6.300 Problem Set 1
Answers

This problem set consists of three problems. Don't forget to complete this week's lab, which can be found on Canvas.

- Problem 1 (Symmetry) starts on page 2.
- Problem 2 (Complex Numbers) starts on page 6.
- Problem 3 (Complex Numbers in Polar Form) starts on page 10.

These problems are intended to be done without the aid of computers, graphing libraries, calculators, and so on, except where those tools are explicitly mentioned. For the problems where they are *not* mentioned, you are welcome to use them to check your work, but you'll generally get more out of the experience if you work the problems by hand.

We have also included some brief notes about complex numbers at the end of this problem set, starting on on page 11. There are no required exercises in those pages; they are simply provided as a reference.

Problem 1: Symmetry

1.1 Part 1

Recall the following definitions:

- a CT function is a *symmetric* function of t if, for all t, $x(t) = x(-t)$
- a CT function is an *antisymmetric* function of t, for all t, $x(t) = -x(-t)$
- a DT function is a *symmetric* function of n if, for all n, $x[n] = x[-n]$
- a DT function is an *antisymmetric* function of n if, for all n, $x[n] = -x[-n]$

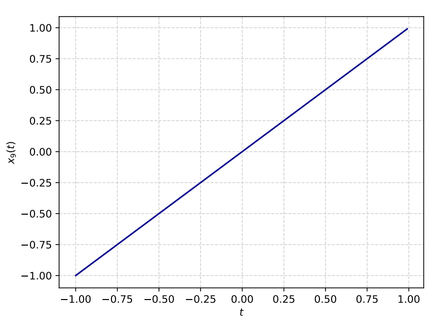
Of the CT signals below, which are symmetric functions of t, which are antisymmetric functions of t, and which are neither? For the functions represented by graphs, assume that $x_i(t) = 0$ for all values of t that are not shown.

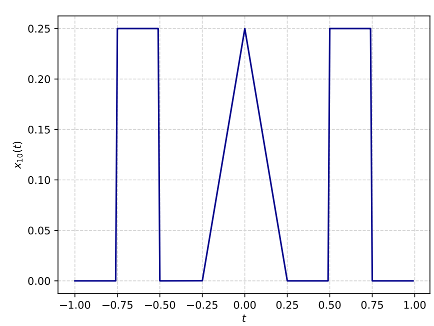
$$x_1(t) = \cos(\pi t) \qquad x_2(t) = \cos(\tfrac{\pi}{2} t)$$

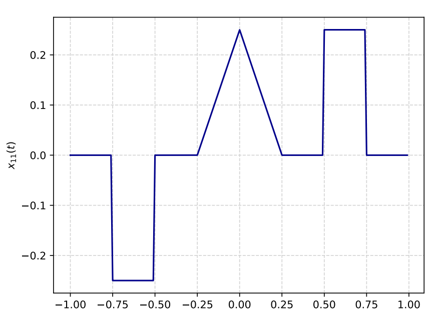
$$x_3(t) = \sin(\pi t) \qquad x_4(t) = \sin(\tfrac{\pi}{2} t)$$

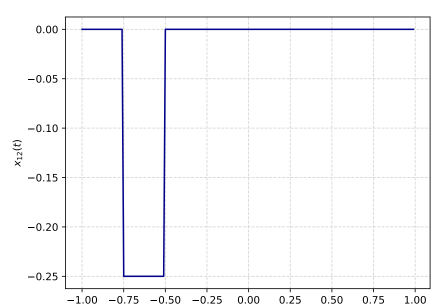
$$x_5(t) = \cos(\pi t) - \cos(2\pi t) \qquad x_6(t) = \sin(\pi t) - \cos(2\pi t)$$

$$x_7(t) = \cos(\pi t) - \sin(2\pi t) \qquad x_8(t) = \sin(\pi t) - \sin(2\pi t)$$









Symmetric: x_1, x_2, x_5, x_{10}
Antisymmetric: x_3, x_4, x_8, x_9
Neither: x_6, x_7, x_{11} (almost antisymmetric but not quite!), x_{12}

1.2 Part 2

Imagine adding two signals $x_1[\cdot]$ and $x_2[\cdot]$ to produce a new signal $x_3[\cdot]$ such that $x_3[n] = (x_1 + x_2)[n] = x_1[n] + x_2[n]$ for all n. If $x_1[n]$ and $x_2[n]$ are both *symmetric* functions of n, which of the following best describes $x_3[\cdot]$?

