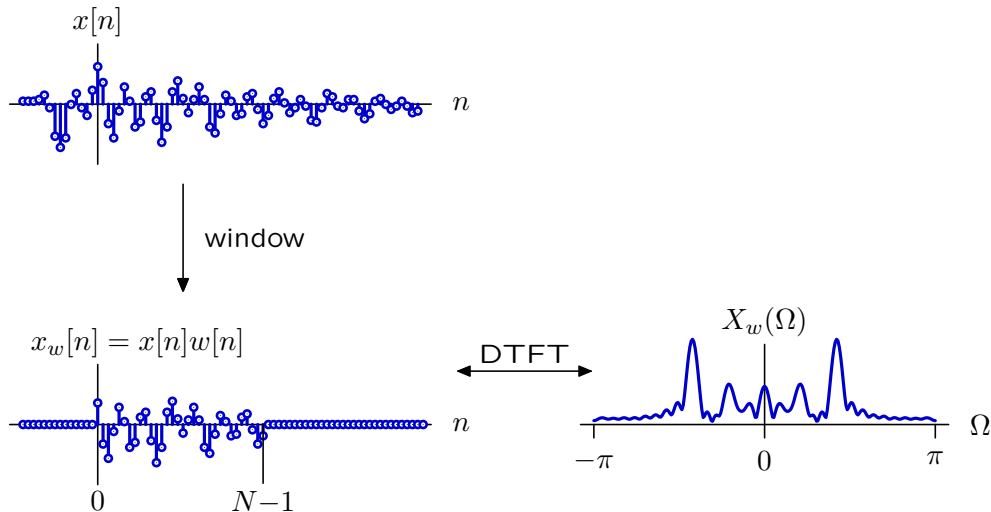
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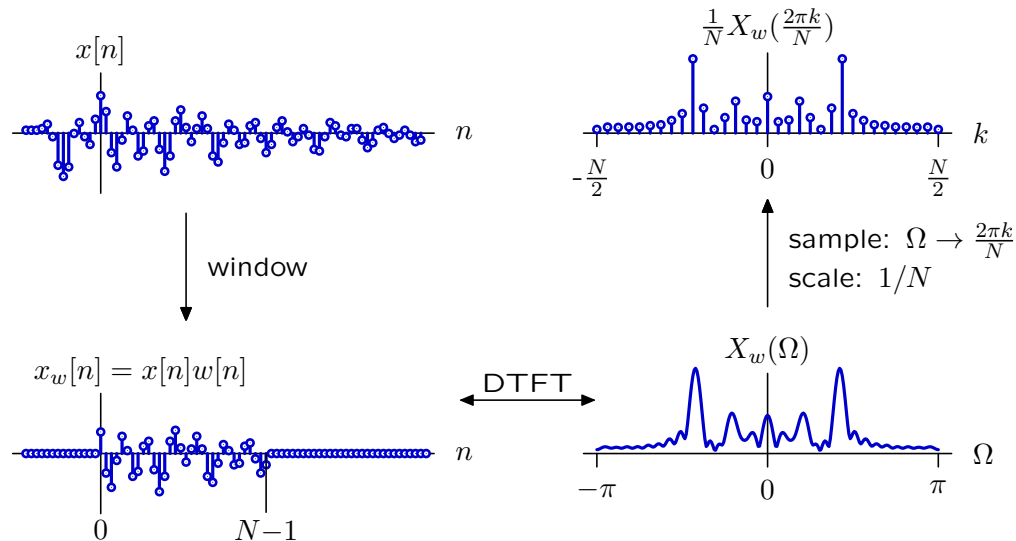
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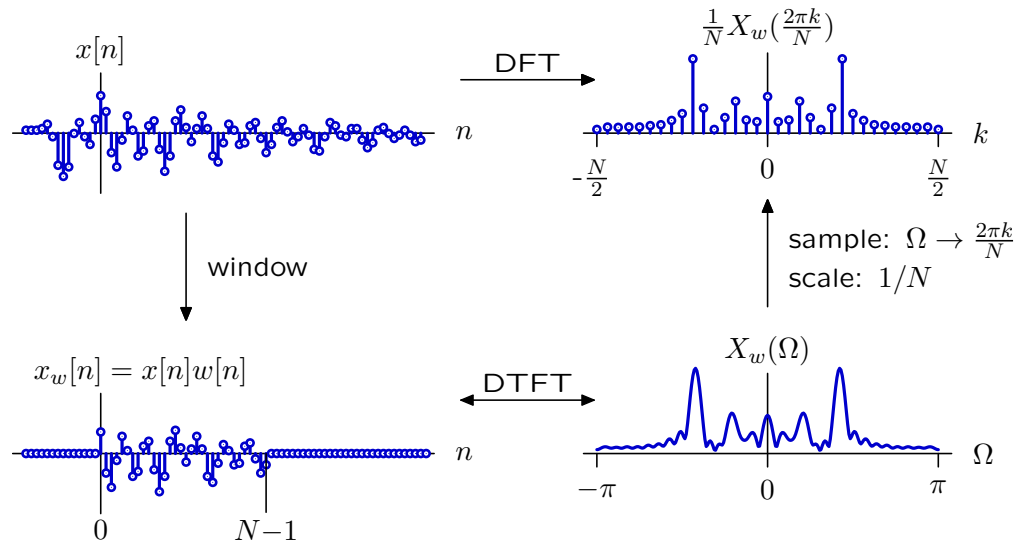
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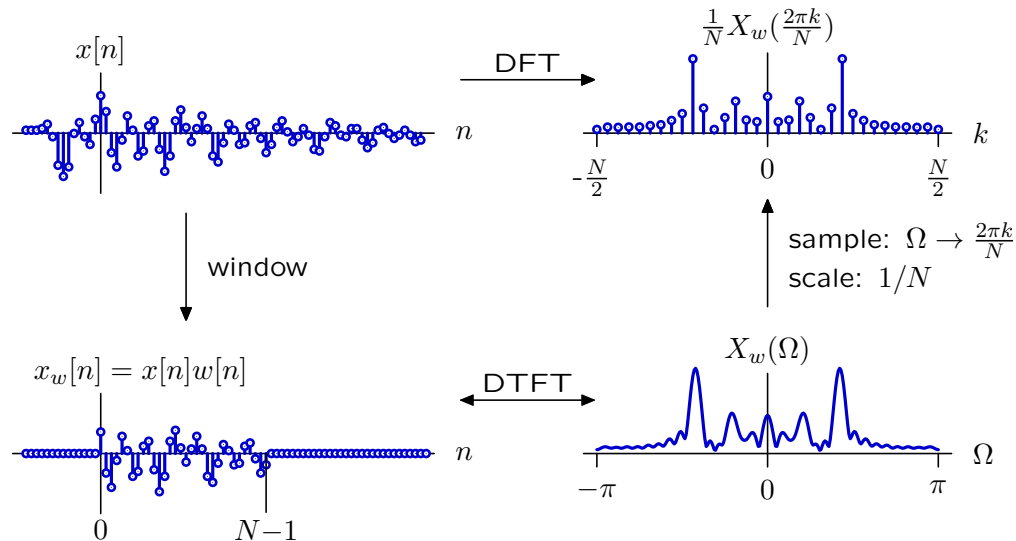
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Graphical depiction of relation between DFT and DTFT.



