6.3000: Signal Processing

Superposition and Convolution

$$y(t) = (h * x)(t) = \int h(\tau)x(t-\tau)\,d\tau$$

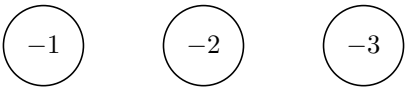
$$y[n] = (h * x)[n] = \sum_m h[m]x[n-m]$$

October 10, 2024

Discrete-Time Example: Solera Process

Aging and blending wines from different crops.

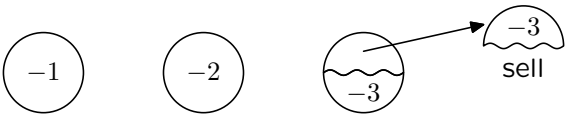
Start with 3 barrels of wine: newest at left, oldest at right.



Discrete-Time Example: Solera Process

Aging and blending wines from different crops.

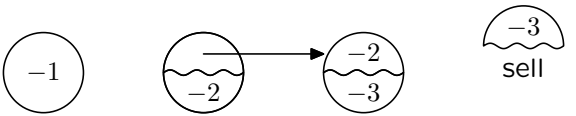
Sell half of the oldest stock.



Discrete-Time Example: Solera Process

Aging and blending wines from different crops.

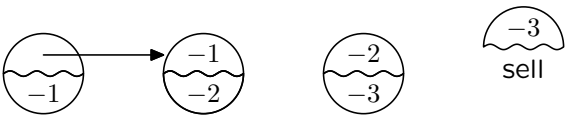
Refill oldest barrel from next-to-oldest barrel.



Discrete-Time Example: Solera Process

Aging and blending wines from different crops.

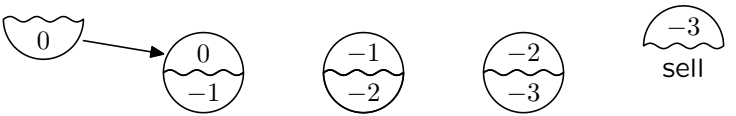
Refill next-to-oldest barrel from youngest barrel.



Discrete-Time Example: Solera Process

Aging and blending wines from different crops.

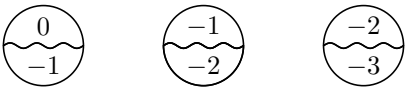
Refill youngest barrel with this year's harvest.



Discrete-Time Example: Solera Process

Aging and blending wines from different crops.

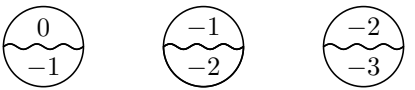
Old and new contents mix; ready for next year.



Discrete-Time Example: Solera Process

Aging and blending wines from different crops.

Old and new contents mix; ready for next year.



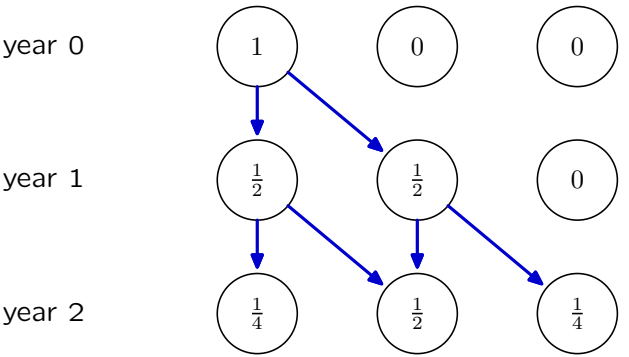
Properties of solera process:

- Mixing produces a more uniform product.
- Mitigates worst-case results of one bad year.
- Blends wines from MANY previous years.

Solera Analysis

We can analyze these effects with a tracer experiment.

Add 1 unit of tracer to new crop; track tracer through the system.

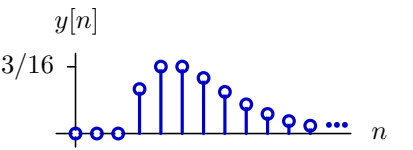


How much tracer will be in each barrel at the end of year 3?

Solera Analysis

Add 1 unit of tracer to new crop; track tracer through the system.

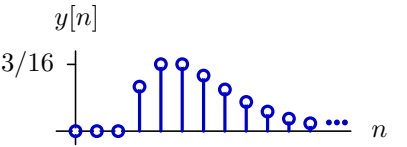
Year n	Tracer in $x[n]$	Barrel #1	Barrel #2	Barrel #3	Tracer out $y[n]$
0	1	1	0	0	0
1	0	1/2	1/2	0	0
2	0	1/4	2/4	1/4	0
3	0	1/8	3/8	3/8	1/8
4	0	1/16	4/16	6/16	3/16
5	0	1/32	5/32	10/32	6/32
6	0	1/64	6/64	15/64	10/64



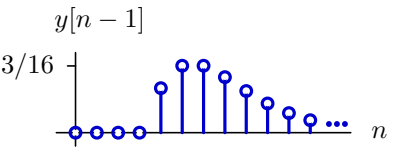
Solera Analysis

How would results change if tracer were added in year 1 (not 0)?