- $x_3[n]$ will always be a symmetric function of n
- $x_3[n]$ will always be an antisymmetric function of n
- $x_3[n]$ will never be symmetric or antisymmetric
- None of the above

Justify your answer in your submission.

We know that $x_3[n] = x_1[n] + x_2[n]$ for all n.
Since $x_1[n] = x_1[-n]$ and $x_2[n] = x_2[-n]$ for all n, we have $x_3[n] = x_1[n] + x_2[n] = x_1[-n] + x_2[-n] = x_3[-n]$.
Therefore, $x_3[n]$ is guaranteed to be a symmetric function of n.

Imagine adding two signals $x_4[\cdot]$ and $x_5[\cdot]$ to produce a new signal $x_6[\cdot]$ such that $x_6[n] = (x_4 + x_5)[n] = x_4[n] + x_5[n]$ for all n. If $x_4[n]$ and $x_5[n]$ are both *antisymmetric* functions of n, which of the following best describes $x_6[\cdot]$?

- $x_6[n]$ will always be a symmetric function of n
- $x_6[n]$ will always be an antisymmetric function of n
- $x_6[n]$ will never be symmetric or antisymmetric
- None of the above

Justify your answer in your submission.

We know that $x_6[n] = x_4[n] + x_5[n]$ for all n.
Since $x_4[n] = -x_4[-n]$ and $x_5[n] = -x_5[-n]$ for all n, we have:
$x_6[n] = x_4[n] + x_5[n] = -x_4[-n] - x_5[-n] = -(x_4[-n] + x_5[-n]) = -x_6[-n]$.
Therefore, $x_6[n]$ is guaranteed to be an antisymmetric function of n.

Imagine adding two signals $x_7[\cdot]$ and $x_8[\cdot]$ to produce a new signal $x_9[\cdot]$ such that $x_9[n] = (x_7 + x_8)[n] = x_7[n] + x_8[n]$ for all n. If $x_7[n]$ is symmetric and $x_8[n]$ is antisymmetric, which of the following best describes $x_9[\cdot]$?

- $x_9[n]$ will always be a symmetric function of n
- $x_9[n]$ will always be an antisymmetric function of n
- $x_9[n]$ will never be symmetric or antisymmetric
- None of the above

Justify your answer in your submission.

We cannot say much about $x_9[\cdot]$, unfortunately. In some general sense, Since $x_9[n] = x_7[-n] - x_8[-n]$, it looks like it is guaranteed that $x_9[\cdot]$ is neither symmetric nor antisymmetric.
However, if $x_7[\cdot]$ or $x_8[\cdot]$ are the zero function (0 for all values of n), then $x_9[\cdot]$ might be symmetric/antisymmetric.

1.3 Part 3

We would like to express an arbitrary discrete-time signal $x[\cdot]$ as the sum of a symmetric part $x_s[\cdot]$ (where $x_s[-n] = x_s[n]$) and an antisymmetric part $x_a[\cdot]$ (where $x_a[-n] = -x_a[n]$) such that $x[n] = x_s[n] + x_a[n]$ for all n.

Is such a decomposition possible for all possible signals $x[\cdot]$? When the decomposition is possible, is the answer always unique?

Assume that $x[n]$ can be written as the sum of symmetric and antisymmetric parts:

$$x[n] = x_s[n] + x_a[n]$$

Then we have:

$$x[-n] = x_s[-n] + x_a[-n] = x_s[n] - x_a[n]$$

For every value of n, the previous two equations provide two constraints on two unknowns: $x_s[n]$ and $x_a[n]$. Solving, we find that

$$x_s[n] = \frac{x[n] + x[-n]}{2} \text{ and}$$
$$x_a[n] = \frac{x[n] - x[-n]}{2}$$

The case when $n = 0$ looks special, but is not. There are still two unknowns, $x_s[0]$ and $x_a[0]$, but now just one known quantity $x[0]$, which appears in two different equations:

$$x[0] = x_s[0] + x_a[0] = x[-0] = x_s[0] - x_a[0]$$

so that there is still a single unique solution:

$$x_s[0] = x[0] \text{ and } x_a[0] = 0$$

Since these solutions always exist (for well-behaved signals), the decomposition into symmetric and antisymmetric parts is always possible. Since the equations are linear and independent, there is a single solution, thus the decomposition is always unique.

