6.300 Problem Set 2

Problem sets and computational labs are both released on Thursdays at 4:00 p.m.

Both are due the following Wednesday at 10:00 p.m.

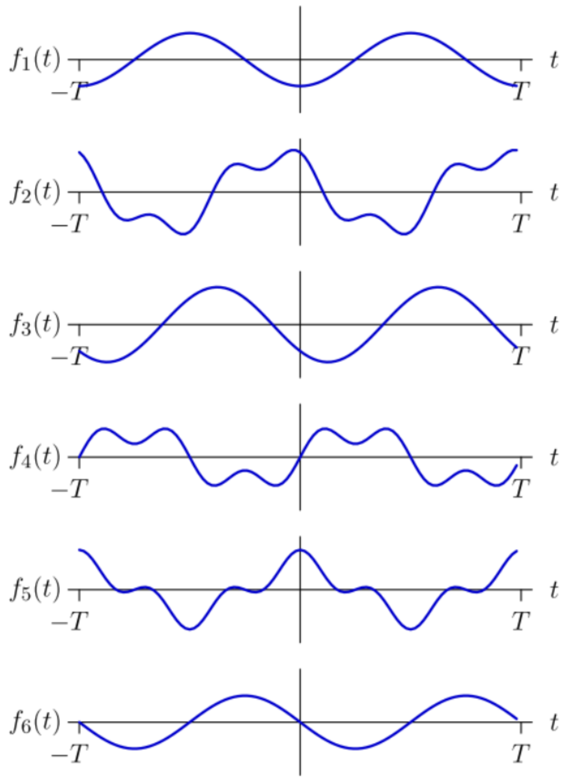
To receive credit for a lab check-in, the check-in must be completed by the following Monday at 9:00 p.m. Lab check-ins may be completed during common hours. A schedule of common hours is posted at `sigproc.mit.edu`.

Problem 1: Graphical Fourier Series

Part A

Each of plots $f_1(\cdot)$ through $f_6(\cdot)$ shows a periodic function with period $t = T$ that can be represented by a Fourier series of the following form:

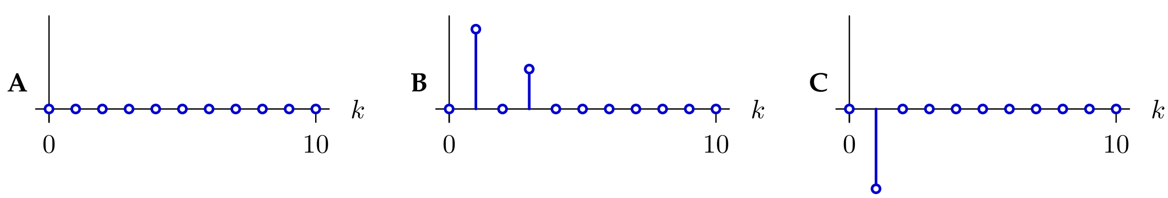
$$f(t) = c_0 + \sum_{k=1}^{10} c_k \cos(2\pi kt/T) + \sum_{k=1}^{10} d_k \sin(2\pi kt/T).$$



Each of the functions $f_i(\cdot)$ can be represented by an expansion of the following form:

$$f_i(t) = \sum_{k=0}^{\infty} c_k \cos\left(\frac{2\pi k}{T}t\right) + \sum_{k=0}^{\infty} d_k \sin\left(\frac{2\pi k}{T}t\right)$$

where c_k and d_k are represented by either **A**, **B**, or **C** (shown below), where values are assumed to be 0 for all values of k not pictured:

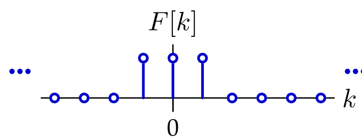


For each $f_i(\cdot)$, indicate which of **A**, **B**, or **C** corresponds to its c_k and d_k coefficients. Justify your answers.

Part B

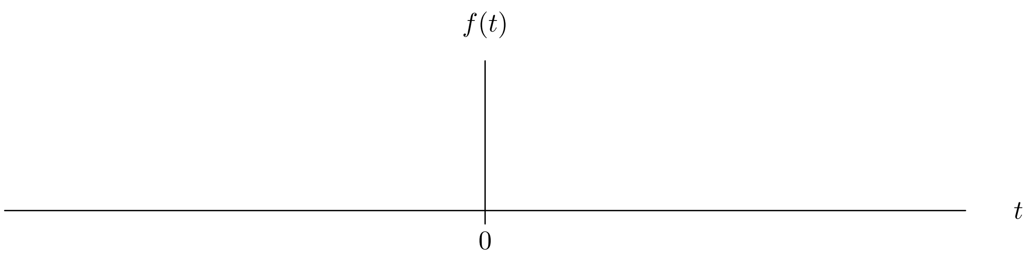
Consider a function $f(\cdot)$ that is periodic in $T = 2$ seconds. Its CTFS coefficients $F[\cdot]$ are shown below:

$$F[k] = \begin{cases} 1 & \text{if } -1 \leq k \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