Relation Between DFT and DTFT

Graphical depiction of relation between DFT and DTFT.



While sampling and scaling are important, it is the **windowing** that most affects frequency content.

Effect of Windowing on Fourier Representations

Determine effects of windowing on signals with a single frequency Ω_o .

Step 1: Find $X(\Omega)$, the DTFT of a complex exponential signal:

$$x[n] = e^{j\Omega_o n}$$

Step 2: Find $X_w(\Omega)$, the DTFT of a windowed version of $x[n]$:

$$x_w[n] = x[n]w[n]$$

Step 3: Compare $X_w(\Omega)$ to $X(\Omega)$.

Effect of Windowing on Fourier Representations

Step 1: Find $X(\Omega)$, the DTFT of a complex exponential signal:

$$x[n] = e^{j\Omega_0 n}$$

Effect of Windowing on Fourier Representations

Step 1: Find $X(\Omega)$, the DTFT of a complex exponential signal:

$$x[n] = e^{j\Omega_o n}$$

The direct form of this transform

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} e^{j\Omega_o n}e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} e^{-j(\Omega-\Omega_o)n}$$

is difficult to evaluate because it requires an infinite sum of nonzero terms.

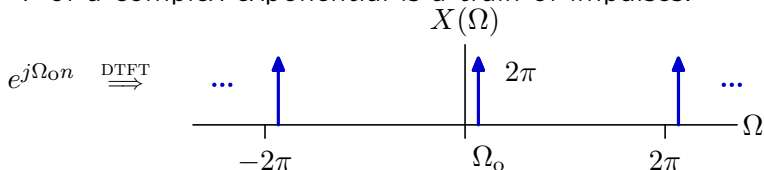
Notice however that all of the energy in this signal is concentrated at a single frequency $\Omega = \Omega_o$. Since there must be an infinite amount of energy concentrated at $\Omega = \Omega_o$, we might suppose that $X(\Omega)$ should contain a delta function at $\Omega = \Omega_o$.

Effect of Windowing on Fourier Representations

Step 1: Find $X(\Omega)$, the DTFT of a complex exponential signal:

$$x[n] = e^{j\Omega_o n}$$

The DTFT of a complex exponential is a train of impulses.