1.4 Part 4

Let $x[\cdot]$ represent the signal whose samples are given by

$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

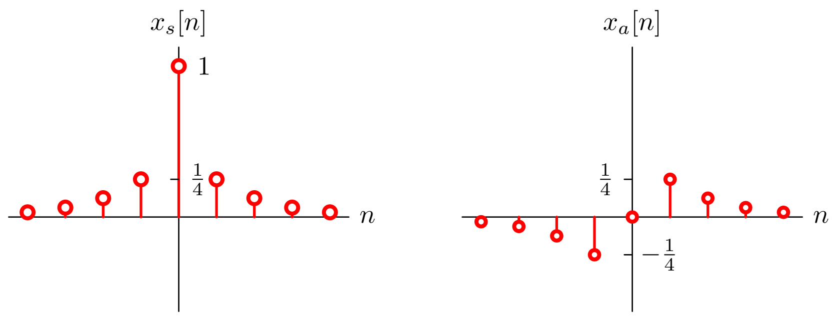
Determine expressions for functions $x_s[\cdot]$ and $x_a[\cdot]$ such that $x_s[\cdot]$ is a symmetric function of n and $x_a[\cdot]$ is an antisymmetric function of n, and $x_a[n] + x_s[n] = x[n]$ for all n. Try to simplify your expressions down, but it is totally OK to leave the end result as a piece-wise function.

Once you have that expression, draw (by hand) a sketch of $x_s[n]$ and $x_a[n]$. Do their shapes make sense given your work from part 3?

Using the result from the last subproblem:

$$x_s[n] = \frac{x[n] + x[-n]}{2} = \begin{cases} \frac{1}{2}\left(\frac{1}{2}\right)^n & \text{if } n > 0 \\ 1 & \text{if } n = 0 \\ \frac{1}{2}\left(\frac{1}{2}\right)^{-n} & \text{if } n < 0 \end{cases}$$

$$x_a[n] = \frac{x[n] - x[-n]}{2} = \begin{cases} \frac{1}{2}\left(\frac{1}{2}\right)^n & \text{if } n > 0 \\ 0 & \text{if } n = 0 \\ -\frac{1}{2}\left(\frac{1}{2}\right)^{-n} & \text{if } n < 0 \end{cases}$$



Problem 2: Complex Numbers

2.1 Part 1

Consider the following series expansions:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{(2n+1)}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

Use these expansions to verify Euler's formula. Show your work.

We want to show that $\cos\theta + j\sin\theta = e^{j\theta}$. Starting with the expansion above, we have:

$$e^{j\theta} = 1 + j\theta + \frac{j^2\theta^2}{2!} + \frac{j^3\theta^3}{3!} + \frac{j^4\theta^4}{4!} + \frac{j^5\theta^5}{5!} + \frac{j^6\theta^6}{6!} + \frac{j^7\theta^7}{7!} + \ldots$$

$$= 1 + j\theta - \frac{\theta^2}{2!} - \frac{j\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{j\theta^5}{5!} - \frac{\theta^6}{6!} - \frac{j\theta^7}{7!} + \ldots$$

Grouping together terms that have a j gives:

$$e^{j\theta} = \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots\right) + \left(j\theta - \frac{j\theta^3}{3!} + \frac{j\theta^5}{5!} - \frac{j\theta^7}{7!} + \ldots\right)$$

Pulling out a factor of j:

$$e^{j\theta} = \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots\right) + j\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots\right)$$

The parenthesized series above are the expansions of $\cos\theta$ and $\sin\theta$, respectively, so we have:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

2.2 Part 2

Use Euler's formula to prove the following:

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Show your work.