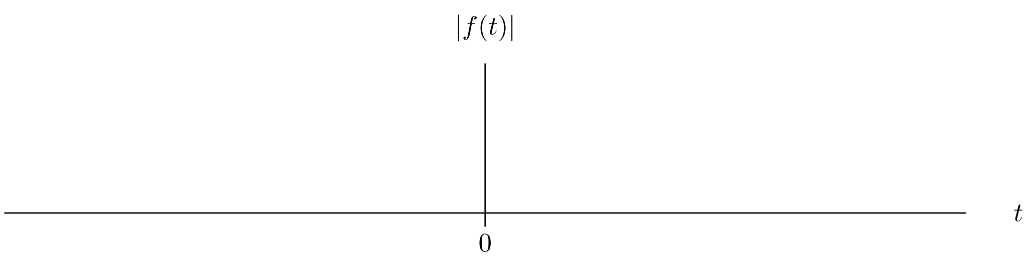


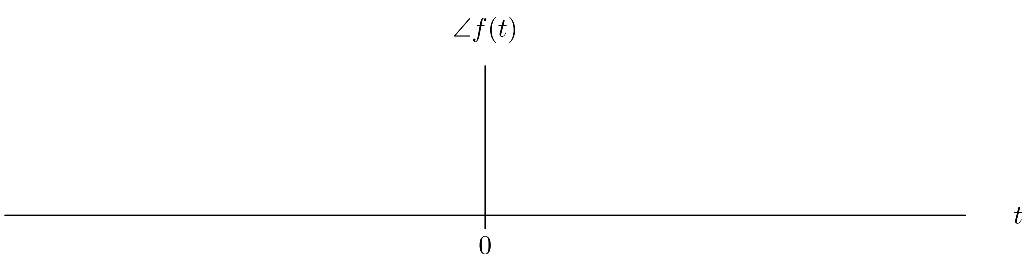
such that $f(t) = \displaystyle\sum_{k=-\infty}^{\infty} F[k] e^{j\frac{2\pi k}{T}t}$

$f(t)$ is purely real. Sketch a plot of $f(t)$ versus t:



Now, thinking of $f(t)$ as a number in the complex plane, sketch the magnitude and angle of $f(t)$ as functions of t:

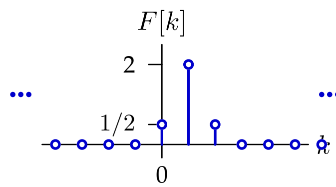




Part C

Consider a function $f(\cdot)$ that is periodic in $T = 3$ seconds. Its CTFS coefficients $F[\cdot]$ are shown below:

$$F[k] = \begin{cases} 1/2 & \text{if } k \in \{0, 2\} \\ 2 & \text{if } k = 1 \\ 0 & \text{otherwise} \end{cases}$$

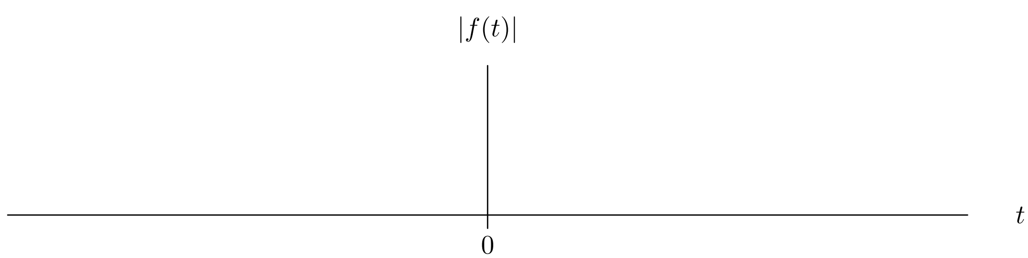


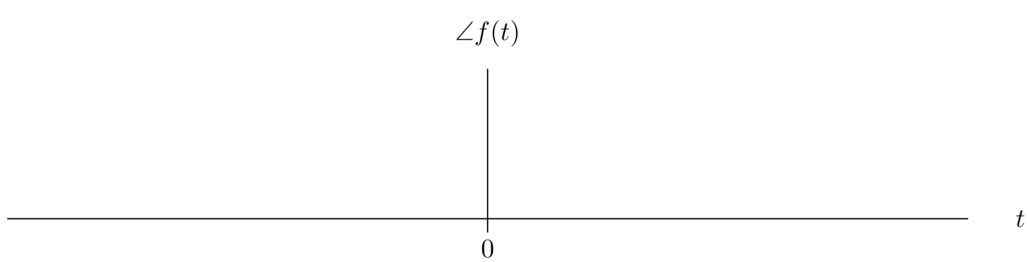
such that $f(t) = \displaystyle\sum_{k=-\infty}^{\infty} F[k] e^{j\frac{2\pi k}{T}t}$

$f(t)$ can be expressed in the following form: $f(t) = \left(e^{j\omega_1 t}\right) \times (\cos(\omega_2 t) + c)$

What are the necessary values of ω_1, ω_2, and c? Show your work.

