6.300: Signal Processing

Quiz #1 (Fall 2025)

Name:			
Kerberos:			
Recitation:	4-237 (with Mark)	4-370 (with Titus)	N/A

Instructions

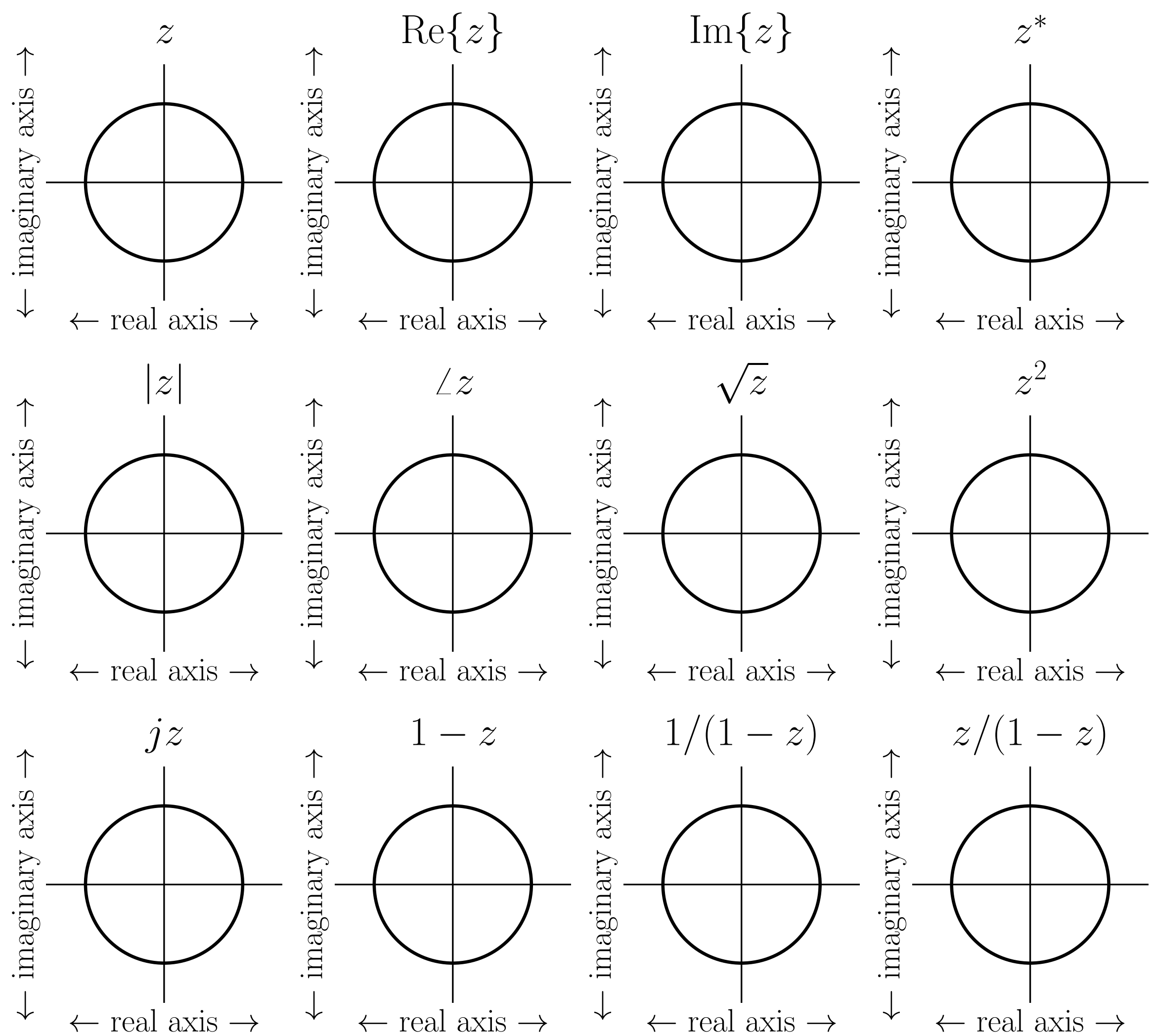
1. Do not write on the QR codes.
2. You may use one 8.5″ × 11″ double-sided sheet of handwritten notes.
   (No phones, laptops, tablets, music players, digital calculators, slide rules, abaci, etc.)
3. If you have a question, come to the front of the room to ask.
4. If you finish the quiz in the last 10 minutes, quietly remain at your desk and wait to hand in your quiz until we call time.
5. Write neatly. We cannot award credit for answers we cannot read.
6. You may not discuss the quiz with anyone other than course instructors until quiz scores have been released.
7. We will report any violations of academic integrity to the MIT Committee on Discipline.

