6.300 Quiz 2

Spring 2023

Name:

Kerberos/Athena Username:

5 questions | 1 hour and 50 minutes

- Please **write your name and your Athena username in the box above**, and please do not write your name on any of the other pages of the exam.
- Please **WAIT** until we tell you to begin.
- This quiz is closed-book, but you may use two 8.5×11 sheets of paper (both sides) as a reference.
- You may **NOT** use any electronic devices (including computers, calculators, phones, etc.).
- If you have questions, please **come to us at the front** to ask them.
- Enter all answers in the boxes provided. Work on other pages with QR codes may be taken into account when assigning partial credit. **Please do not write on the QR codes.**
- If you finish the exam more than 10 minutes before the end time, please quietly bring your exam to us at the front of the room. If you finish within 10 minutes of the end time, please remain seated so as not to disturb those who are still finishing their quizzes.
- You may not discuss the details of the quiz with anyone other than course staff until final quiz grades have been assigned and released.

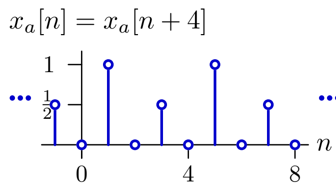
Worksheet (intentionally blank)

1 Summing Signals

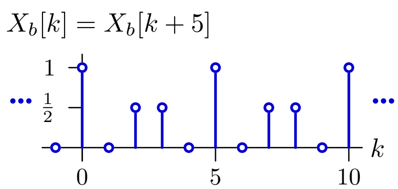
Let $x[n]$ represent the sum of two discrete-time signals:

$$x[n] = x_a[n] + x_b[n]$$

The signal $x_a[n]$, shown in the graph below, is periodic with a period of 4:



The signal $x_b[n]$ is represented by its DTFS coefficients $X_b[k]$, which are periodic with a period of 5 and which are shown in the graph below:



Find $x[-17]$.

$x[-17] =$

Worksheet (intentionally blank)

2 Sampling

Consider taking a signal $x(\cdot)$ that is periodic in $T = 1$ second and sampling at a sampling rate of 6 samples per second to obtain DT signal $x[\cdot]$ that is periodic in $N = 6$. Analyzing the resulting DT signal, you find that:

- $x[\cdot]$ is a symmetric function of n.
- $x[n]$ is positive for all values of n.
- $x[0] + x[1] + x[2] + x[3] + x[4] + x[5] = 3$.
- $x[0] - x[1] + x[2] - x[3] + x[4] - x[5] = 1$.
- most of the Fourier series coefficients $X[\cdot]$ are 0; only two out of every 6 coefficients are nonzero.

What are two distinct CT functions $x(\cdot)$ that could have produced the results shown above?

