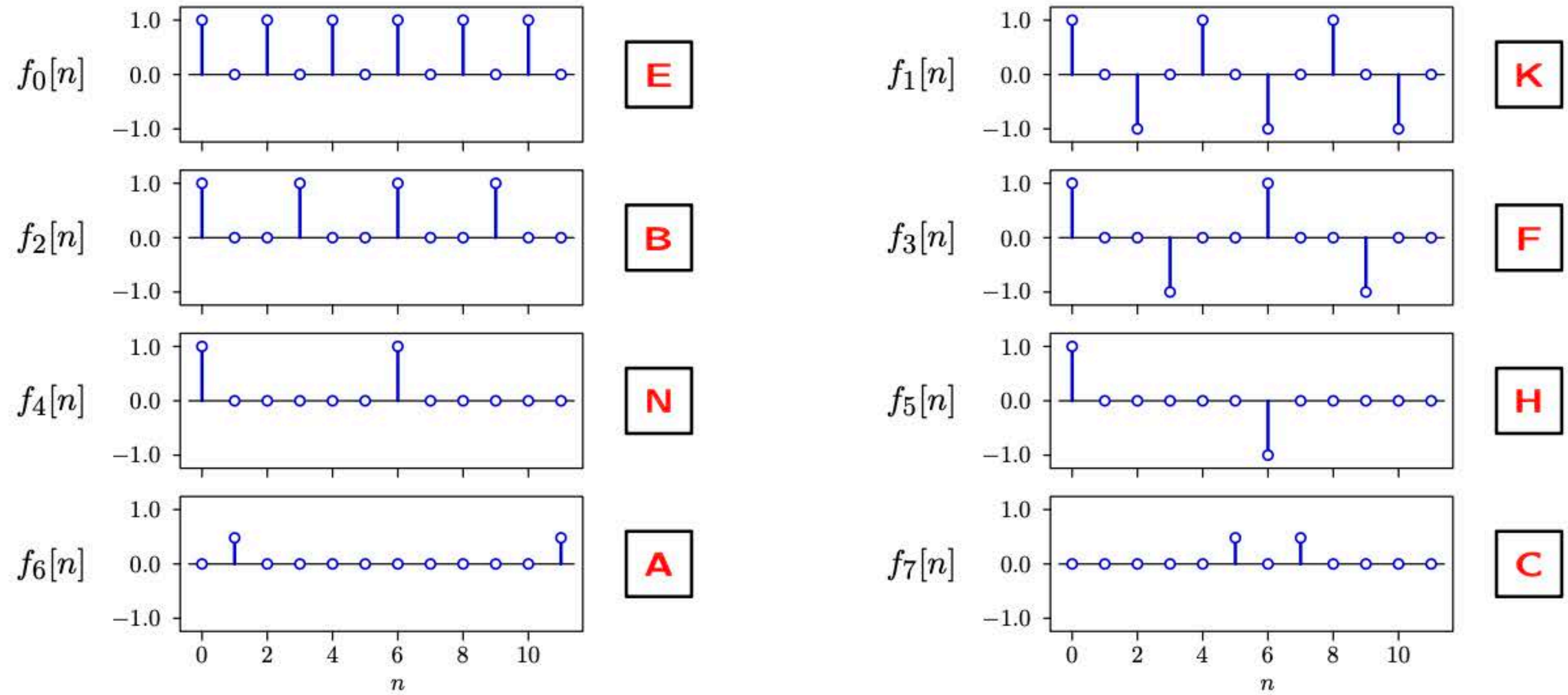
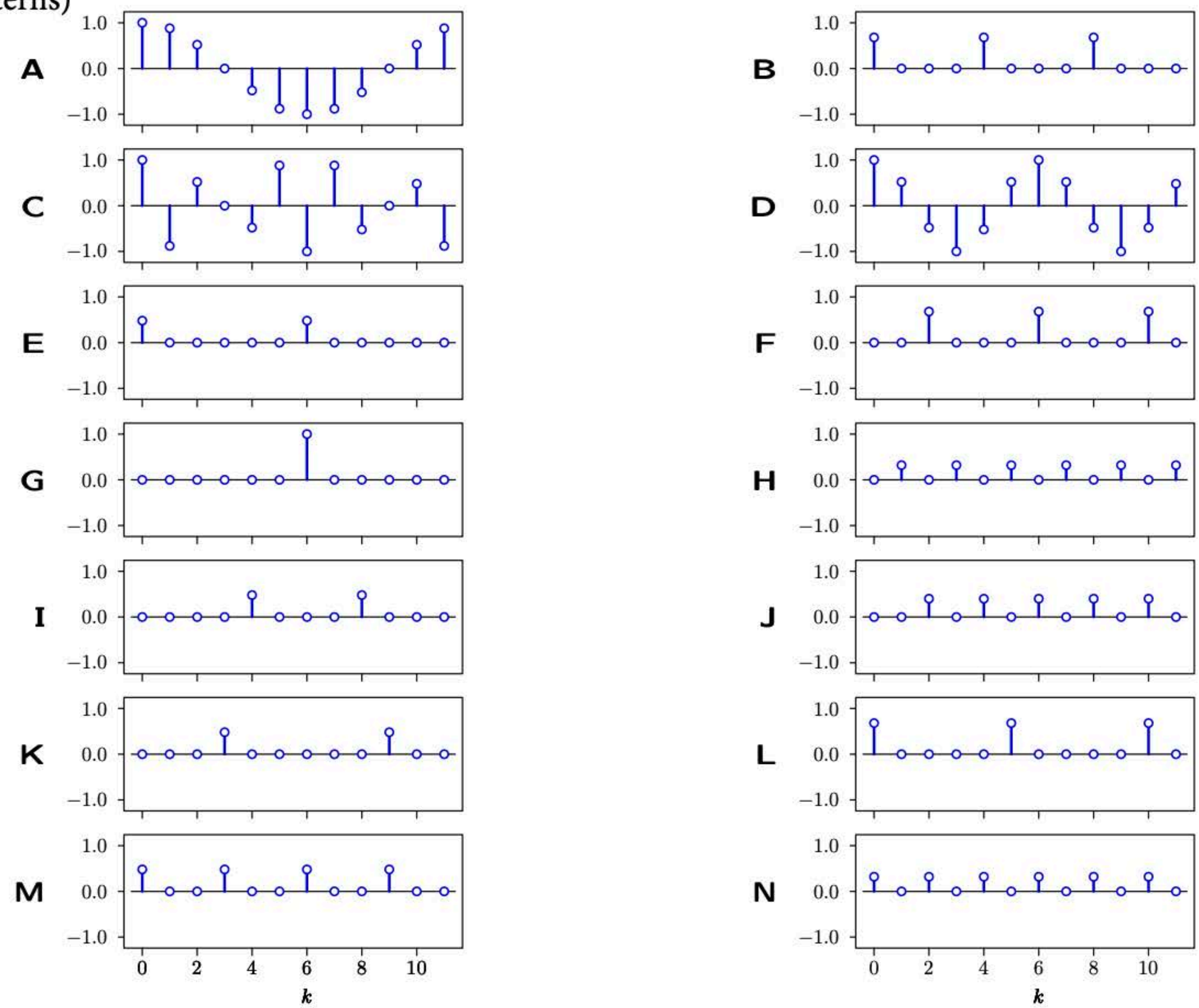
Each of the following plots shows samples n=0 through n=11 of a discrete-time signal $f_i[n]$.