1 Complex Numbers (25%)

Consider the complex number

$$z = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}.$$

On each plot below, clearly sketch the indicated transformation of z in the complex plane. The unit circle is shown for reference.



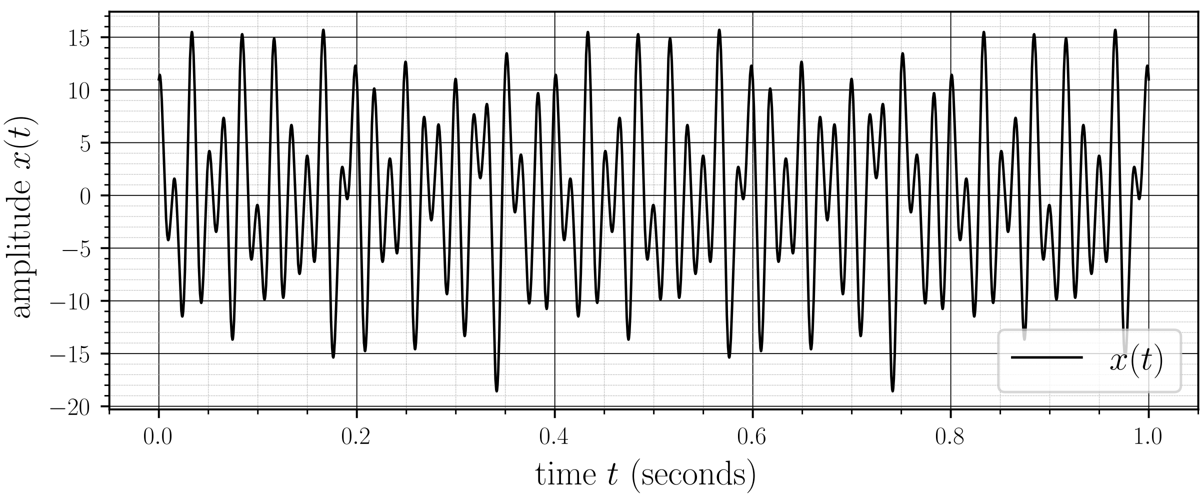
This worksheet is for your scratch work. It has been left blank intentionally.

2 Continuous-Time Fourier Series (25%)

Consider the continuous-time (CT) signal

$$x(t) = x(t+T) = 3\cos(50\pi t) - 2\sin(60\pi t) + 6\sin(75\pi t) + 8\cos(120\pi t)$$

which is periodic with fundamental period T. Suppose that T is measured in seconds. A plot of $x(t)$ over $0 \le t \le 1$ is shown below. You should not necessarily assume that $T = 1$ second.



2.1

Determine the fundamental angular frequency ω_0 (radians per second) of $x(t)$. Specify an exact numerical answer, not an approximation based solely on the plot shown above. You may express your answer in terms of familiar constants such as π and e.

$\boldsymbol{\omega_0} =$

This worksheet is for your scratch work. It has been left blank intentionally.

2.2

Recall that $x(t)$ is given by

$$x(t) = x(t+T) = 3\cos(50\pi t) - 2\sin(60\pi t) + 6\sin(75\pi t) + 8\cos(120\pi t).$$

Determine the coefficients $\{c_k, d_k\}_{k=0}^{\infty}$ for a Fourier series expansion of the form

$$x(t) = \sum_{k=0}^{\infty} c_k \cos(k\omega_0 t) + \sum_{k=0}^{\infty} d_k \sin(k\omega_0 t)$$

where, as before, ω_0 denotes the fundamental angular frequency (radians per second) of $x(t)$.

