6.300: Signal Processing

Final Exam Practice Problems #1

- Final Exam: Johnson Track on Friday, 12/19 at 1:30 p.m.
- Bring three 8.5″ × 11.0″ pages (six sides) of handwritten notes.

Fill out the survey on the course website.
`https://sigproc.mit.edu/fall25/survey`

Fill out a subject evaluation for this class.
`https://registrar.mit.edu/classes-grades-evaluations/subject-evaluation`

December 2, 2025

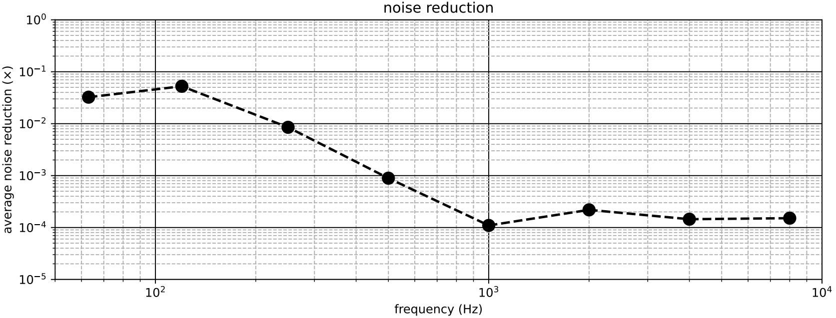
Noise Reduction

My neighbor plays loud music at night while I'm trying to read. So, I bought a pair of noise-reducing ear muffs at Micro Center last weekend. Here are numbers from the back of the package:

frequency	average reduction	
63 Hz	14.9 dB	$\times 10^{-1.49}$
120 Hz	12.8 dB	$\times 10^{-1.28}$
250 Hz	20.7 dB	$\times 10^{-2.07}$
500 Hz	30.5 dB	$\times 10^{-3.05}$
1,000 Hz	39.6 dB	$\times 10^{-3.96}$
2,000 Hz	36.6 dB	$\times 10^{-3.66}$
4,000 Hz	38.4 dB	$\times 10^{-3.84}$
8,000 Hz	38.2 dB	$\times 10^{-3.82}$

Noise Reduction

Magnitude of frequency response on log-log scale:



Approximately a low-pass filter? Approximately a high-pass filter?

More quantitative problems up next:

- Impulsive Images
- Plaid Patterns

Impulsive Images

Pixel values for $f_1[r,c]$, $f_2[r,c]$, and $f_3[r,c]$ are shown below for $|r| \le 2$ and $|c| \le 2$. Pixel values outside the indicated regions are zero.

$f_1[r,c]$

r \ c	−2	−1	0	1	2
−2	0	0	0	1	0
−1	0	0	0	0	0
0	0	0	0	0	0
1	0	0	0	0	0
2	0	1	0	0	0

$\left|F_1[k_r,k_c]\right| =$ ☐

$f_2[r,c]$

r \ c	−2	−1	0	1	2
−2	0	0	0	0	0
−1	0	−1	0	1	0
0	0	−2	0	2	0
1	0	−1	0	1	0
2	0	0	0	0	0

