

6.300: Signal Processing

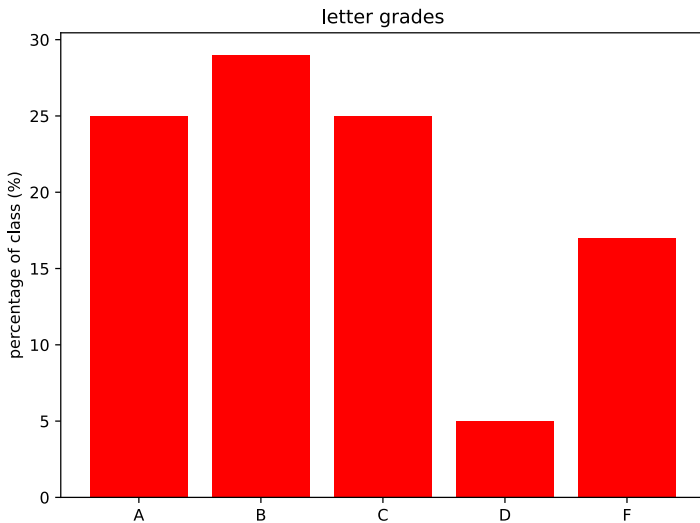
Quiz #2 Retrospective

- Very short Homework #10 is out; due next Thursday, 11/13.
- Office hours after recitation today (until 6:00 p.m.) in 24-310.
- Office hours canceled next Monday, 11/10 and Tuesday, 11/11.
- Veterans Day: No lecture or recitation next Tuesday, 11/11.

If you're concerned about your performance in the course for any reason or just want to talk with someone, we're here for you. We can make a plan for the rest of the term. (Talk to us in person or send an e-mail to sigproc-instructors@mit.edu.)

November 6, 2025

Quiz #2 Statistics



Letter Grades

A: **Exceptionally good performance** demonstrating a superior understanding of the subject matter, a foundation of extensive knowledge, and a skillful use of concepts and/or materials.

B: **Good performance** demonstrating capacity to use the appropriate concepts, a good understanding of the subject matter, and an ability to handle the problems and materials encountered in the subject.

C: **Adequate performance** demonstrating an adequate understanding of the subject matter, an ability to handle relatively simple problems, and adequate preparation for moving on to more advanced work in the field.

D: **Minimally acceptable performance** demonstrating at least partial familiarity with the subject matter and some capacity to deal with relatively simple problems, but also demonstrating deficiencies serious enough to make it inadvisable to proceed further in the field without additional work.

F: Failed.

Progress

Overall course grade:

• Participation (5%)	70% complete
• Exercises (5%)	70% complete
• Problems (20%)	70% complete
• Labs (15%)	70% complete
• Quiz #1 (10%)	100% complete
• Quiz #2 (20%)	100% complete
• Final Exam (25%)	0% complete

“The process of assigning the final grade involves a lot of discussion among the staff and very often a careful review of the final exam to look behind the numbers to understand better the kinds of mistakes that were made. We know that the final grade is important to you and we take the process seriously.”

CT Signals and Systems

#1.1. Let $X_1(\omega)$ denote the Fourier transform of $x_1(t)$.

$$x_1(t) = e^{-t}u(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Determine a closed-form expression for $X_1(\omega)$.

$X_1(\omega) =$

#1.2. Sketch $|X_1(\omega)|$, the magnitude of $X_1(\omega)$. Sketch $\angle X_1(\omega)$, the phase of $X_1(\omega)$. Label the key features of your plots.

CT Signals and Systems

#1.3. Let $X_2(\omega)$ denote the Fourier transform of $x_2(t)$.

$$x_2(t) = e^{-|t|} = \begin{cases} e^{-t} & t \geq 0 \\ e^t & t < 0 \end{cases}$$

Determine an expression for $X_2(\omega)$ in terms of $X_1(\omega)$.

$$X_2(\omega) =$$

#1.4. Sketch $|X_2(\omega)|$, the magnitude of $X_2(\omega)$. Sketch $\angle X_2(\omega)$, the phase of $X_2(\omega)$. Label the key features of your plots.

CT Signals and Systems

#1.5. Let $X_3(\omega)$ denote the Fourier transform of $x_3(t)$.

$$x_3(t) = 2e^{-|t|} \cos(5t) = \begin{cases} 2e^{-t} \cos(t) & t \geq 0 \\ 2e^t \cos(5t) & t < 0 \end{cases}$$

Determine an expression for $X_3(\omega)$ in terms of $X_2(\omega)$.