By Euler's formula,

$$e^{j\theta} + e^{-j\theta} = \cos(\theta) + j\sin(\theta) + \cos(-\theta) + j\sin(-\theta)$$

Since cosine is symmetric, $\cos(-\theta) = \cos(\theta)$. And since sine is antisymmetric, $\sin(-\theta) = -\sin(\theta)$. Substituting:

$$e^{j\theta} + e^{-j\theta} = \cos(\theta) + j\sin(\theta) + \cos(\theta) - j\sin(\theta) = 2\cos(\theta)$$

$$e^{j\theta} + e^{-j\theta} = \cos(\theta) + j\sin(\theta) - \cos(\theta) + j\sin(\theta) = 2j\sin(\theta)$$

2.3 Part 3

Express each of the following numbers in rectangular form. Note that you may find it useful to convert some or all parts of some expressions to polar form as you're working toward an answer.

$3e^{j\pi/3} + 4e^{-j\pi/6}$

Adding complex numbers tends to be easiest in rectangular form, so we will first convert the two numbers to rectangular form.

$$3e^{j\pi/3} = 3\cos(\pi/3) + 3j\sin(\pi/3) = \frac{3}{2} + \frac{3\sqrt{3}}{2}j$$

$$4e^{-j\pi/6} = 4\cos(\pi/6) - 4j\sin(\pi/6) = \frac{4\sqrt{3}}{2} - 2j$$

Their sum, therefore, is:

$$3e^{j\pi/3} + 4e^{-j\pi/6} = \frac{4\sqrt{3}+3}{2} + \frac{3\sqrt{3}-4}{2}j$$

$\left(\sqrt{3}+j\right)^{11}$

Exponentiation tends to be easiest in polar form. So we'll start by converting this number to polar form. This is a number with magnitude 2, and its angle in the complex plane is $\tan^{-1}(1/\sqrt{3}) = \pi/6$.
Therefore, we have:

$$\left(\sqrt{3}+j\right)^{11} = \left(2e^{j\pi/6}\right)^{11} = 2^{11}e^{j11\pi/6} = 2048e^{-j\pi/6}$$

Converting back to rectangular form, we have:

$$\left(\sqrt{3}+j\right)^{11} = 2048e^{-j\pi/6} = 2048\left(\cos(-\pi/6) + j\sin(-\pi/6)\right) = 2048\left(\cos(\pi/6) - j\sin(\pi/6)\right)$$

$$= 2048\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) = 1024\sqrt{3} - 1024j$$

$\sqrt{-j}$

This is difficult to see at first, perhaps, but we can start by noting that $-j = e^{-j\frac{\pi}{2}}$.
Then, to find the square root, we can raise this to the power $1/2$, which gives:

$$\sqrt{-j} = \left(e^{-j\frac{\pi}{2}}\right)^{1/2} = e^{-j\frac{\pi}{4}} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

Notice that this is the same as halving the angle of $-j$.

$\left(\sqrt{2}e^{j\pi/2} + \sqrt{2}e^{j\pi/4} + \frac{2}{\sqrt{2}}e^{j5\pi/4} + \frac{1}{\sqrt{2}}e^{j\pi} - \frac{1}{\sqrt{2}}j\right)^{218}$

Firstly, let's figure out an equivalent expression for the value in the parentheses.
The $\sqrt{2}e^{j\pi/4}$ and $\frac{2}{\sqrt{2}}e^{j5\pi/4}$ terms have equal magnitude and complementary angles, so they entirely cancel each other out when added together. After this cancellation, we are interested in:

$$\left(\sqrt{2}e^{j\pi/2} + \frac{1}{\sqrt{2}}e^{j\pi} - \frac{1}{\sqrt{2}}j\right)^{218}$$

Since $e^{j\pi} = -1$ and $e^{j\pi/2} = j$, we can write the above as:

$$\left(-\frac{1}{\sqrt{2}} + \sqrt{2}j - \frac{1}{\sqrt{2}}j\right)^{218}$$

$$= \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j\right)^{218}$$

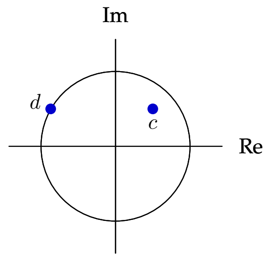
The value in the parentheses is a complex number with a magnitude of 1 and an angle of $3\pi/4$. Thus, the expression about can be rewritten as:

$$\left(e^{j3\pi/4}\right)^{218}$$

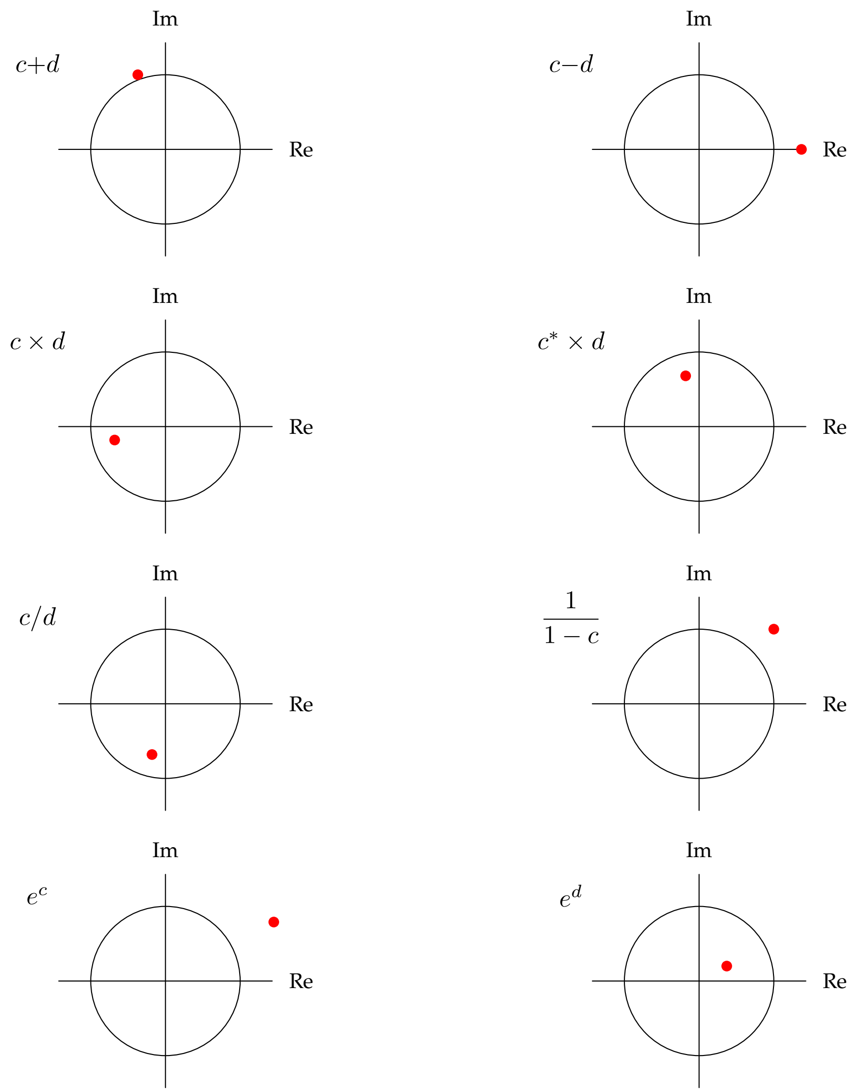
This is a complex number whose magnitude is 1 and whose angle is $\frac{\pi\times3\times218}{4}$. We can ignore full revolutions (multiples of 2π in the angle), so the angle can also be written as $\frac{\pi\times((3\times218) \mod 8)}{4} = \frac{\pi\times((654) \mod 8)}{4} = \frac{6\pi}{4} = \frac{3\pi}{2}$.
So ultimately, we have a number whose magnitude is 1 and whose angle is $\frac{3\pi}{2}$. In rectangular form, this number is $-j$.

Problem 3: Complex Numbers in Polar Form

Let c and d represent the complex numbers shown by filled dots in the following diagram, where the real and imaginary parts of the complex numbers are shown on the horizontal and vertical axes, respectively, and the circle has a radius of 1.



Below are eight complex-valued functions of c and d, each paired with a depiction of the complex plane demarked by the unit circle. Evaluate each expression and mark its value on the complex plane with a dot. Note that e represents Euler's number (2.71828...) and c^* represents the complex conjugate of c.