$x(t) =$ $\boxed{}$

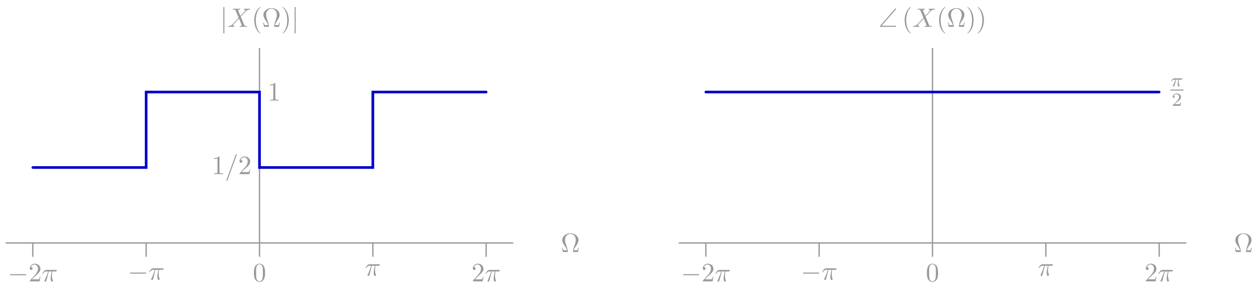
or

$x(t) =$ $\boxed{}$

3 Filtering

Part A

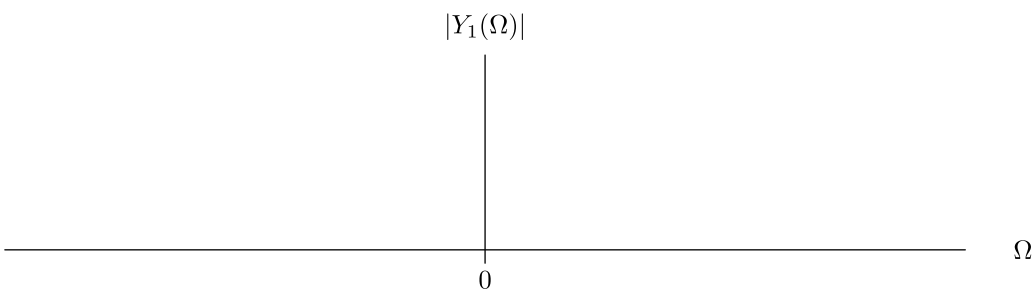
Consider a signal $x[n]$, whose DTFT magnitude and phase are shown below:

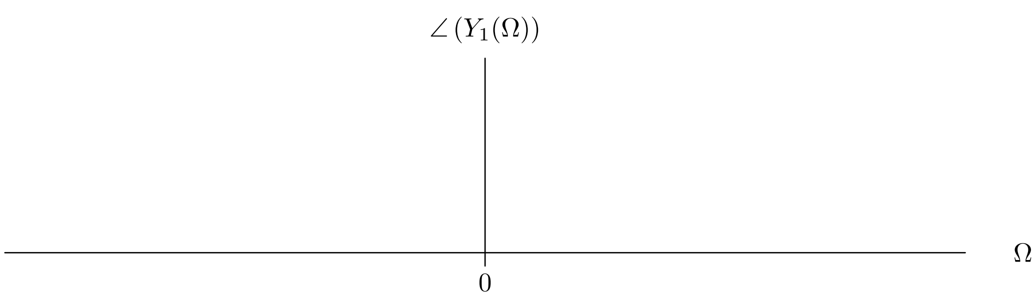


Assume that when $x[n]$ is the input to an LTI system whose frequency response is $H(\Omega) = e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} + e^{-j\frac{\Omega}{2}}\right)$, the output is a new signal $y_1[n]$:

$$x[n] \longrightarrow \boxed{\text{LTI}} \longrightarrow y_1[n]$$

Sketch the magnitude and phase of $Y_1(\Omega)$, the DTFT of $y_1[n]$, on the axes below, and label all key points:



