

# 6.3000: Signal Processing

## Sinusoids and Fourier Series

## Fourier Series (Trigonometric Form)

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If  $f(t)$  is expressed as a Fourier series

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} \left( c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t) \right)$$

the Fourier coefficients are given by

$$c_0 = \frac{1}{T} \int_T f(t) dt$$

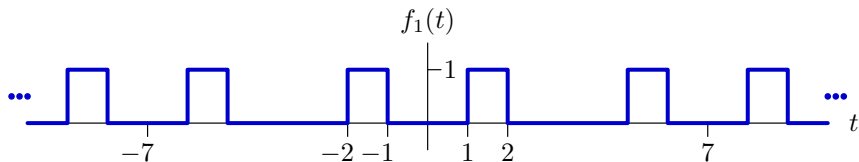
$$c_k = \frac{2}{T} \int_T f(t) \cos(k\omega_0 t) dt; \quad k = 1, 2, 3, \dots$$

$$d_k = \frac{2}{T} \int_T f(t) \sin(k\omega_0 t) dt; \quad k = 1, 2, 3, \dots$$

## Two Pulses

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Let  $f_1(t)$  represent the following function, which is periodic in  $T = 7$ :



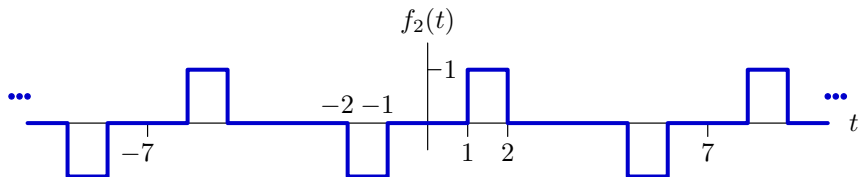
Determine a Fourier series of the following form for  $f_1(t)$ .

$$f_1(t) = c_0 + \sum_{k=1}^{\infty} \left( c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t) \right)$$

## Opposite Pulses

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Let  $f_2(t)$  represent the following function, which is periodic in  $T = 7$ :



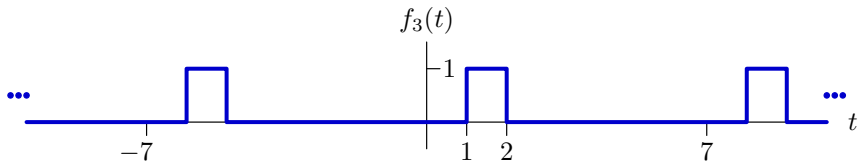
Find  $\omega_o$  and the Fourier series coefficients  $c_k$  and  $d_k$  so that

$$f_2(t) = \sum_{k=0}^{\infty} \left( c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \right)$$

## Single Pulse

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Let  $f_3(t)$  represent the following function, which is periodic in  $T = 7$ :



Determine the Fourier series coefficients for  $f_3(t)$ .

Discuss the relation(s) among the Fourier series coefficients of  $f_1(t)$ ,  $f_2(t)$ , and  $f_3(t)$ .

Discuss the relation(s) among  $f_1(t)$ ,  $f_2(t)$ , and  $f_3(t)$ .

## Check With Python

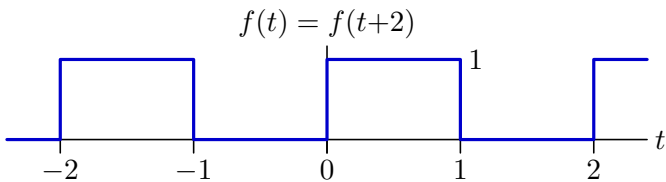
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It's interesting that overshoots move around. They are "glued" to the discontinuities.

## Fourier Analysis of a Square Wave

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Find the Fourier series coefficients for the following square wave:



Can we tell about the  $c_k$  and  $d_k$  coefficients by looking at the signal?

## Trig Table

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$$\sin(a+b) = \sin(a) \cos(b) + \cos(a) \sin(b)$$

$$\sin(a-b) = \sin(a) \cos(b) - \cos(a) \sin(b)$$

$$\cos(a+b) = \cos(a) \cos(b) - \sin(a) \sin(b)$$

$$\cos(a-b) = \cos(a) \cos(b) + \sin(a) \sin(b)$$

$$\tan(a+b) = (\tan(a)+\tan(b))/(1-\tan(a) \tan(b))$$

$$\tan(a-b) = (\tan(a)-\tan(b))/(1+\tan(a) \tan(b))$$

$$\sin(A) + \sin(B) = 2 \sin((A+B)/2) \cos((A-B)/2)$$

$$\sin(A) - \sin(B) = 2 \cos((A+B)/2) \sin((A-B)/2)$$

$$\cos(A) + \cos(B) = 2 \cos((A+B)/2) \cos((A-B)/2)$$

$$\cos(A) - \cos(B) = -2 \sin((A+B)/2) \sin((A-B)/2)$$

$$\sin(a+b) + \sin(a-b) = 2 \sin(a) \cos(b)$$

$$\sin(a+b) - \sin(a-b) = 2 \cos(a) \sin(b)$$

$$\cos(a+b) + \cos(a-b) = 2 \cos(a) \cos(b)$$

$$\cos(a+b) - \cos(a-b) = -2 \sin(a) \sin(b)$$

$$2 \cos(A) \cos(B) = \cos(A-B) + \cos(A+B)$$

$$2 \sin(A) \sin(B) = \cos(A-B) - \cos(A+B)$$

$$2 \sin(A) \cos(B) = \sin(A+B) + \sin(A-B)$$

$$2 \cos(A) \sin(B) = \sin(A+B) - \sin(A-B)$$