Original response:



Delayed input $\rightarrow$ delayed output:

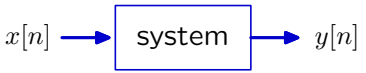


Delaying the input by a year simply delays the outputs by one year.

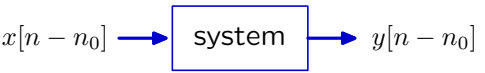
Time-Invariance

A system is time-invariant if delaying the input to the system simply delays the output by the same amount of time.

Given



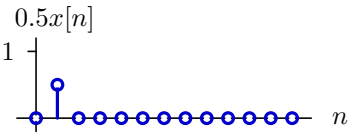
the system is **time invariant** if

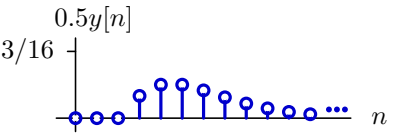


is true for all n_0.

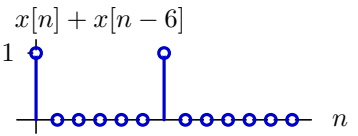
Solera Analysis

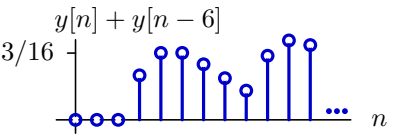
Scaling the input amplitudes:



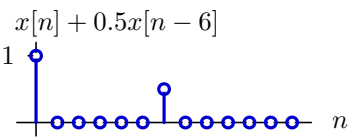


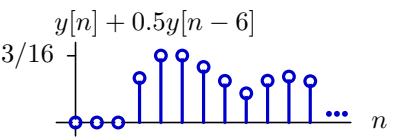
Adding two inputs:





Linearly combining two inputs:

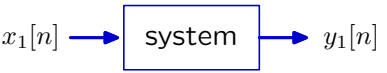




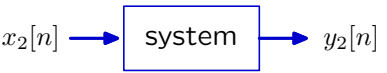
Linearity

A system is linear if its response to a weighted sum of inputs is equal to the weighted sum of its responses to each of the inputs.

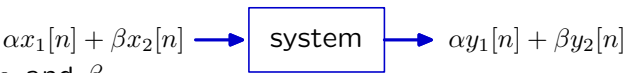
Given



and



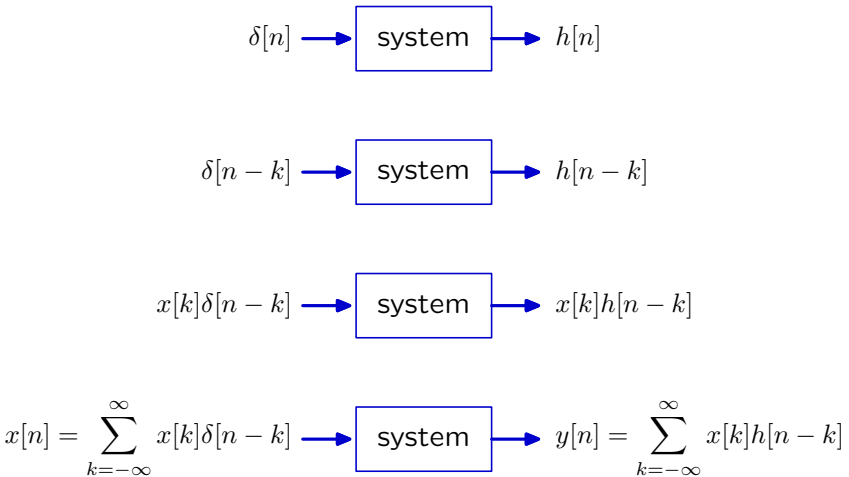
the system is linear if



is true for all α and β.

Convolution

If a system is linear and time invariant, the response to an arbitrary input is the convolution of that input with the unit-sample response.



Check Yourself

For solera process ...

Year n	Tracer in $x[n]$	Barrel #1	Barrel #2	Barrel #3	Tracer out $y[n]$
0	1	1	0	0	0
1	0	1/2	1/2	0	0
2	0	1/4	2/4	1/4	0
3	0	1/8	3/8	3/8	1/8
4	0	1/16	4/16	6/16	3/16
5	0	1/32	5/32	10/32	6/32
6	0	1/64	6/64	15/64	10/64

How much tracer would be found in output from year 5, if one unit of tracer is added in years 0, 1, and 2?

1. $\frac{21}{32}$ 2. $\frac{1}{2}$ 3. $\frac{3}{16}$ 4. $\frac{9}{16}$ 5. none of above