5 Tetra (16 points)

Let $f[r, c]$ represent the following image as an array of pixel brightnesses in the range $[-\frac{1}{2}, \frac{1}{2}]$ where black corresponds to $-1/2$, white corresponds to $+1/2$, and the background grey corresponds to 0. The pixels are indexed by their row number r (with $-\frac{1}{2}R \leqslant r < \frac{1}{2}R$) and column number c (with $-\frac{1}{2}C \leqslant c < \frac{1}{2}C$), and $R = C = 200$.

Note: High-quality images of this figure as well as those on the following page have been provided on a separate sheet. Use the high-quality images to answer the questions below, but record your answers on this page (which has QR codes).

Also consider four additional images, two of which are defined in the spatial domain:

$$h_1[r, c] = \sin\left(\frac{40\pi r}{R}\right)$$

$$h_2[r, c] = \begin{cases} 1 & \text{if } r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

and two defined by their two-dimensional Fourier transforms:

$$H_3[k_r, k_c] = j \sin\left(\frac{40\pi k_r}{R}\right)$$

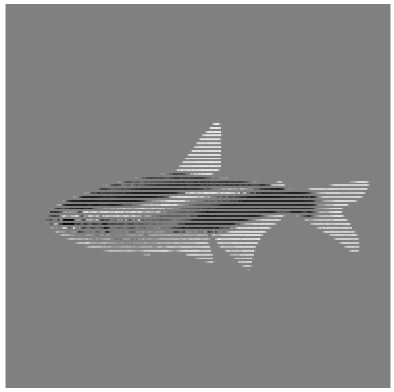
$$H_4[k_r, k_c] = \begin{cases} 1 & \text{if } k_r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

Determine which of the images on the next page corresponds to each of the following image combinations, and enter the appropriate label (A-L) in each of the boxes below.

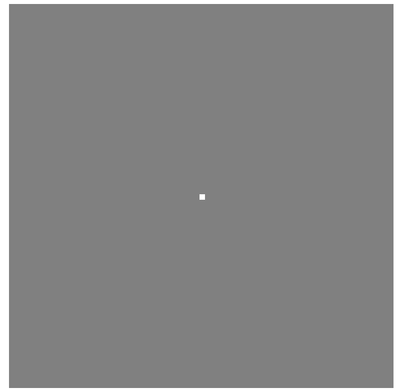
$(f \times h1)[r, c]$	E	$(f \circledast h1)[r, c]$	A
$(f \times h2)[r, c]$	B	$(f \circledast h2)[r, c]$	F
$(f \times h3)[r, c]$	G	$(f \circledast h3)[r, c]$	I
$(f \times h4)[r, c]$	C	$(f \circledast h4)[r, c]$	K

A:

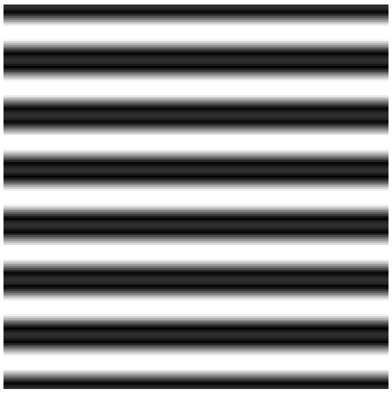
B:



C:



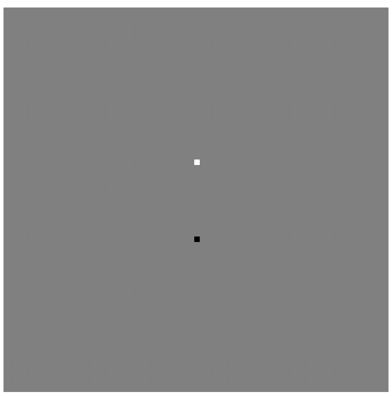
D:



E:

F:

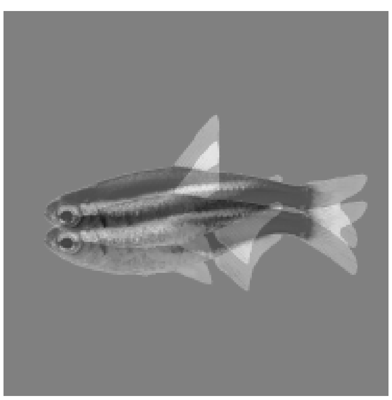
G:



H:

I:

J:



K:

L:

$$h_1[r,c] = \sin\left(\frac{40\pi r}{R}\right)$$

This image consists of 20 horizontal stripes with brightnesses changing sinusoidally along the vertical dimension. Multiplying the fish image by this signal puts horizontal stripes on the fish but does not affect the background since its brightness is zero.

Thus the answer is **E**.

Convolving the fish image with h_1 is equivalent to multiplying their 2D DFTs. The 2D DFT of h_1 is a positive dot at $r = -20, c = 0$ and a negative dot at $r = 20, c = 0$. Multiplying these dots times the 2D DFT of the fish selects only the two basis functions at the dots location.