$\left|F_2[k_r,k_c]\right| =$ ☐

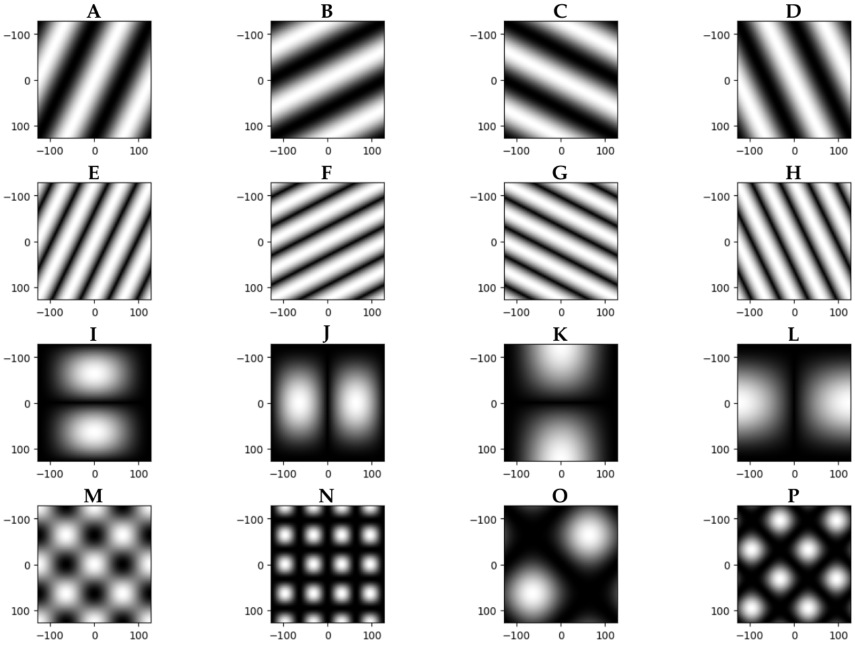
$f_3[r,c]$

r \ c	−2	−1	0	1	2
−2	0	0	1	0	0
−1	0	0	0	0	0
0	−1	0	4	0	1
1	0	0	0	0	0
2	0	0	−1	0	0

$\left|F_3[k_r,k_c]\right| =$ ☐

Match $f_i[r,c]$ to the image on the next page that shows $|F_i[k_r,k_c]|$, the magnitude of its 2D DFT, computed for $-128 \le r, c < 128$.

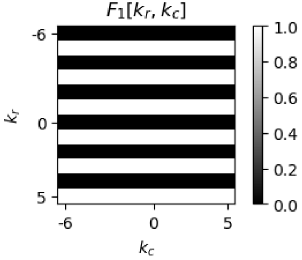
In each image, black represents the minimum magnitude and white represents the maximum magnitude.


A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
−100
0
100

Plaid Patterns

(**a**) Consider the 12×12 2D DFT $F_1[k_r, k_c]$ shown below.

- white = 1, black = 0



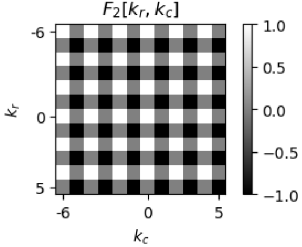
Determine a closed-form expression for the 2D signal $f_1[r, c]$.

$f_1[r, c] =$

Plaid Patterns

(**b**) Consider the 12×12 2D DFT $F_2[k_r, k_c]$ shown below.

- white = 1, gray = 0, black = -1



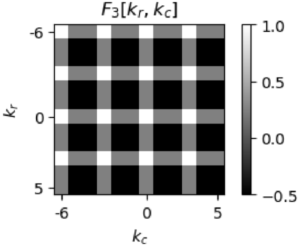
Determine a closed-form expression for the 2D signal $f_2[r, c]$.

$f_2[r, c] =$

Plaid Patterns

(**c**) Consider the 12×12 2D DFT $F_3[k_r, k_c]$ shown below.

- white = 1, gray = 1/4, black = $-1/2$



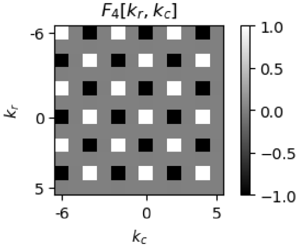
Determine a closed-form expression for the 2D signal $f_3[r, c]$.

$f_3[r, c] =$

Plaid Patterns

(**d**) Consider the 12×12 2D DFT $F_4[k_r, k_c]$ shown below.

- white $= 1$, gray $= 0$, black $= -1$



Determine a closed-form expression for the 2D signal $f_4[r, c]$.

$f_4[r, c] =$