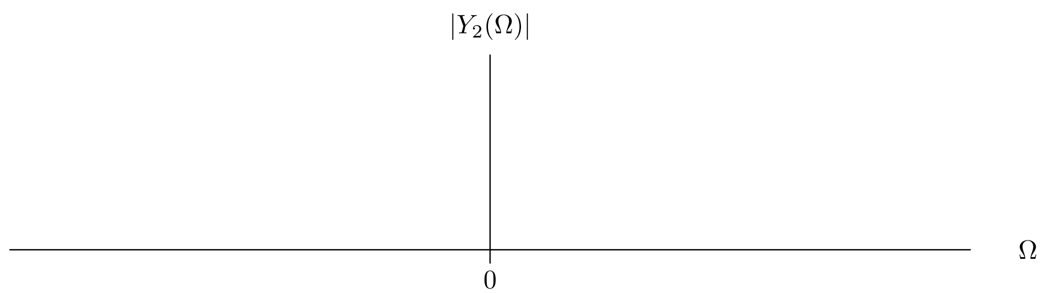


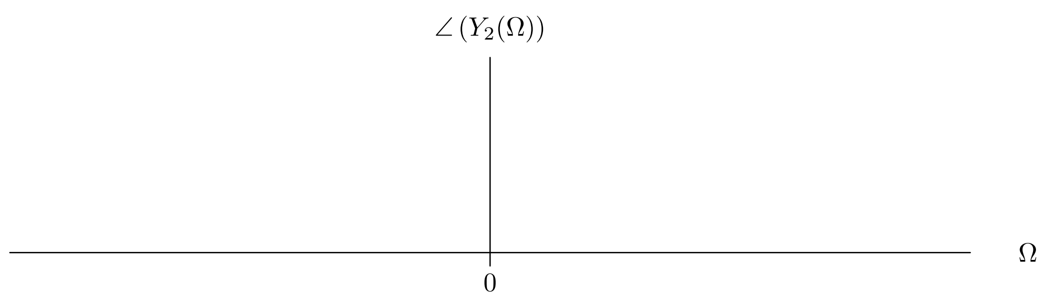
Part B

Now consider feeding $y_1[n]$ (the result from the last part) through the same system to produce a new result $y_2[n]$:

$$x[n] \rightarrow \boxed{\text{LTI}} \rightarrow y_1[n] \qquad\qquad y_1[n] \rightarrow \boxed{\text{LTI}} \rightarrow y_2[n]$$

Sketch the magnitude and phase of $Y_2(\Omega)$, the DTFT of $y_2[n]$, on the axes below, and label all key points:





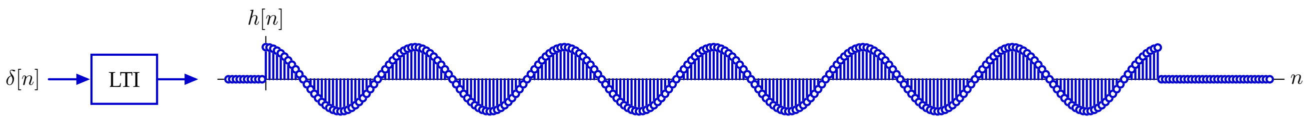
4 Sinusoidal Pulse

Part A

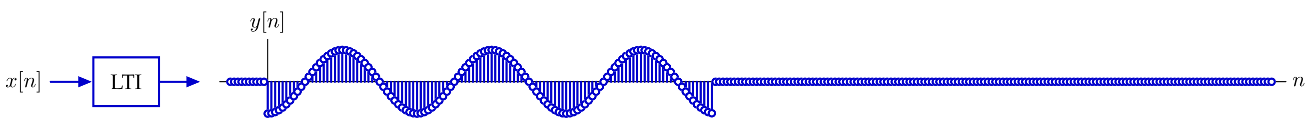
Consider an LTI system whose unit-sample response is given by:

$$h[n] = \begin{cases} \cos\left(\frac{\pi}{20}n\right) & \text{if } 0 \le n < 6N \\ 0 & \text{otherwise} \end{cases}$$

where N is the fundamental period of the cosine wave, so that exactly six full periods of the cosine can be seen in the unit sample response:



For some other input $x[n]$, this system's output will consist of **three** full periods of an **upside-down** cosine at the same frequency, followed by zeros for all time:



Determine this signal $x[n]$. In the boxes below, enter both the value of n and the associated value of $x[n]$ for the first 10 nonzero values of $x[n]$. The boxes below should contain **only numbers**. If there are fewer than 10 nonzero values in $x[n]$, write `None` in the remaining boxes.

n	$x[n]$

Part B

We could compute this response by multiplying the DTFTs of these two signals, $X(\Omega)$ and $H(\Omega)$. Find closed-form expressions for $X(\Omega)$ and $H(\Omega)$ and enter them in the boxes below.

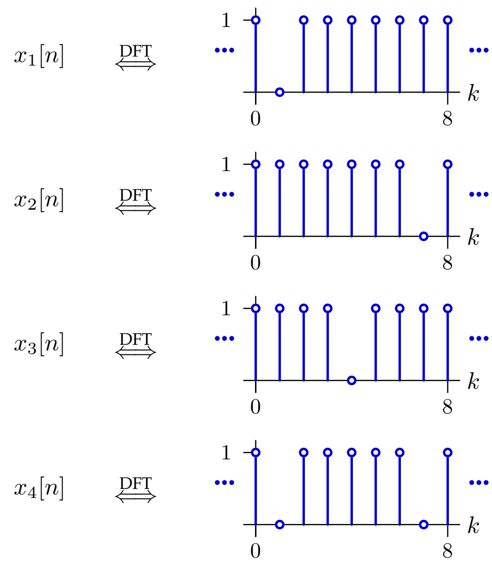
Hint: $\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$