The result is a striped image shown in **A**.

$$h_2[r,c] = \begin{cases} 1 & \text{if } r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

This image consists of 100 alternating horizontal stripes of white and black. Multiplying the fish image by this signal puts finely spaced horizontal stripes on the fish but does not affect the background since its brightness is zero.

Thus the answer is **B**.

Convolving the image with h_2 is equivalent to multiplying their 2D DFTs. The 2D DFT of h_1 is a positive dot at $r = -1, c = 0$ and a negative dot at $r = 1, c = 0$. Multiplying these dots times the 2D DFT of the fish selects only the two basis functions, corresponding to the dots' locations. However, the amplitudes of these components in the fish image are small. Thus the resulting stripes are not visible at the output.

The result is shown in **F**.

$$H_3[k_r,k_c] = j\sin\left(\frac{40\pi k_r}{R}\right)$$

This 2D DFT consists of 20 horizontal stripes with amplitudes changing sinusoidally along the vertical dimension. The inverse DFT of this pattern is two dots, much like image G. Multiplying the fish image by this signal creates an image that has the background gray color (which represents 0) everywhere except for the two dots in G.

Thus the answer is **G**.

Convolving the fish image with the two dots that comprise h_3 produces a positive image of the fish that is shifted up 20 pixels plus a negative image of the fish that is shifted down 20 pixels.

The result is a striped image shown in **I**.

$$H_4[k_r,k_c] = \begin{cases} 1 & \text{if } k_r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

This 2D DFT consists of 100 horizontal stripes that alternate between white and black along the vertical dimension. The inverse DFT of this pattern is two dots: one at the origin and one in the center of the top row of the image. Multiplying the fish image times this signal sets the top dot to zero, making it blend in with the background. Only the bottom dot remains.

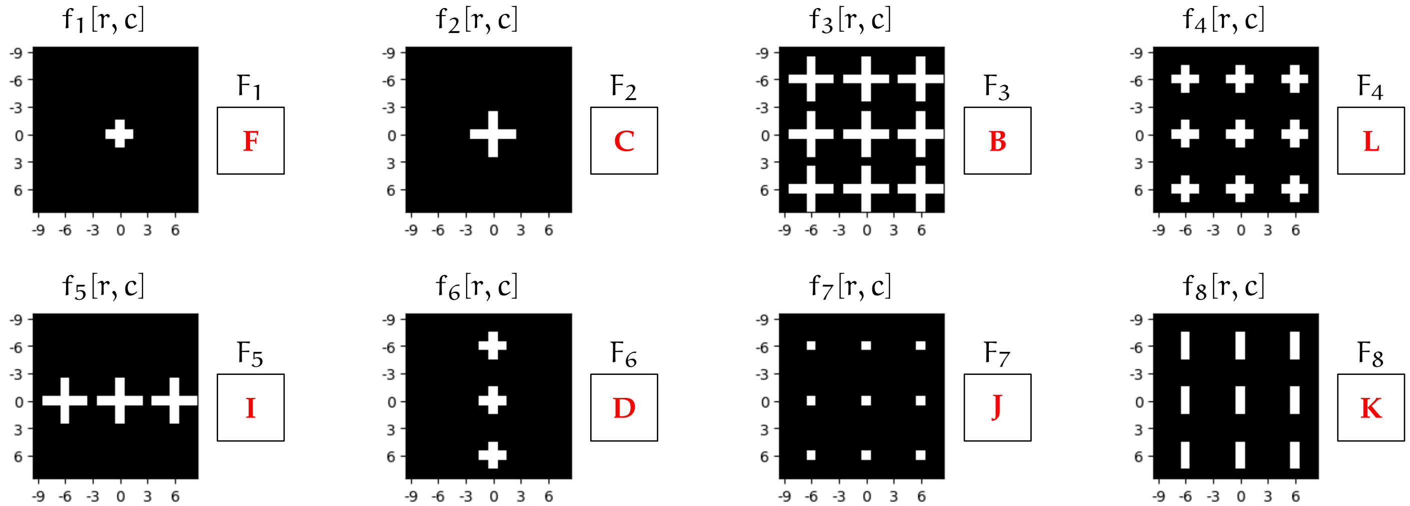
Thus the answer is **C**.

Convolving the fish image with the two dots that comprise h_4 produces two images of the fish. The first is at the original location of the fish. The second is shifted up by half the frame, with the upper part of this shifted fish circularly shifted into the bottom part of the new image.