This is easy to verify using the DTFT synthesis equation.

$$\begin{aligned} x[n] &= \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} 2\pi \delta(\Omega - \Omega_o) e^{j\Omega n} d\Omega \\ &= \int_{-\pi}^{\pi} \delta(\Omega - \Omega_o) e^{j\Omega_o n} d\Omega = e^{j\Omega_o n} \int_{-\pi}^{\pi} \delta(\Omega - \Omega_o) d\Omega \\ &= e^{j\Omega_o n} \end{aligned}$$

Effect of Windowing on Fourier Representations

Step 2: Find $X_w(\Omega)$, the DTFT of a windowed version of $x[n]$:

$$x_w[n] = x[n]w[n] = e^{j\Omega_o n}w[n]$$

Effect of Windowing on Fourier Representations

Step 2: Find $X_w(\Omega)$, the DTFT of a windowed version of $x[n]$:

$$x_w[n] = x[n]w[n] = e^{j\Omega_o n}w[n]$$

$$X_w(\Omega) = \sum_{n=-\infty}^{\infty} e^{j\Omega_o n}w[n]e^{-j\Omega n} \quad \text{DTFT analysis equation}$$

$$= \sum_{n=-\infty}^{\infty} w[n]e^{-j(\Omega - \Omega_o)n} \quad \text{combine exponential terms}$$

$$= W(\Omega - \Omega_o) \quad \text{DTFT of } w[n], \text{ shifted in frequency}$$

The DTFT of a windowed version of a complex exponential signal is a shifted version of the DTFT of the window signal.

$$e^{j\Omega_o n}w[n] \xrightarrow{\text{DTFT}} W(\Omega - \Omega_o)$$

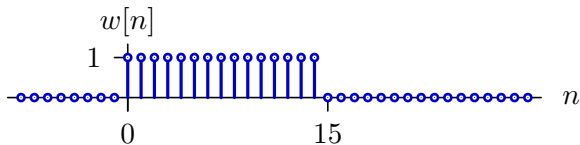
→ Need to know $W(\Omega)$.

Effect of Windowing on Fourier Representations

Simplest window is rectangular, with width of N (length of DFT analysis)

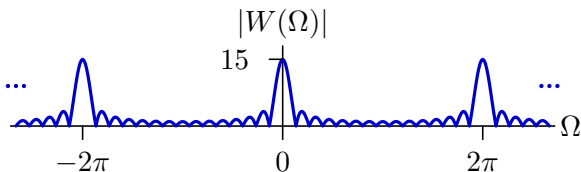
$$w[n] = \begin{cases} 1 & 0 \leq n < N \\ 0 & \text{otherwise} \end{cases}$$

as shown below for $N = 15$.



The DTFT of $w[n]$ is

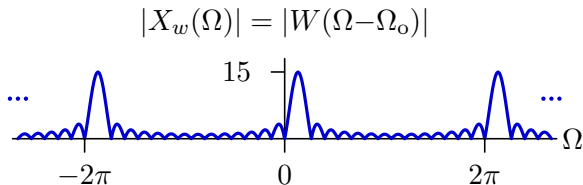
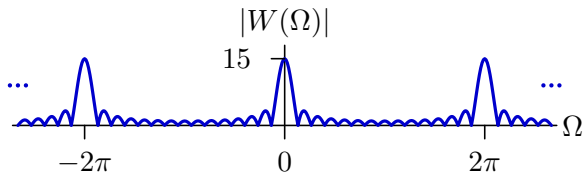
$$W(\Omega) = \sum_{n=-\infty}^{\infty} w[n]e^{-j\Omega n} = \sum_{n=0}^{N-1} e^{-j\Omega n} = \frac{1 - e^{-j\Omega N}}{1 - e^{-j\Omega}} = \frac{\sin \frac{N\Omega}{2}}{\sin \frac{\Omega}{2}} e^{-j\Omega \frac{N-1}{2}}$$



Effect of Windowing on Fourier Representations

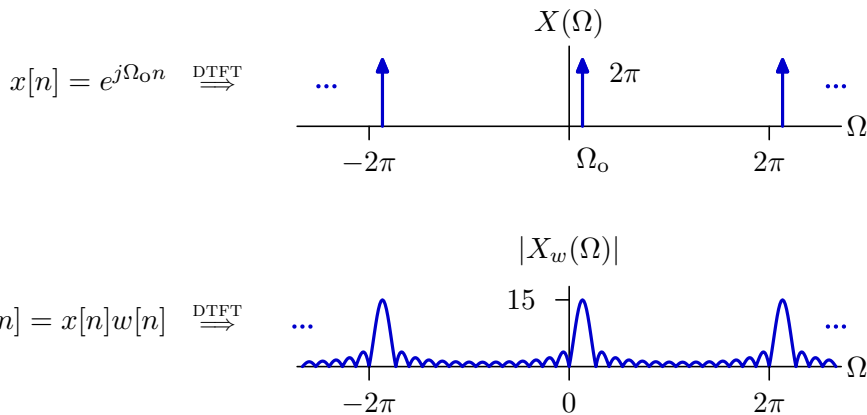
The DTFT of a windowed version of a complex exponential signal is a shifted version of the DTFT of the window signal.

$$x_w[n] = e^{j\Omega_o n} w[n] \xrightarrow{\text{DTFT}} X_w(\Omega) = W(\Omega - \Omega_o)$$



Effect of Windowing on Fourier Representations

Step 3: Compare $X_w(\Omega)$ to $X(\Omega)$.