1. Specify c_k and d_k for five values of k for which either $c_k \neq 0$ or $d_k \neq 0$.
2. If there are fewer than five such values of k, enter an **X** in the remaining boxes.
3. If there are more than five such values of k, enter any five.

Specify numerical values. You may express your answer in terms of familiar constants such as π and e. This problem can be answered with relatively few calculations.

$\boldsymbol{k}$	$\boldsymbol{c_k}$	$\boldsymbol{d_k}$

This worksheet is for your scratch work. It has been left blank intentionally.

2.3

Recall that $x(t)$ is given by

$$x(t) = x(t+T) = 3\cos(50\pi t) - 2\sin(60\pi t) + 6\sin(75\pi t) + 8\cos(120\pi t).$$

Determine the coefficients $\{a_k\}_{k=-\infty}^{\infty}$ for a Fourier series expansion of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where, as before, ω_0 denotes the fundamental angular frequency (radians per second) of $x(t)$.

1. Specify a_k for five values of k for which $a_k \neq 0$ and $a_{-k} \neq 0$.
2. If there are fewer than five such values of k, enter an **X** in the remaining boxes.
3. If there are more than five such values of k, enter any five.

Specify numerical values. You may express your answer in terms of familiar constants such as j, π, and e. This problem can be answered with relatively few calculations.

$\boldsymbol{k}$	$\boldsymbol{a_k}$	$\boldsymbol{a_{-k}}$

This worksheet is for your scratch work. It has been left blank intentionally.

2.4

Recall that $x(t)$ is given by

$$x(t) = x(t+T) = 3\cos(50\pi t) - 2\sin(60\pi t) + 6\sin(75\pi t) + 8\cos(120\pi t).$$

Determine the coefficients $\{m_k, \phi_k\}_{k=0}^{\infty}$ for a Fourier series expansion of the form

$$x(t) = \sum_{k=0}^{\infty} m_k \cos(k\omega_0 t + \phi_k)$$

where, as before, ω_0 denotes the fundamental angular frequency (radians per second) of $x(t)$, and where $m_k \geq 0$ and $-\pi \leq \phi_k < \pi$ for all k.

1. Specify m_k and ϕ_k for five values of k for which either $m_k \neq 0$ or $\phi_k \neq 0$.
2. If there are fewer than five such values of k, enter an **X** in the remaining boxes.
3. If there are more than five such values of k, enter any five.

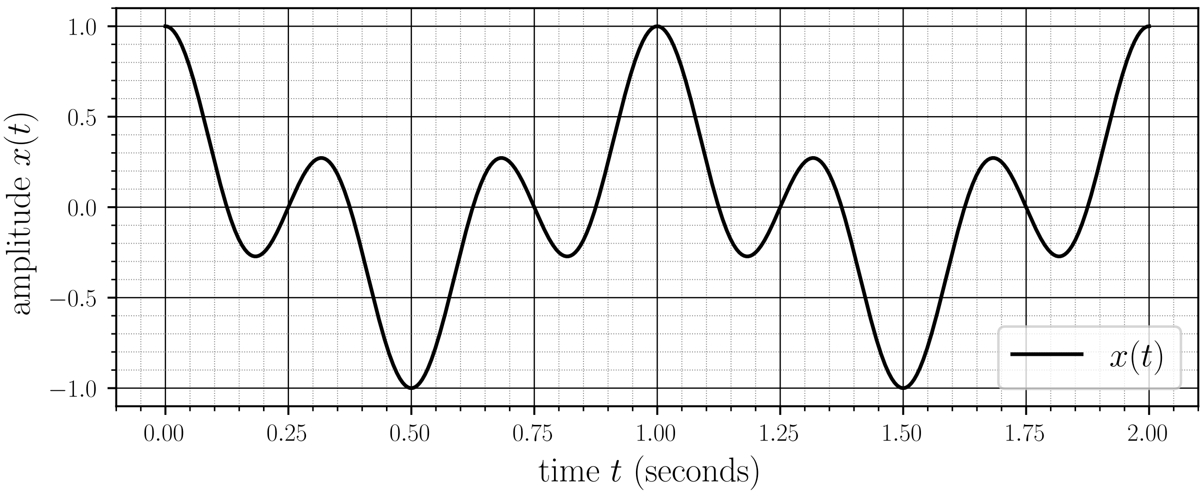
Specify numerical values. You may express your answer in terms of familiar constants such as π and e. This problem can be answered with relatively few calculations.

k	m_k	ϕ_k

This worksheet is for your scratch work. It has been left blank intentionally.

