

# 6.3000: Signal Processing

## Systems

*October 8, 2024*

## Linearity and Time Invariance

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Determine which of the following systems are **linear**.

Determine which of the following systems are **time-invariant**.

1.  $y[n] = x[-n]$

2.  $y[n] = x[2n]$

3.  $y[n] = |x[n]|$

## Linearity and Time Invariance

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1.  $y[n] = x[-n]$

Start with **homogeneity**: would the output samples  $y[n]$  be doubled (for example) if the input samples  $x[n]$  are doubled?

Intuitively, the answer is yes. More formally ...

Assume that an input  $x_1[n]$  produces an output  $y_1[n]$ :

$$y_1[n] = x_1[-n]$$

Find the output  $y_2[n]$  when the input  $x_2[n] = 2x_1[n]$ :

$$y_2[n] = x_2[-n] = 2x_1[-n]$$

But  $x_1[-n]$  is the same as  $y_1[n]$ , so

$$y_2[n] = x_2[-n] = 2x_1[-n] = 2y_1[n]$$

We just showed that doubling the input  $x_1[n]$  doubles the output  $y_1[n]$ . The same argument applies when the input is multiplied by any constant  $\alpha$ .

## Linearity and Time Invariance

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1.  $y[n] = x[-n]$

To test **additivity**, assume that the input  $x_1[n]$  generates output  $y_1[n]$ :

$$y_1[n] = x_1[-n]$$

and  $x_2[n]$  generates output  $y_2[n]$ :

$$y_2[n] = x_2[-n]$$

Find the response  $y_3[n]$  to  $x_3[n] = x_1[n] + x_2[n]$ . We know that

$$y_3[n] = x_3[-n]$$

We can find  $x_3[-n]$  by substituting  $-n$  for  $n$  in the definition of  $x_3[n]$ :

$$y_3[n] = x_3[-n] = x_1[-n] + x_2[-n]$$

But  $x_1[-n]$  is the same as  $y_1[n]$  and  $x_2[-n]$  is the same as  $y_2[n]$ .

$$y_3[n] = x_3[-n] = x_1[-n] + x_2[-n] = y_1[n] + y_2[n]$$

We just showed that the response to the sum of two signals is the sum of the responses to each signal taken separately.

Since the system is homogeneous and additive, it is also linear.

## Linearity and Time Invariance

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1.  $y[n] = x[-n]$

A system is **time invariant** if shifting the input by some number of samples shifts the output by the same number of samples.

Assume that the input  $x_1[n]$  generates output  $y_1[n]$ :

$$y_1[n] = x_1[-n]$$

Find the output  $y_2[n]$  when the input  $x_2[n] = x_1[n - n_0]$ .

$$y_2[n] = x_2[-n] = x_1[-n - n_0] = x_1[-(n + n_0)]$$

But  $x_1[-(n + n_0)]$  is the same as  $y_1[n + n_0]$ :

$$y_2[n] = x_2[-n] = x_1[-n - n_0] = x_1[-(n + n_0)] = y_1[n + n_0]$$

The shift is in the wrong direction!

This system is **not time invariant**.

This is easy to see by example. If the input  $x_1[n] = \delta[n]$  then the output  $y_1[n] = \delta[n]$ . Shifting the input to the right is not equivalent to shifting the output to the right!

## Answers

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system	linear?	time invariant?
1. $y[n] = x[-n]$	Y	N
2. $y[n] = x[2n]$	Y	N
3. $y[n] =  x[n] $	N	Y

## Linearity and Time Invariance

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Determine which of the following systems are **linear**.

Determine which of the following systems are **time-invariant**.

$$4. y(t) = \frac{d}{dt}x(t)$$

$$5. y(t) = \int_0^t x(\tau)d\tau$$

$$6. y(t) = \int_{-\infty}^t x(\tau)d\tau$$

$$7. y(t) = \int_t^{\infty} x(\tau)d\tau$$

$$8. y(t) = \int_{-\infty}^{\infty} x(\tau)d\tau$$

## Answers

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system	linear?	time invariant?
4. $y(t) = \frac{d}{dt}x(t)$	Y	Y
5. $y(t) = \int_0^t x(\tau)d\tau$	Y	N
6. $y(t) = \int_{-\infty}^t x(\tau)d\tau$	Y	Y
7. $y(t) = \int_t^{\infty} x(\tau)d\tau$	Y	Y
8. $y(t) = \int_{-\infty}^{\infty} x(\tau)d\tau$	Y	Y