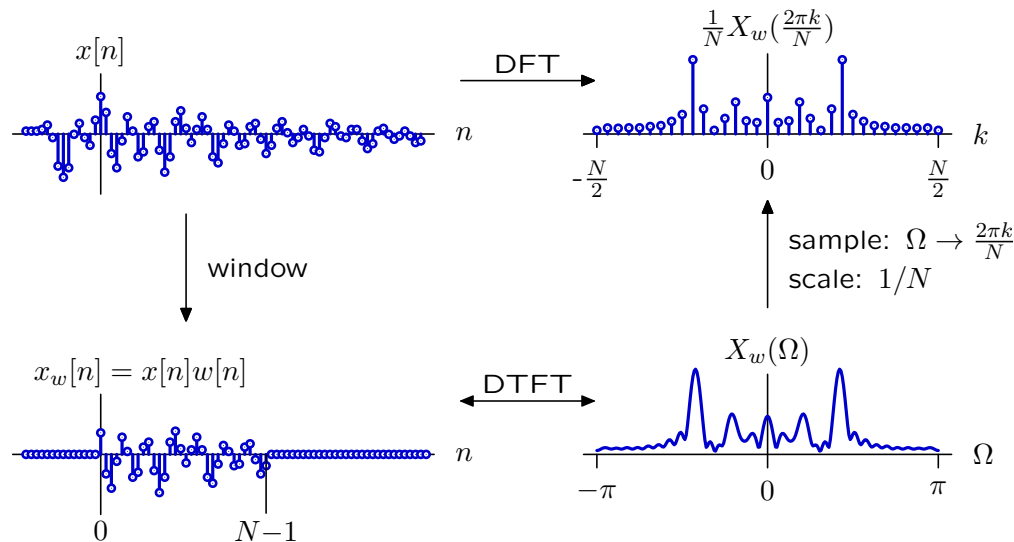


The frequency content of $X(\Omega)$ is at discrete frequencies $\Omega = \Omega_o + 2\pi m$.

The frequency content of $X_w(\Omega)$ is most dense at these same frequencies, but is spread out over almost all other frequencies as well.

Relation Between DFT and DTFT

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$.



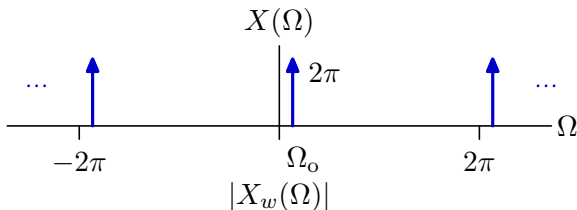
Next: apply these steps to a sinusoidal input.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{2\pi}{15}$.

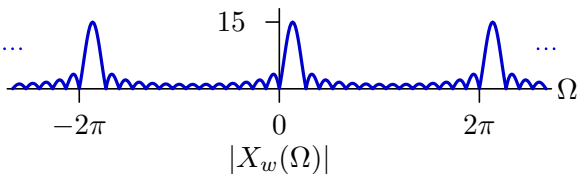
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



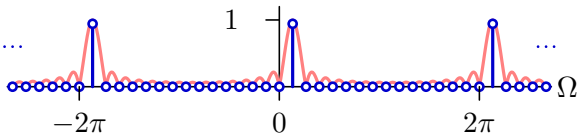
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



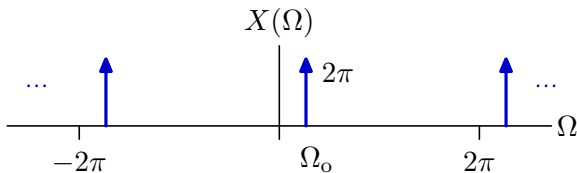
One sample is taken at the peak, and the others fall on zeros.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{4\pi}{15}$.