$X_3(\omega) =$

#1.6. Sketch $|X_3(\omega)|$, the magnitude of $X_3(\omega)$. Sketch $\angle X_3(\omega)$, the phase of $X_3(\omega)$. Label the key features of your plots.

CT Signals and Systems

#1.7. Consider a continuous-time LTI system. The system is initially at rest: $y(t) = 0$ for $t < 0$. Suppose the impulse response is

$$h(t) = x_1(t) = e^{-t}u(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0. \end{cases}$$

Determine numerical values for a_1, a_2, b_0, b_1, b_2 so that the linear differential equation

$$y(t) + a_1 \frac{dy(t)}{dt} + a_2 \frac{d^2y(t)}{dt^2} = b_0 x(t) + b_1 \frac{dx(t)}{dt} + b_2 \frac{d^2x(t)}{dt^2}$$

has impulse response $h(t)$.

$a_1 =$	$a_2 =$	$b_0 =$	$b_1 =$	$b_2 =$
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DT Signals and Systems

#2.1. Let $\mathcal{S}_1$ represent a DT LTI system, and let $H_1(\Omega)$ denote the frequency response of system $\mathcal{S}_1$.

$$H_1(\Omega) = \frac{1}{1 - \frac{1}{5}e^{-j\Omega}}$$

Determine an expression for $h_1[n]$, the unit sample response of system $\mathcal{S}_1$.

$h_1[n] =$

DT Signals and Systems

#2.2. Let $\mathcal{S}_2$ represent a DT LTI system, and let $H_2(\Omega)$ denote the frequency response of system $\mathcal{S}_2$.

$$H_2(\Omega) = \frac{e^{-j2\Omega}}{1 - \frac{1}{3}e^{-j\Omega}}$$

Determine numerical values for a_1, a_2, b_0, b_1, b_2 so that the linear difference equation

$$y[n] + a_1y[n-1] + a_2y[n-2] = b_0x[n] + b_1x[n-1] + b_2x[n-2]$$

has frequency response $H_2(\Omega)$. Assume the system is initially at rest: $y[n] = 0$ for $n < 0$.

$a_1 =$	$a_2 =$	$b_0 =$	$b_1 =$	$b_2 =$
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DT Signals and Systems

#2.3. Let $\mathcal{S}_{12}$ represent the LTI system that results from connecting systems $\mathcal{S}_1$ and $\mathcal{S}_2$ in cascade.

Determine an expression for $H_{12}(\Omega)$, the frequency response of system $\mathcal{S}_{12}$, in terms of $H_1(\Omega)$ and $H_2(\Omega)$.

$H_{12}(\Omega) =$

#2.4. Let $h_{12}[n]$ denote the unit sample response of system $\mathcal{S}_{12}$.

Determine the values of $h_{12}[n]$ for $n \in \{0, 1, 2, 3\}$, and enter those values in the boxes below.

$h_{12}[0] =$	$h_{12}[1] =$	$h_{12}[2] =$	$h_{12}[3] =$
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DT Signals and Systems

#2.5. Let $\mathcal{S}_{1+2}$ represent the LTI system that results from connecting the systems $\mathcal{S}_1$ and $\mathcal{S}_2$ in parallel.

The unit sample response of system $\mathcal{S}_{1+2}$ is

$$h_{1+2}[n] = h_1[n] + h_2[n],$$

where $h_i[n]$ denotes the unit sample response of system $\mathcal{S}_i$.

Determine an expression for $H_{1+2}(\Omega)$, the frequency response of system $\mathcal{S}_{1+2}$, in terms of $H_1(\Omega)$ and $H_2(\Omega)$.

$$H_{1+2}(\Omega) =$$

DT Signals and Systems

#2.6. If the input to system $\mathcal{S}_{1+2}$ is

$$x[n] = (-1)^n$$

the output may be expressed as

$$y[n] = \lambda(-1)^n$$

where λ is a complex number. Determine a numerical value for λ .