$X(\Omega) =$

$H(\Omega) =$

5 Missing Components

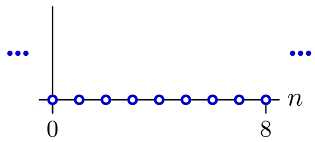
Consider a set of discrete-time signals $x_i[n]$ represented by the DFT coefficients (computed with $N = 8$) shown below:



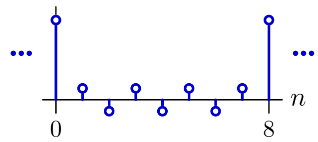
Fill in the table below by writing a single letter indicating the plot on the facing page (page 11) that matches the shape of the indicated signal. Note that the plots are not necessarily all on the same vertical scale.

Signal	Real Part	Imaginary Part	Magnitude	Phase
$x_1[n]$				
$x_2[n]$				
$x_3[n]$				
$x_4[n]$				

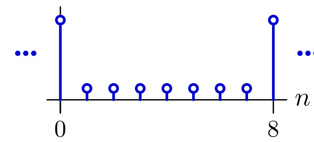
Plot A



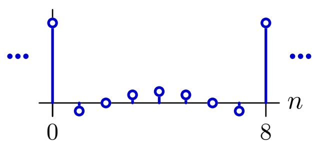
Plot B



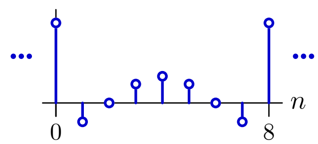
Plot C



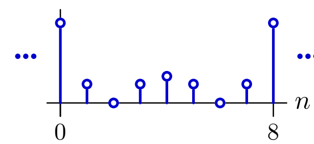
Plot D



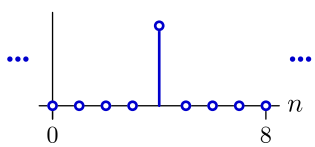
Plot E



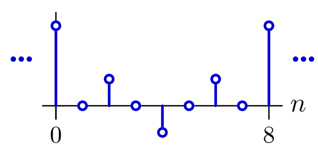
Plot F



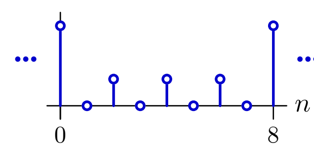
Plot G



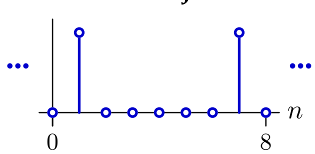
Plot H



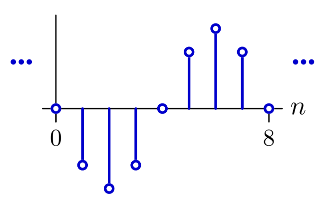
Plot I



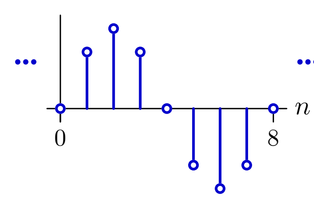
Plot J



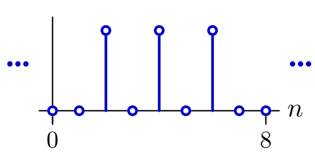
Plot K



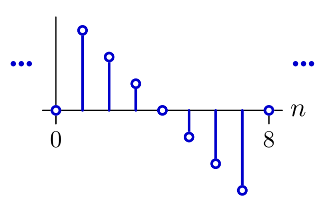
Plot L



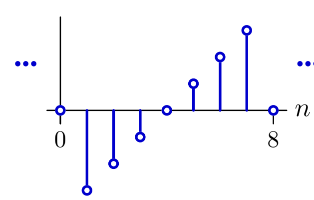
Plot M



Plot N



Plot O



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