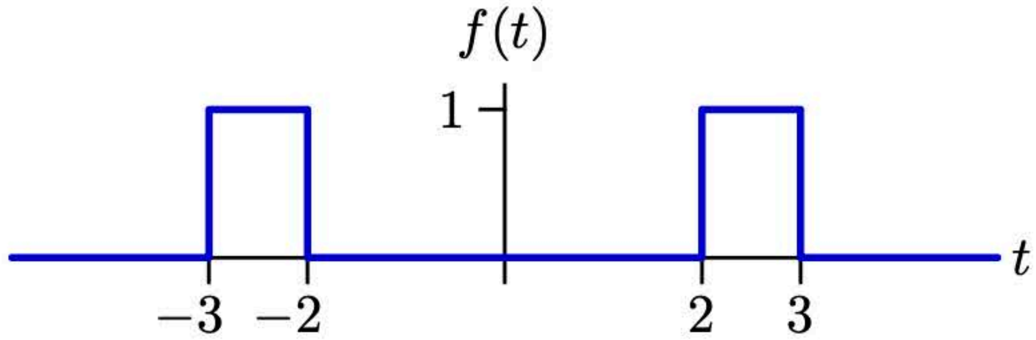


Determine which of the following plots (**A-N**) shows the real part of the 12-point DFT of each of the preceding signals ($f_0[n]$–$f_7[n]$), and enter that letter in the corresponding box above. DFT values have been scaled/normalized (only match patterns)



Part a. Determine the frequency response $F(\omega)$ of a linear, time-invariant system with the following impulse response:

$$f(t) = \begin{cases} 1 & \text{if } 2 < |t| < 3 \\ 0 & \text{otherwise} \end{cases}$$



Enter a closed form expression for $F(\omega)$ in the box below.

$$F(\omega) = 2\frac{\sin 3\omega}{\omega} - 2\frac{\sin 2\omega}{\omega}$$

The signal $f(t)$ can be written as the difference between a rectangular pulse that extends from −3 to 3 and a rectangular pulse that extends from −2 to 2. The Fourier transform of the former is

$$\int_{-3}^{3} e^{-j\omega t} dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-3}^{3} = 2\frac{\sin 3\omega}{\omega}$$

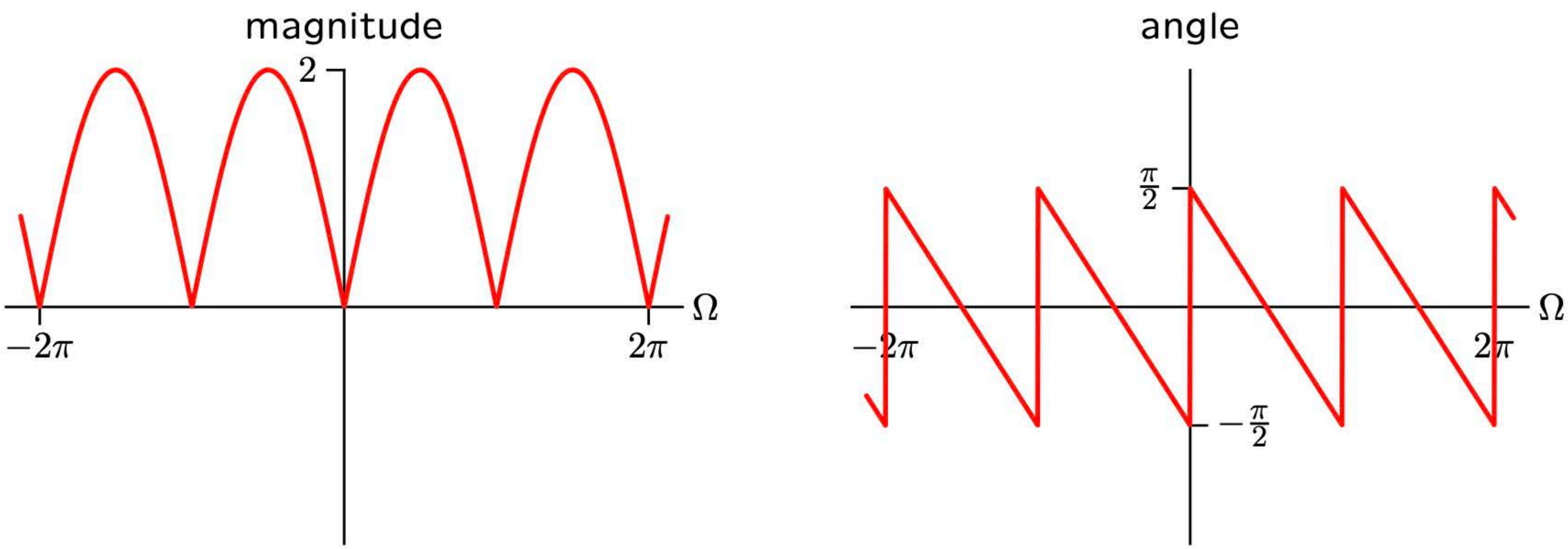
Similarly, the Fourier transform of the latter is $2\frac{\sin 2\omega}{\omega}$. Thus the total answer is

$$F(\omega) = 2\frac{\sin 3\omega}{\omega} - 2\frac{\sin 2\omega}{\omega}$$

Part b. Determine the frequency response of a linear, time-invariant system with the following unit-sample response.

