6.3000: Signal Processing

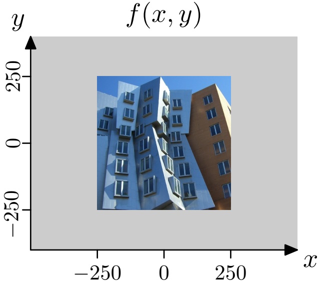
Fourier Series Trig Form

Representing Signals as Fourier Series

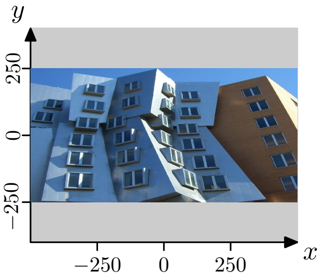
- Synthesis: making a signal from components
- Analysis: finding the components

September 04, 2025

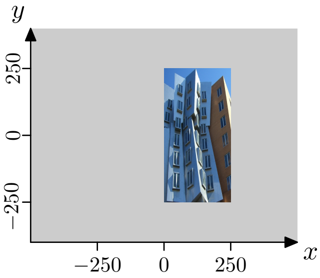
Start With Some Basic Transformations



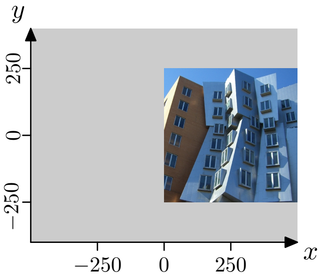
How many images match the expressions beneath them?



$f_1(x,y) = f(2x,y)$?



$f_2(x,y) = f(2x-250,y)$?



$f_3(x,y) = f(-x-250,y)$?

Fourier Series

Fourier representations are a major theme of this subject.

The basic ideas were described in lecture:

Synthesis Equation (making a signal from components):

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} c_k \cos\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{\infty} d_k \sin\left(\frac{2\pi kt}{T}\right)$$

Analysis Equations (finding the components):

$$c_0 = \frac{1}{T}\int_T f(t)\, dt$$

$$c_k = \frac{2}{T}\int_T f(t) \cos\left(\frac{2\pi kt}{T}\right) dt \ ; \quad k \geq 1$$

$$d_k = \frac{2}{T}\int_T f(t) \sin\left(\frac{2\pi kt}{T}\right) dt \ ; \quad k \geq 1$$

Warm Up

Find the Fourier series coefficients (c_k and d_k) for

$$f(t) = \cos(t)$$

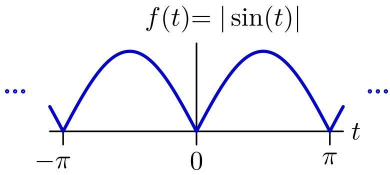
Fourier Series Coefficients

How many of the following functions have **exactly one** non-zero Fourier series coefficient?

- $f_1(t) = \cos^2 t$
- $f_2(t) = \sin t \cos t$
- $f_3(t) = 4\cos^3 t - 3\cos t$
- $f_4(t) = \cos(12t)\cos(4t)\cos(2t)$

Rectified Sine Wave

Consider a Fourier series representation of the following function.



- What is the approximate value of c_0?
- Which non-DC Fourier coefficient has the largest absolute value? What's the sign of that coefficient?
- Determine an expression for the Fourier coefficients of $f(t)$.
- Compute the sum of the first 100 terms in the Fourier series of $f(t)$.

Verify Fourier Series of Rectified Sine Wave Numerically

Compute the sum of the first 100 terms in the Fourier series of $f(t)$.

Trig Table

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sin(a+b) = sin(a) cos(b) + cos(a) sin(b)
sin(a-b) = sin(a) cos(b) - cos(a) sin(b)
cos(a+b) = cos(a) cos(b) - sin(a) sin(b)
cos(a-b) = cos(a) cos(b) + sin(a) sin(b)
tan(a+b) = (tan(a)+tan(b))/(1-tan(a) tan(b))
tan(a-b) = (tan(a)-tan(b))/(1+tan(a) tan(b))

sin(A) + sin(B) =  2 sin((A+B)/2) cos((A-B)/2)
sin(A) - sin(B) =  2 cos((A+B)/2) sin((A-B)/2)
cos(A) + cos(B) =  2 cos((A+B)/2) cos((A-B)/2)
cos(A) - cos(B) = -2 sin((A+B)/2) sin((A-B)/2)

sin(a+b) + sin(a-b) =  2 sin(a) cos(b)
sin(a+b) - sin(a-b) =  2 cos(a) sin(b)
cos(a+b) + cos(a-b) =  2 cos(a) cos(b)
cos(a+b) - cos(a-b) = -2 sin(a) sin(b)

2 cos(A) cos(B) = cos(A-B) + cos(A+B)
2 sin(A) sin(B) = cos(A-B) - cos(A+B)
2 sin(A) cos(B) = sin(A+B) + sin(A-B)
2 cos(A) sin(B) = sin(A+B) - sin(A-B)
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