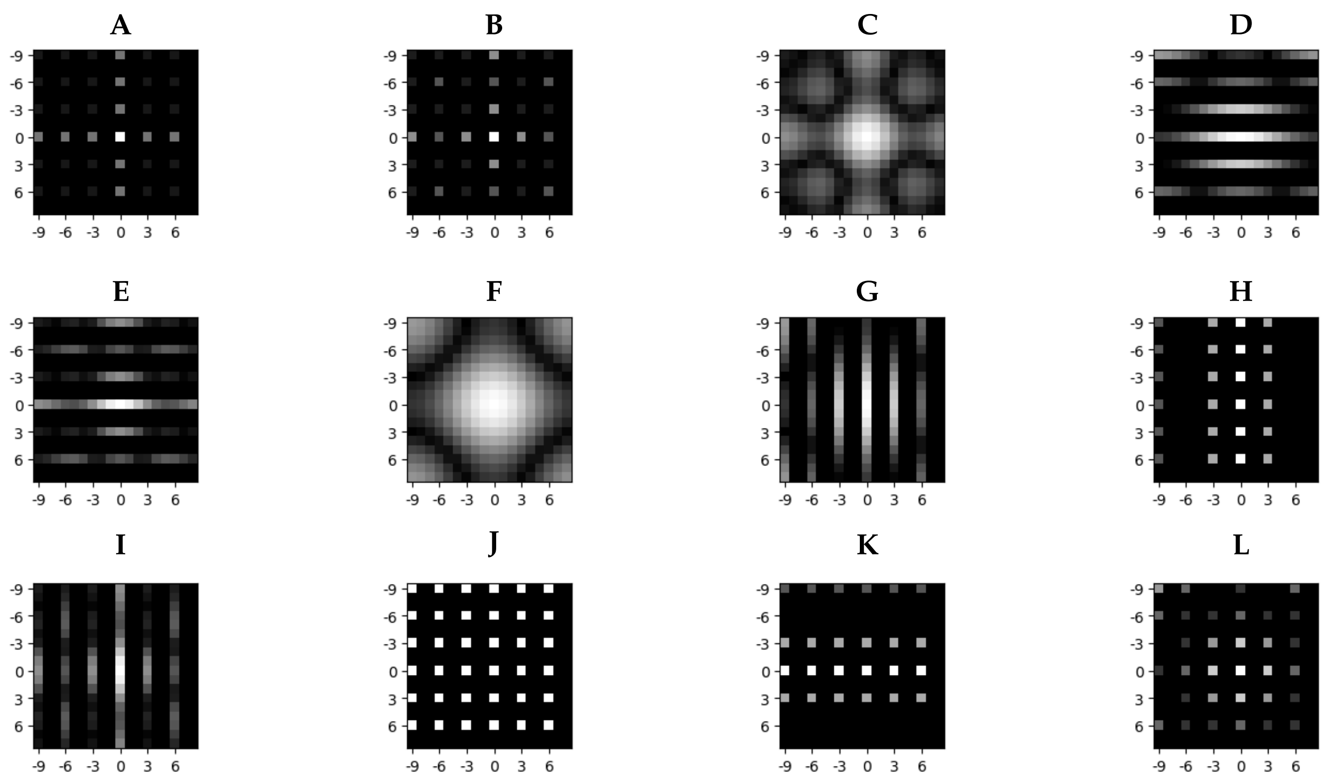
The result is **K**.

6 Pluses and Minuses (16 points)

The panels below show eight images ($f_1[r,c]$ to $f_8[r,c]$ with black (0) or white (1) pixels that are indexed by row number r and column number c where $-9 \leqslant r < 9$ and $-9 \leqslant c < 9$.



Each of the following panels shows the magnitude of a 2D DFT. For each panel, black represents a value of 0 and white represents the largest magnitude in that panel (which may be different for each panel). Determine which of the panels shows the magnitude of the 2D DFT of each of the images above, and enter the corresponding letter (A-L) in the appropriate box.



$f_1[r, c]$

This signal contains a DC component (due to the white square at $r = c = 0$, as well as a horizontal fundamental component (due to white squares at $c = \pm 1$, and a vertical fundamental component (due to white squares at $r = \pm 1$. The periods of the fundamental components are equal to the image width $C = 18$ and height $R = 18$. Thus the 2D DFT is panel **F**.

$f_2[r, c]$

This signal includes not only fundamental components (as in f_1 but also second harmonic terms (due to the which squares at $c = \pm 2$ and $r = \pm 2$). Thus the 2D DFT is panel **C**.

$f_7[r, c]$

This image is periodic in r and c, so its 2D DFT will be composed of isolated dots. Since there are 3 dots across the field in both the r and c directions, the lowesst, non-zero harmonic in each direction will be the third. Thus the 2D DFT is panel **J**.

$f_3[r, c]$

The f_3 image is the convolution of f_2 with f_7. Therefore F_3 will be the product of F_2 (C) with F_7 (J). Thus the 2D DFT is panel **B**.

$f_4[r, c]$

The f_4 image is the convolution of f_1 with f_7. Therefore F_4 will be the product of F_1 (F) with F_7 (J). Thus the 2D DFT is panel **L**.

$f_8[r, c]$

The f_8 image is the convolution of a large, rotated minus sign with f_7. The rotated minus sign is skinny and tall. Therefore, its 2D DFT would be short and fat. The only pattern of isolated dots that fits this description is panel **K**.

$f_5[r, c]$

This image is periodic in c but not in r. Therefore its 2D DFT is dots in k_c continuous in k_r. This description could fit with I or G. The big plus signs produces the shorter vertical bars in panel **I**.

$f_6[r, c]$

This image is a rotated version of f_5 with small plus signs, which produces the longer horizontal bars in panel **D**.