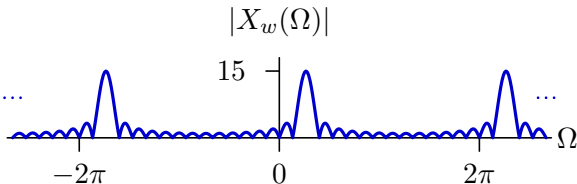
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



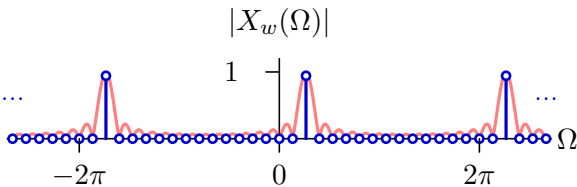
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

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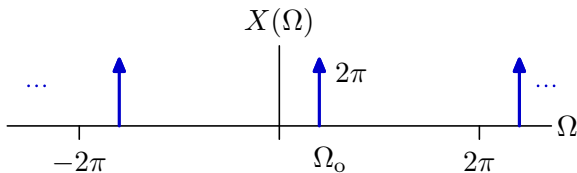
One sample is taken at the peak, and the others fall on zeros.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{6\pi}{15}$.

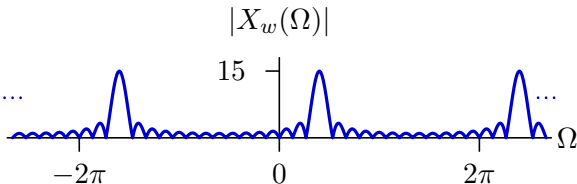
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



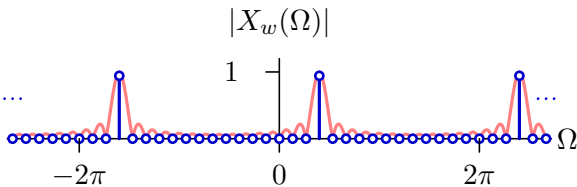
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



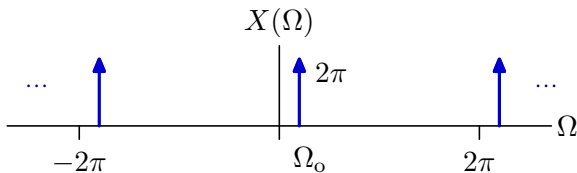
One sample is taken at the peak, and the others fall on zeros.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{3\pi}{15}$.

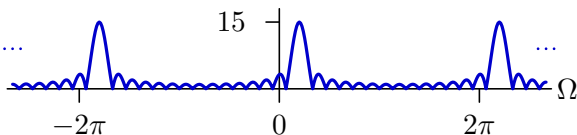
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



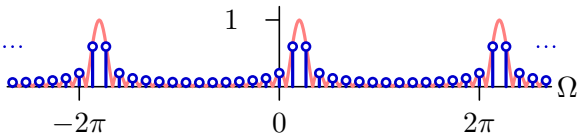
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



scaled and sampled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



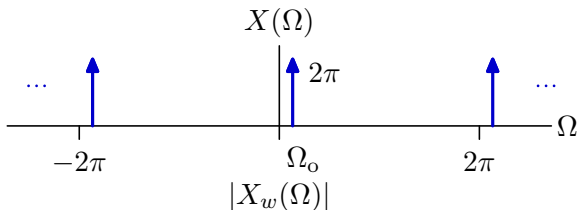
Now none of the samples fall on zeros.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{2\pi}{15}$.

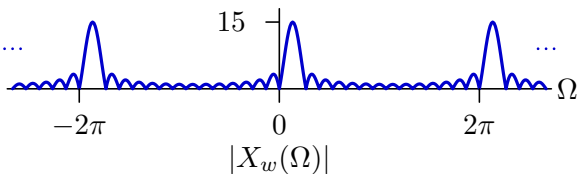
original signal

$$x[n] = e^{j\Omega_o n} \xRightarrow{\text{DTFT}}$$



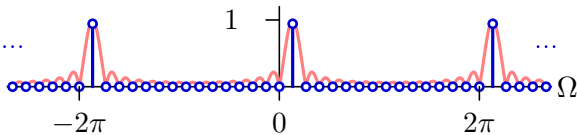
windowed

$$x_w[n] = x[n]w[n] \xRightarrow{\text{DTFT}}$$



sampled and scaled

$$x_w[n] = x[n]w[n] \xRightarrow{\text{DFT}}$$



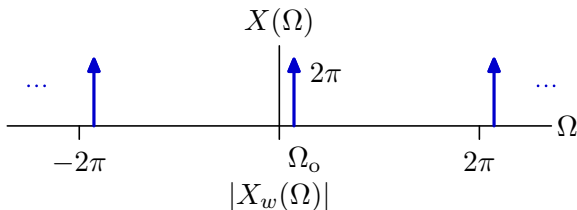
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{2.2\pi}{15}$.