3 Sampling (25%)

A periodic continuous-time (CT) signal, $x(t)$, is shown in the plot below. While $x(t)$ is plotted over $0 \leq t \leq 2$, you should not necessarily assume that the fundamental period is $T = 2$.



3.1

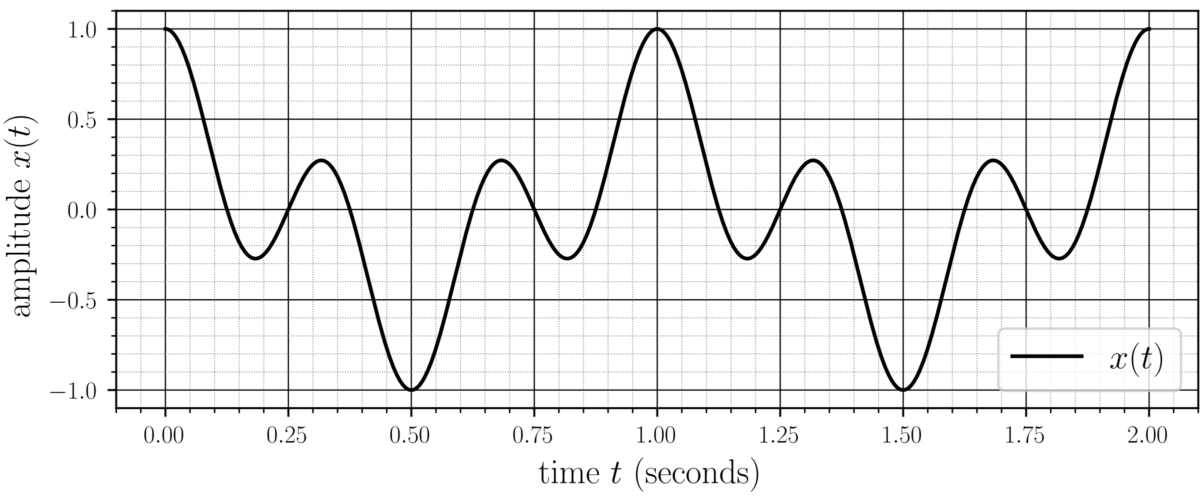
Based on the plot of $x(t)$ shown above, answer the following questions. Specify numerical values. You may express these numerical values in terms of constants such as π and e.

1. Determine the fundamental period T (seconds) of $x(t)$.
2. Determine the fundamental cyclical frequency f_0 (cycles per second, i.e., hertz) of $x(t)$.
3. Determine the fundamental angular frequency ω_0 (radians per second) of $x(t)$.

$\boldsymbol{T}$	$\boldsymbol{f_0}$	$\boldsymbol{\omega_0}$

3.2

For convenience, the plot of $x(t)$ is reproduced below.

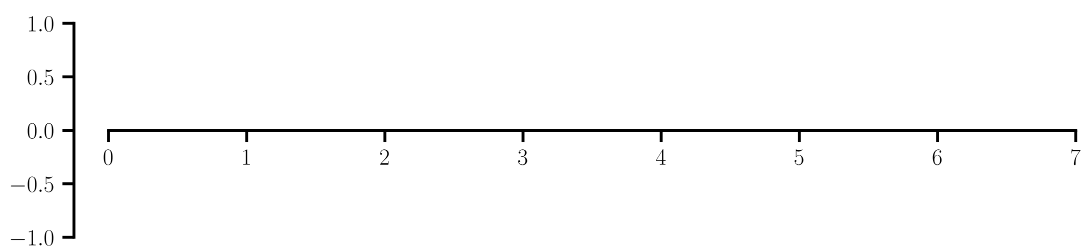


Suppose we sample $x(t)$ every $\Delta_1 = 0.25$ seconds — or, equivalently, with a sampling rate of $f_1 = 4$ cycles per second (i.e., hertz) — to produce a discrete-time (DT) signal $x_1[n]$.

