

6.3000: Signal Processing

Sinusoids and Fourier Series

September 09, 2025

Fourier Series (Trigonometric Form)

If $f(t)$ is expressed as a Fourier series

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} \left(c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \right)$$

the Fourier coefficients are given by

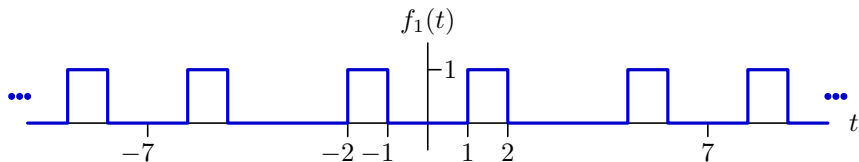
$$c_0 = \frac{1}{T} \int_T f(t) dt$$

$$c_k = \frac{2}{T} \int_T f(t) \cos(k\omega_o t) dt; \quad k = 1, 2, 3, \dots$$

$$d_k = \frac{2}{T} \int_T f(t) \sin(k\omega_o t) dt; \quad k = 1, 2, 3, \dots$$

Two Pulses

Let $f_1(t)$ represent the following function, which is periodic in $T = 7$:

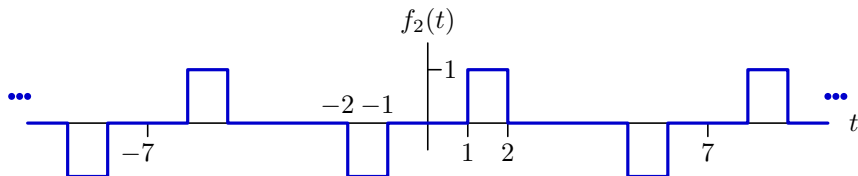


Determine a Fourier series of the following form for $f_1(t)$.

$$f_1(t) = c_0 + \sum_{k=1}^{\infty} \left(c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \right)$$

Opposite Pulses

Let $f_2(t)$ represent the following function, which is periodic in $T = 7$:

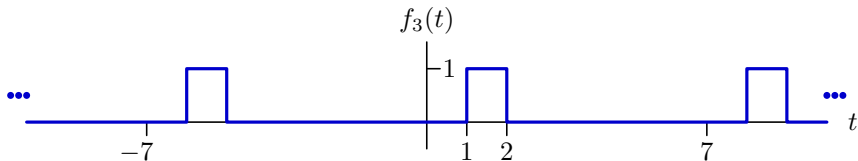


Find ω_o and the Fourier series coefficients c_k and d_k so that

$$f_2(t) = \sum_{k=0}^{\infty} \left(c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \right)$$

Single Pulse

Let $f_3(t)$ represent the following function, which is periodic in $T = 7$:



Determine the Fourier series coefficients for $f_3(t)$.

Discuss the relation(s) among the Fourier series coefficients of $f_1(t)$, $f_2(t)$, and $f_3(t)$.

Discuss the relation(s) among $f_1(t)$, $f_2(t)$, and $f_3(t)$.

Trig Table

$$\sin(a+b) = \sin(a) \cos(b) + \cos(a) \sin(b)$$

$$\sin(a-b) = \sin(a) \cos(b) - \cos(a) \sin(b)$$

$$\cos(a+b) = \cos(a) \cos(b) - \sin(a) \sin(b)$$

$$\cos(a-b) = \cos(a) \cos(b) + \sin(a) \sin(b)$$

$$\tan(a+b) = (\tan(a)+\tan(b))/(1-\tan(a) \tan(b))$$

$$\tan(a-b) = (\tan(a)-\tan(b))/(1+\tan(a) \tan(b))$$

$$\sin(A) + \sin(B) = 2 \sin((A+B)/2) \cos((A-B)/2)$$

$$\sin(A) - \sin(B) = 2 \cos((A+B)/2) \sin((A-B)/2)$$

$$\cos(A) + \cos(B) = 2 \cos((A+B)/2) \cos((A-B)/2)$$

$$\cos(A) - \cos(B) = -2 \sin((A+B)/2) \sin((A-B)/2)$$

$$\sin(a+b) + \sin(a-b) = 2 \sin(a) \cos(b)$$

$$\sin(a+b) - \sin(a-b) = 2 \cos(a) \sin(b)$$

$$\cos(a+b) + \cos(a-b) = 2 \cos(a) \cos(b)$$

$$\cos(a+b) - \cos(a-b) = -2 \sin(a) \sin(b)$$

$$2 \cos(A) \cos(B) = \cos(A-B) + \cos(A+B)$$

$$2 \sin(A) \sin(B) = \cos(A-B) - \cos(A+B)$$

$$2 \sin(A) \cos(B) = \sin(A+B) + \sin(A-B)$$

$$2 \cos(A) \sin(B) = \sin(A+B) - \sin(A-B)$$