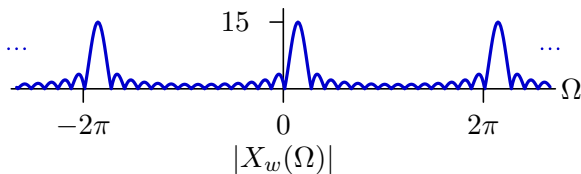
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



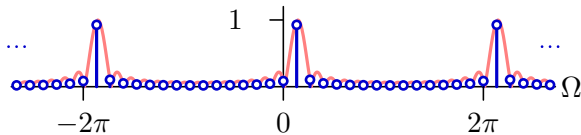
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



scaled and sampled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



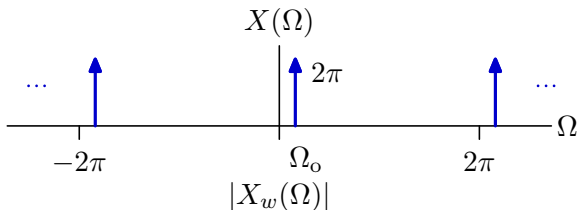
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{2.4\pi}{15}$.

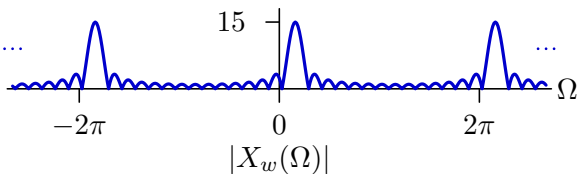
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



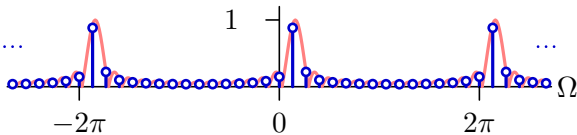
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



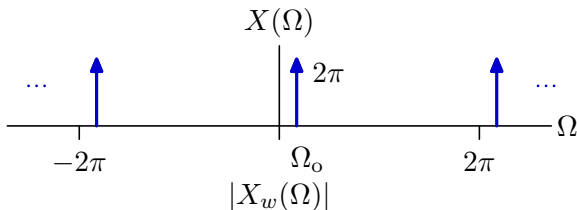
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{2.6\pi}{15}$.

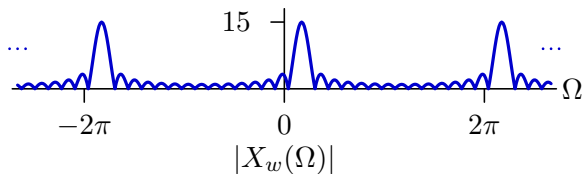
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



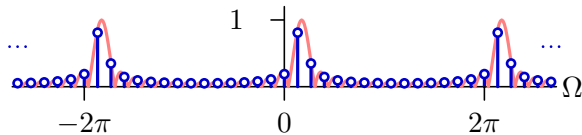
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



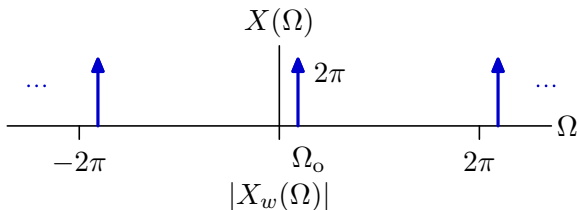
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{2.8\pi}{15}$.

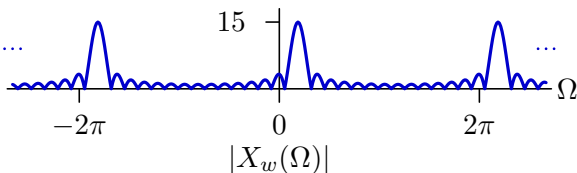
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



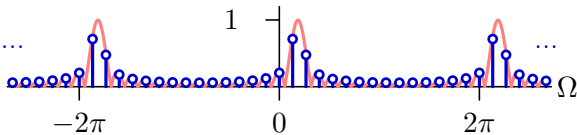
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



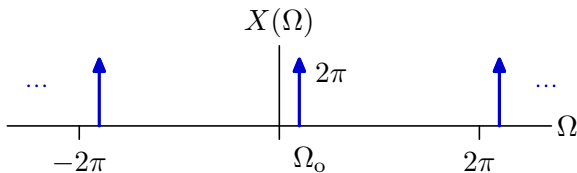
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{3\pi}{15}$.

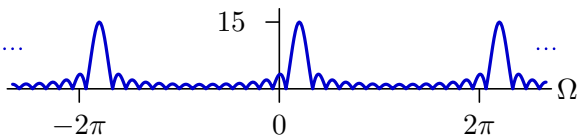
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



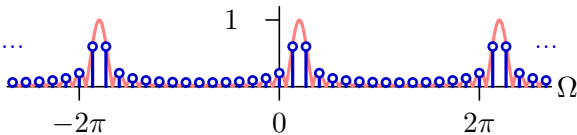
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



scaled and sampled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



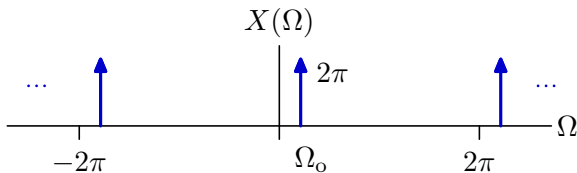
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{3.2\pi}{15}$.

