

# 6.3000: Signal Processing

## Fourier Series – Complex Form

**Synthesis Equation** (making a signal from components):

$$f(t) = f(t + T) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

**Analysis Equation** (finding the components)

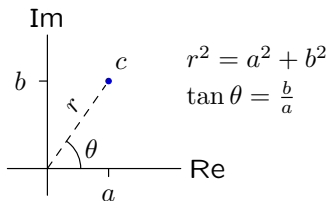
$$a_k = \frac{1}{T} \int_T f(t) e^{-jk\omega_0 t} dt$$

where  $\omega_0 = \frac{2\pi}{T}$

## Representations of Complex Numbers

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Let  $c$  represent a complex number.



rectangular form:  $c = a + jb$

polar (phasor) form:  $r \angle \theta$

Euler form:  $r e^{j\theta}$

## Complex Numbers

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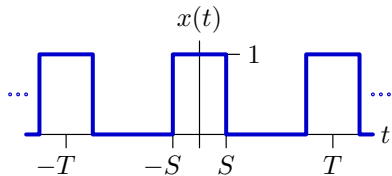
How many of the following are true?

- $\frac{1}{\cos \theta + j \sin \theta} = \cos \theta - j \sin \theta$
- $(\cos \theta + j \sin \theta)^n = \cos(n\theta) + j \sin(n\theta)$
- $|2 + j2 + e^{\frac{j\pi}{4}}| = |2 + j2| + |e^{\frac{j\pi}{4}}|$
- $\text{Im}(j^j) > \text{Re}(j^j)$
- $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1} 1$

## Pulse Train

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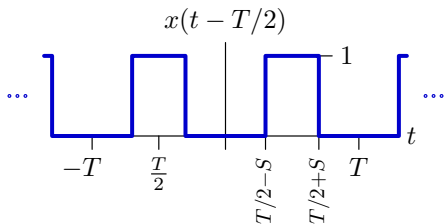
Find the Fourier series coefficients  $a_k$  for  $x(t)$ :



## Pulse Train

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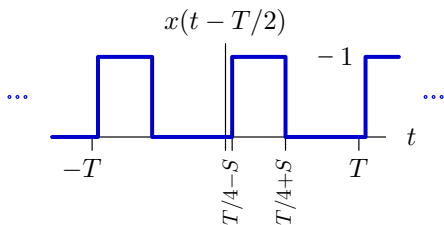
What would happen to Fourier series if you delayed  $x(t)$  by  $T/2$ ?



## Pulse Train

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What would happen if you delayed  $x(t)$  by  $T/4$ ?



# Fourier Series Matching

Match the signals (left column) to Fourier series coefficients (right).