1. Specify N_1, the fundamental period (in samples) of $x_1[n]$. In addition, specify Ω_1 (where $0 \le \Omega_1 \le \pi$), the fundamental angular frequency (in radians per sample) of $x_1[n]$. If $x_1[n]$ does not have a fundamental period, write **None**.
2. Sketch $x_1[n]$ over $0 \le n \le 7$.

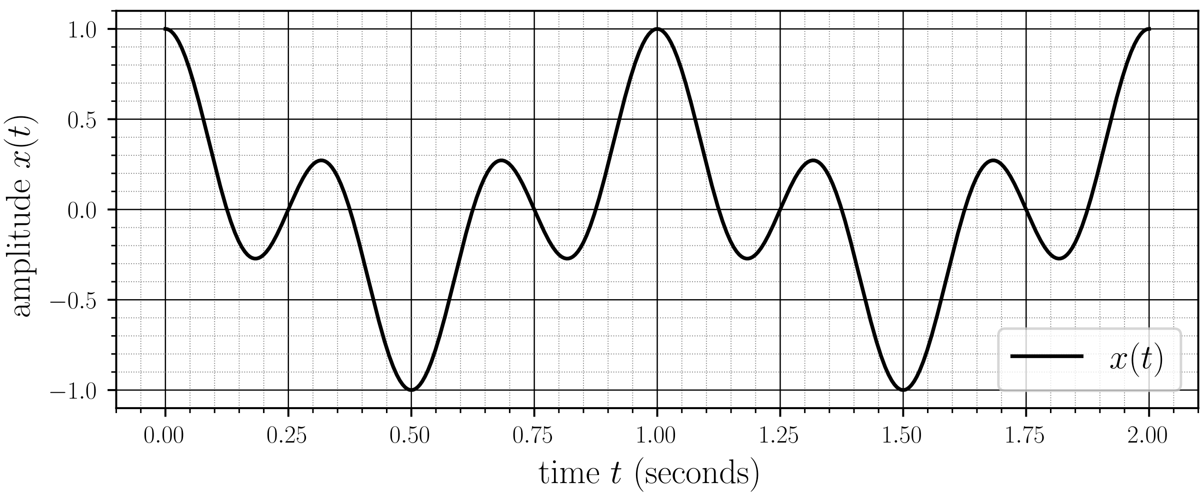
You may express numerical values in terms of constants such as π and e.

$\boldsymbol{N_1}$	$\boldsymbol{\Omega_1}$



3.3

For convenience, the plot of $x(t)$ is reproduced below.



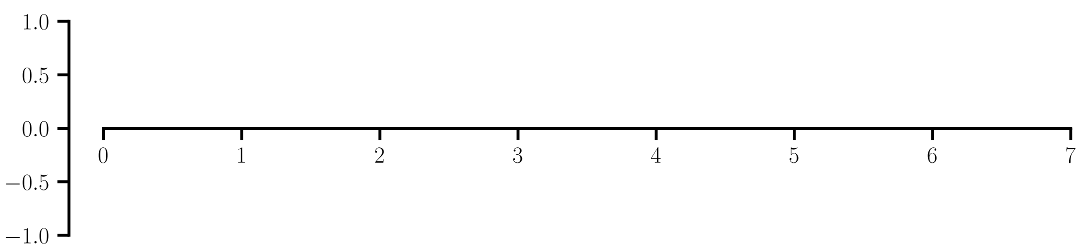
Suppose we sample $x(t)$ every $\Delta_2 = 0.5$ seconds — or, equivalently, with a sampling rate of $f_2 = 2$ cycles per second (i.e., hertz) — to produce a discrete-time (DT) signal $x_2[n]$.