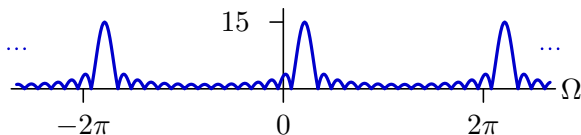
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



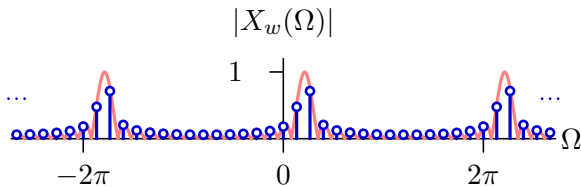
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



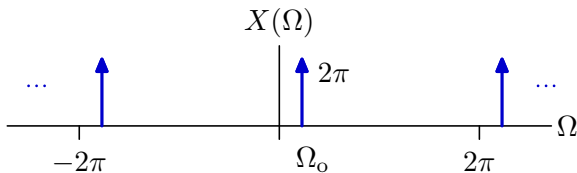
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{3.4\pi}{15}$.

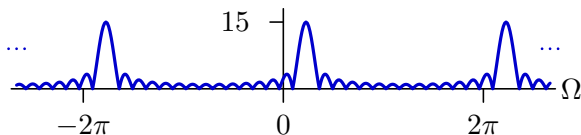
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



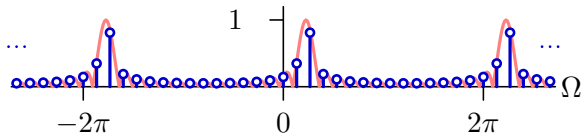
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



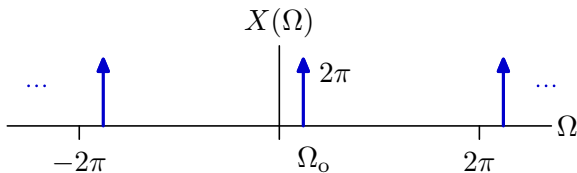
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{3.6\pi}{15}$.

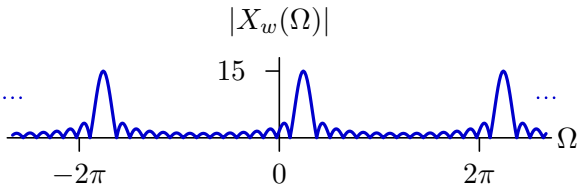
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



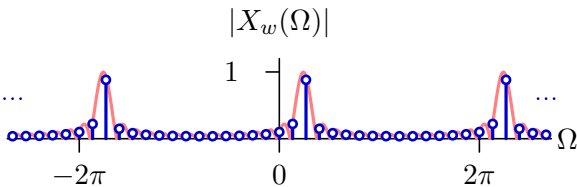
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



scaled and sampled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



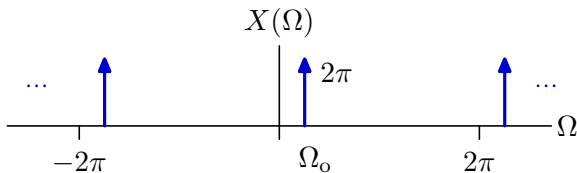
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{3.8\pi}{15}$.

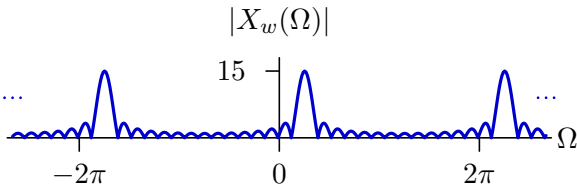
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



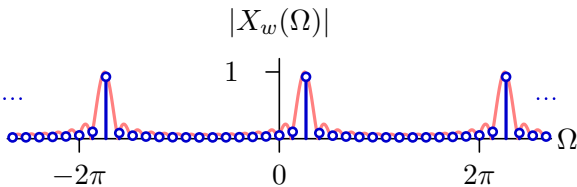
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



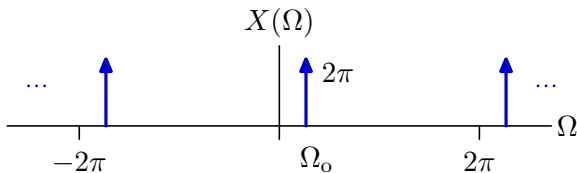
Generally, the relation between the samples is complicated.

Effect of Windowing on Fourier Representations

The DFT can be thought of as **samples** of the DTFT of a **windowed** version of $x[n]$ **scaled** by $1/N$. Here $\Omega_o = \frac{4\pi}{15}$.