Now, thinking of $f(t)$ as a number in the complex plane, sketch the magnitude and angle of $f(t)$ as functions of t:



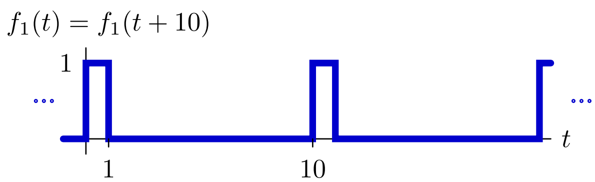


Notice that, unlike the function we looked at in a similar problem last week, $f(t)$ is not purely real-valued! What feature(s) of the coefficients are consequences of this fact?

Problem 2: Fourier Series

Part A

Let $f_1(t)$ represent the following function, which is periodic in t with period $T = 10$:

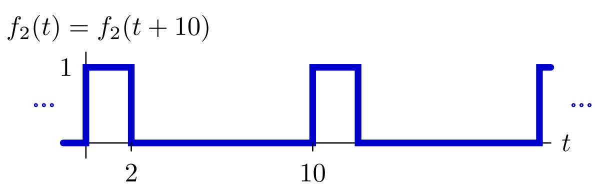


Find the coefficients $F_1[k]$ for a Fourier series expansion of $f_1(t)$ in complex-exponential form:

$$f_1(t) = \sum_{k=-\infty}^{\infty} F_1[k] e^{j\frac{2\pi k}{T}t}$$

Part B

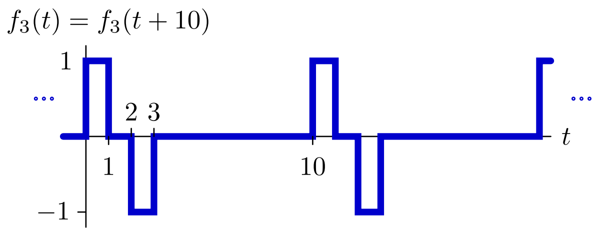
Let f_2 represent the following function that is periodic in t with period $T = 10$:



Find the Fourier series coefficients associated with this function in complex-exponential form.

Part C

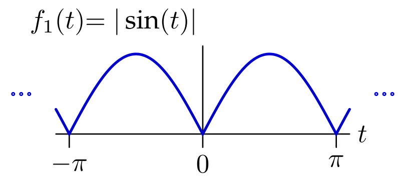
Let f_3 represent the following function that is periodic in t with period $T = 10$:



Find the Fourier series coefficients associated with this function in complex-exponential form.

Part D

Find the complex-exponential Fourier series coefficients $F_1[k]$ for the following function:



Part E

Find the complex-exponential Fourier series coefficients $F_2[k]$ for the following function:

$$f_2(t) = \cos\left(\frac{2t\pi}{3}\right)\sin\left(\frac{2t\pi}{9}\right)$$

Problem 3: Fourier Series — Trigonometric Form vs. Complex Form

In this problem, we compare two methods for expanding a function $f(\omega_0 t)$ as a series of the following form:

$$f(\omega_0 t) = c_0 + \sum_{k=1}^{\infty} c_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} d_k \sin(k\omega_0 t)$$

Part A

Use trigonometric identities and the rules of ordinary algebra to determine the values of the non-zero coefficients c_k and d_k needed to expand the function $f_1(\omega_0 t) = \cos^5(\omega_0 t)$. Show your work.

Part B

An alternative to trigonometric identities is to use complex exponentials. Determine the non-zero coefficients c_k and d_k as in the previous part – but this time use Euler's formula and complex numbers, but no trigonometric identifies.

Part C

Use trigonometric identities plus the rules of ordinary algebra to determine the values of the non-zero coeeficients c_k and d_k needed to expand the function $f_2(\omega_0 t) = \sin^5(\omega_0 t)$. Show your work.

Part D

Determine the non-zero coefficients c_k and d_k as in the previous part – but this time use Euler's formula and complex numbers, but no trigonometric identities.

Part E

List the mathematical relations that you used in each of the previous parts. Briefly describe the pros and cons of using trigonometric identities versus Euler's formula.