1. Specify N_2, the fundamental period (in samples) of $x_2[n]$. In addition, specify Ω_2 (where $0 \le \Omega_2 \le \pi$), the fundamental angular frequency (in radians per sample) of $x_2[n]$. If $x_2[n]$ does not have a fundamental period, write **None**.

2. Sketch $x_2[n]$ over $0 \le n \le 7$.

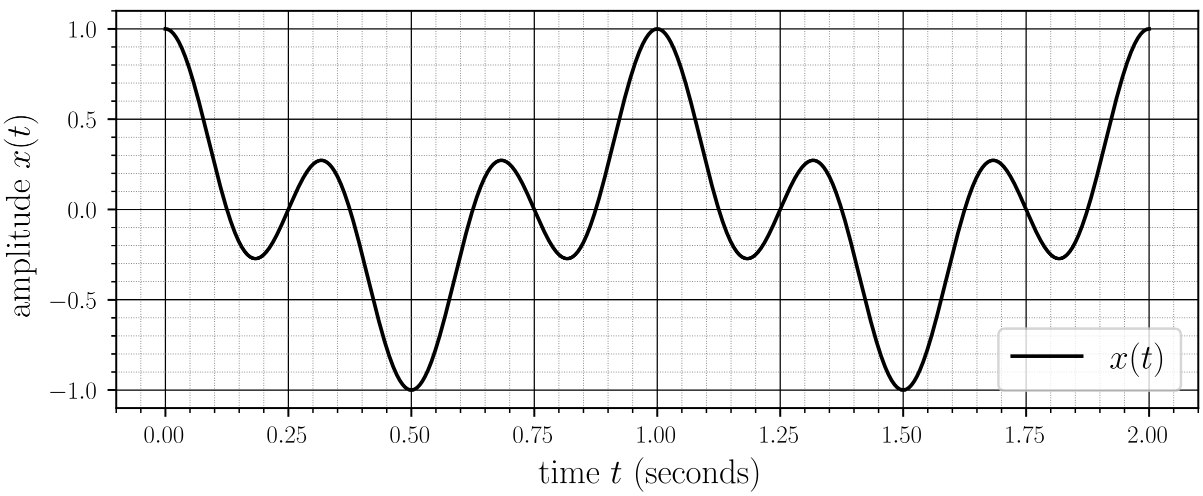
You may express numerical values in terms of constants such as π and e.

$\boldsymbol{N_2}$	$\boldsymbol{\Omega_2}$



3.4

For convenience, the plot of $x(t)$ is reproduced below.

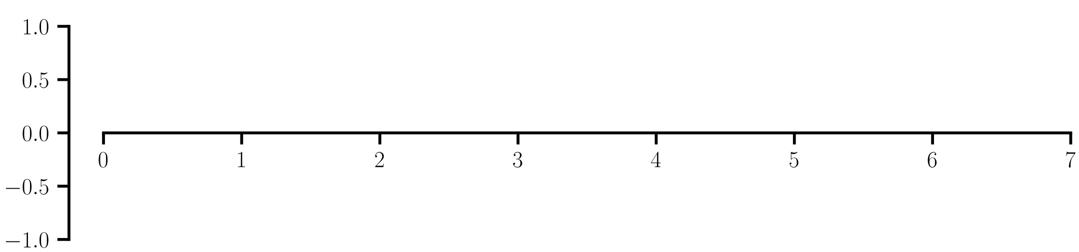


Suppose we sample $x(t)$ every $\Delta_3 = 1$ second — or, equivalently, with a sampling rate of $f_3 = 1$ cycle per second (i.e., hertz) — to produce a discrete-time (DT) signal $x_3[n]$.

1. Specify N_3, the fundamental period (in samples) of $x_3[n]$. In addition, specify Ω_3 (where $0 \le \Omega_3 \le \pi$), the fundamental angular frequency (in radians per sample) of $x_3[n]$. If $x_3[n]$ does not have a fundamental period, write **None**.
2. Sketch $x_3[n]$ over $0 \le n \le 7$.

You may express numerical values in terms of constants such as π and e.