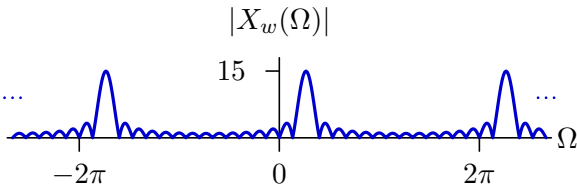
original signal

$$x[n] = e^{j\Omega_o n} \xrightarrow{\text{DTFT}}$$



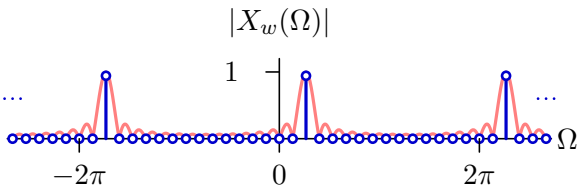
windowed

$$x_w[n] = x[n]w[n] \xrightarrow{\text{DTFT}}$$



sampled and scaled

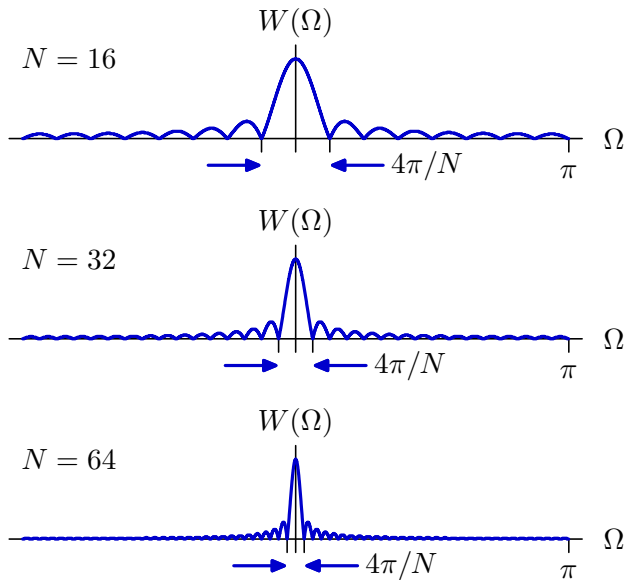
$$x_w[n] = x[n]w[n] \xrightarrow{\text{DFT}}$$



Generally, the relation between the samples is complicated.

Spectral Blurring introduces a Time/Frequency Tradeoff

Longer windows provide finer frequency resolution.

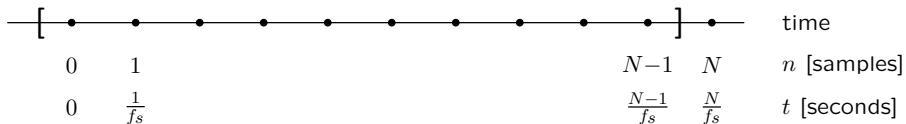


The width of the central lobe is inversely related to window length.

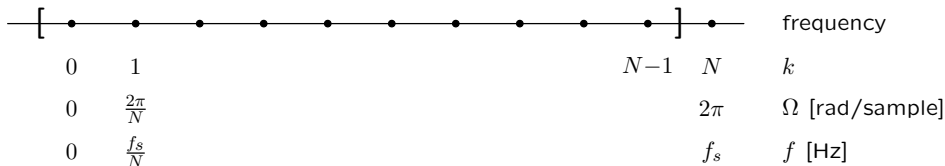
Frequency Resolution

The DFT analysis period N determines both the window in time that is analyzed and the frequency resolution of the result.

The time window is divided into N samples numbered $n = 0$ to $N-1$.



Discrete frequencies are similarly numbered as $k = 0$ to $N-1$.



Two Ways to Think About the DFT

Compare to DTFS:

1. The DFT of a signal $x[n]$ is equal to the **DTFS** of a version of $x[n]$ that is periodically extended so that it is periodic in N .

→ emphasizes the importance of **periodicity** in time.

Compare to DTFT:

2. The DFT is equal to samples of the **DTFT** of a windowed version of the original signal.

→ emphasizes the importance of **spectral smear** in frequency.

Summary

Today we introduced a new Fourier representation for DT signals: the Discrete Fourier Transform (DFT).

The DFT has a number of features that make it particularly convenient.

- It is not limited to periodic signals.
- It has discrete domain (k instead of Ω) and finite length: convenient for numerical computation.

The finite analysis window of the DFT can smear the resulting spectral representation.

- The DFT is equivalent to the DTFS of a periodically extended version of the input signal. The frequency smear results because of discontinuities introduced by periodic extension.
- The DFT is equivalent to the DTFT of a windowed version of the input signal that is then sampled and scaled in amplitude. The frequency smear results because of the limited duration of the analysis window.