$$g[n] = \delta[n] - \delta[n-2]$$

Sketch the magnitude and angle of the frequency response on the axes below. Label the key points.



The frequency response is the Fourier transform of the unit-sample response:

$$G(\Omega) = \sum_{n=-\infty}^{\infty} g[n]e^{-j\Omega n} = 1 - e^{-j2\Omega}$$

This frequency response can be simplified by realizing that $g[n]$ is a time-delayed version of $\delta[n+1]-\delta[n-1]$, which would correspond to a sinusoid in frequency. We can take advantage of this fact by factoring the time delay term out of the expression for $G(\Omega)$, as follows.

$$G(\Omega) = 1 - e^{-j2\Omega} = e^{-j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right) = j2\sin(\Omega)e^{-j\Omega}$$

The magnitude of the frequency response is the magnitude of $2\sin(\Omega)$.

The angle of the frequency response is determined by three factors. First, the j in $j2\sin(\Omega)e^{-j\Omega}$ contributes $+\pi/2$. Second, $\sin(\theta)$ is negative when $\pi < \theta < 2\pi$. Third, the phase term $e^{-j\Omega}$ contributes a linear term that decreases in proportion to Ω. When Ω is a small positive number, only the first factor contributes, so the angle of the frequency response is $\pi/2$. As Ω increases, the linear term decreases the angle of the frequency response until $\Omega = \pi/2$. At that point, the sign of $\sin(\Omega)$ is negative, so the phase jumps by π. Then the cycle repeats. Thus these three factors combine to give rise to the sawtooth function above.

Part a. Let $\mathcal{S}$ represent a linear, time-invariant system with unit-sample response

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & \text{if } n \text{ is both even and greater than or equal to } 0 \\ 0 & \text{otherwise} \end{cases}$$

If the input to $\mathcal{S}$ is

$$x[n] = \cos(\pi n/4)$$

then the output can be written in the following form:

$$y[n] = A\cos(\pi n/4) + B\sin(\pi n/4)$$

Determine A and B and enter these numbers in the following boxes.

A =	$\frac{16}{17}$
B =	$\frac{4}{17}$

The frequency response of $\mathcal{S}$ is

$$H(\Omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\Omega n} = \sum_{\substack{n=0 \\ n \text{ even}}}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\Omega n} = \sum_{m=0}^{\infty} \left(\frac{1}{2}\right)^{2m} e^{-j\Omega 2m} = \frac{1}{1-\frac{1}{4}e^{-j2\Omega}}$$

The input signal can be written as

$$x[n] = \cos(\pi n/4) = \text{Re}\left(e^{j\pi n/4}\right)$$

and the corresponding output is then

$$y[n] = \text{Re}\left(H\left(\frac{\pi}{4}\right)e^{j\pi n/4}\right) = \text{Re}\left(\frac{1}{1-\frac{1}{4}e^{-j\pi/2}}e^{j\pi n/4}\right)$$

$$= \text{Re}\left(\frac{1}{1+\frac{1}{4}j}e^{j\pi n/4}\right) = \text{Re}\left(\frac{4}{4+j}e^{j\pi n/4}\right)$$

$$= \text{Re}\left(\left(\frac{16-4j}{17}\right)\left(\cos\left(\frac{\pi n}{4}\right) + j\sin\left(\frac{\pi n}{4}\right)\right)\right)$$

$$= \frac{16}{17}\cos\left(\frac{\pi n}{4}\right) + \frac{4}{17}\sin\left(\frac{\pi n}{4}\right)$$