$\lambda =$

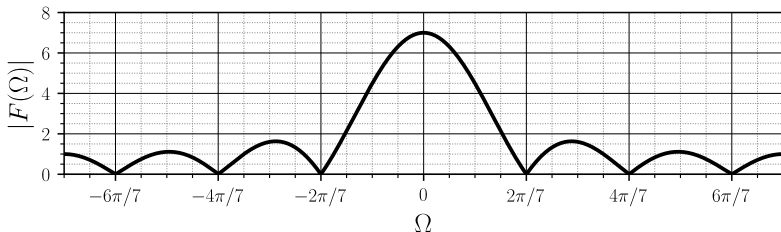
Sinc

Consider the aperiodic discrete-time signal

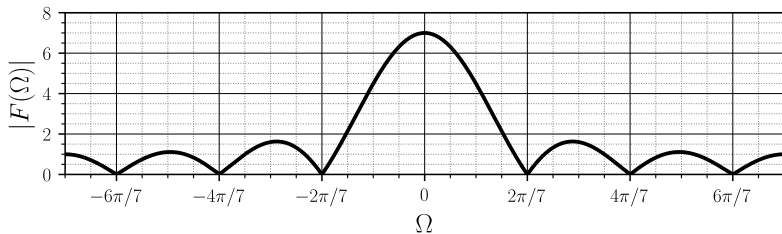
$$f[n] = \begin{cases} 1 & 0 \leq n \leq M-1 \\ 0 & \text{otherwise.} \end{cases}$$

Here, M is a positive integer.

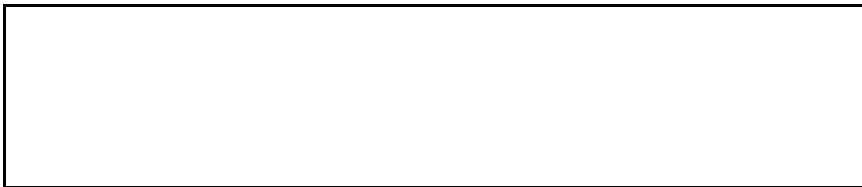
#3.1. From the plot of $|F(\Omega)|$, determine a numerical value for M .



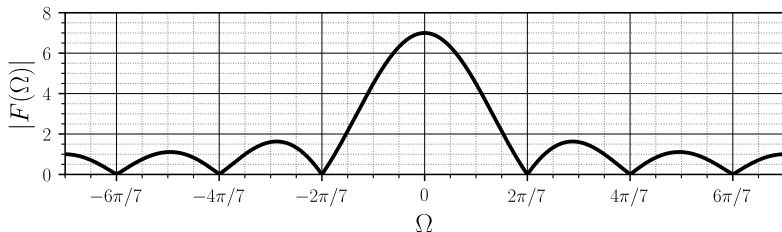
Sinc



#3.2. Sketch $\angle F(\Omega)$, the phase of $F(\Omega)$, over $-\pi \leq \Omega \leq \pi$.



Sinc



#3.3. Suppose that $F(\Omega)$ is the frequency response of an LTI system, as depicted below.

$$x[n] \rightarrow \boxed{\text{LTI}} \rightarrow y[n]$$

Determine the output $y[n]$ that results when $x[n] = \delta[n \bmod 7]$.

$$y[n] =$$

Sinc

#3.4. Let $F_{12}[k]$ denote the DFT of $f[n]$ computed with an analysis window of length $N = 12$.

Determine the values of $F_{12}[k]$ for $k \in \{0, 3, 6\}$.

$F_{12}[0] =$	$F_{12}[3] =$	$F_{12}[6] =$
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#3.5. Let $g_{12}[n]$ denote the signal with DFT $G_{12}[k] = F_{12}[k] \times F_{12}[k]$. As before, the analysis window is of length $N = 12$. Determine the values of $g_{12}[n]$ for $n \in \{0, 1, 2\}$.

$g_{12}[0] =$	$g_{12}[1] =$	$g_{12}[2] =$
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Discrete Fourier Transform

#4. Consider the three time-domain signals given below.

$$f_1[n] = \delta[n \bmod 4] \quad f_2[n] = \cos\left(\frac{5\pi}{6}n\right) \quad f_3[n] = \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)^n$$

Twelve signals $g_1[n], \dots, g_{12}[n]$ are derived from $f_1[n], f_2[n], f_3[n]$.

$g_1[n] = (f_1[n])^2$	$g_2[n] = (f_2[n])^2$	$g_3[n] = (f_3[n])^2$
$g_4[n] = (f_1 \circledast f_1)[n]$	$g_5[n] = (f_1 \circledast f_3)[n]$	$g_6[n] = (f_2 \circledast f_3)[n]$
$g_7[n] = \frac{1}{2} + f_2[n]$	$g_8[n] = (-1)^n f_2[n]$	$g_9[n] = f_2[n] f_3[n]$
$g_{10}[n] = (-j)^n f_3[n]$	$g_{11}[n] = f_3[n + 1]$	$g_{12}[n] = f_3^*[n]$

Match each $g_i[n]$ to the plot that shows $|G_i[k]|$, the magnitude of its DFT computed with analysis window length $N = 24$. Multiple $g_i[n]$ may match to the same plot. If there is no match, write **None**.