$\boldsymbol{N_3}$	$\boldsymbol{\Omega_3}$



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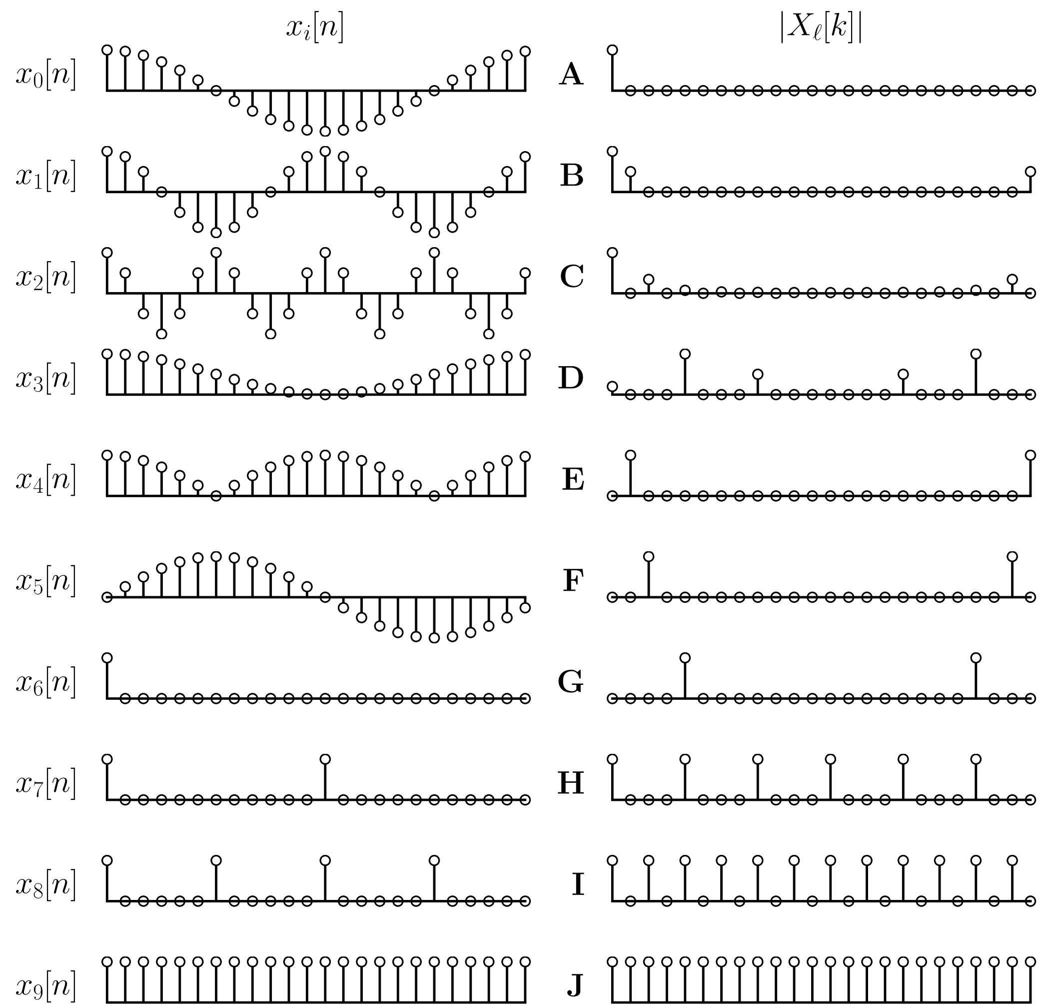
4 Discrete-Time Fourier Series (25%)

Match each periodic discrete-time sequence $x_i[n]$ shown on the left to the corresponding plot on the right (**A** – **J**) that shows the magnitude of its Fourier series coefficients $|X_i[k]|$ computed with a period of $N = 24$ samples. Multiple $x_i[n]$'s may match to the same plot on the right.

Left	$x_0[n]$	$x_1[n]$	$x_2[n]$	$x_3[n]$	$x_4[n]$	$x_5[n]$	$x_6[n]$	$x_7[n]$	$x_8[n]$	$x_9[n]$
Right										



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