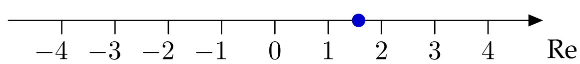


Tutorial: Complex Numbers and Complex Exponentials

Complex numbers are central to a number of techniques discussed in this class. As such, we'll use the next few pages to talk a bit about the nature of complex numbers, but with a particular focus on algebraic and geometric interpretations of operations on complex numbers. Some of the problems in the pset will give you an opportunity to practice with operations on complex numbers.

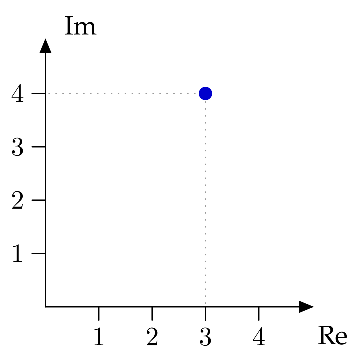
Graphical Representations of Real and Complex Numbers

To start discussing complex numbers, we'll first consider the real numbers (such as 10, -4, 0.3, -2.2, π, etc.). A real number can be represented as a point along an infinite one-dimensional number line, which we'll call the *real axis*, shown below:



Any real number can be represented by a single point on this line. For example, the blue dot above represents the real number $\frac{\pi}{2}$.

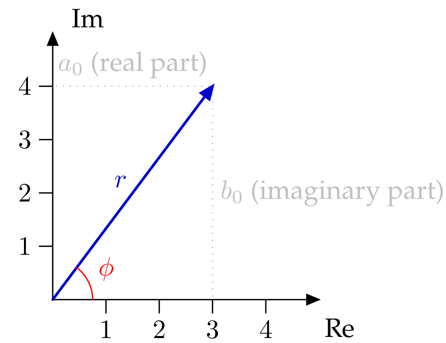
You may already be familiar with complex numbers written in their *rectangular form* (also called the *Cartesian form*), which separates a complex number into its real and imaginary parts. Complex numbers in rectangular form are written as $a_0 + b_0 j$, where both a_0 and b_0 are themselves real numbers, and j is the imaginary unit[1] ($\sqrt{-1}$). A complex number can be represented using two real numbers; as such, we can represent a complex number as a point in a two-dimensional space that we'll refer to as the *complex plane*. Below is a diagram showing a portion of the complex plane, with the number $3 + 4j$ indicated as a point:



[1]Throughout this course, as in many engineering disciplines, we will use the letter j to represent the imaginary unit $\sqrt{-1}$. Some other disciplines use the letter i to represent the imaginary unit, but in engineering, the letter i is traditionally reserved for electrical current.

Polar Form

Importantly, we can think of this number as a vector in the complex plane, as shown below[2]. This vector has a magnitude (let's call it r) and an angle (ϕ) in the complex plane.



This representation opens the door to some interesting geometric/graphical interpretation, which can help us reason about operations on complex numbers. To start with, we can define r and ϕ in terms of a_0 and b_0:

$$r = \sqrt{a_0^2 + b_0^2}$$
$$\phi = \tan^{-1}\left(\frac{b_0}{a_0}\right)$$

We can also define a_0 and b_0 in terms of r and ϕ:

$$a_0 = r\cos(\phi)$$
$$b_0 = r\sin(\phi)$$

From there, we can rewrite our rectangular form as $r(\cos\phi + j\sin\phi)$, which is called the *trigonometric form*.

From there, we can use Euler's equation ($e^{j\theta} = \cos\theta + j\sin\theta$) to express our complex number as $re^{j\phi}$. This form, where we express a complex number in terms of its magnitude and angle (often referred to as its *phase*) in the complex plane, is referred to as *polar form* (also sometimes called the *exponential form*).

Polar form is the primary form we'll use throughout this class. However, it is worth noting that both rectangular and polar forms have their uses: certain operations are easier to perform and/or to understand when working with one form, versus the other, as we'll see in the following sections.

Note also that the polar representation of a number is not unique; that is to say that there are many different ways to represent the same number in polar form. Since increasing ϕ involves increasing the angle, any number $re^{j\phi}$ could also (equivalently) be represented by $re^{j(\phi+2\pi)}$. One way to see this is to note that both the real part $a_0 = r\cos(\phi)$ and $b_0 = r\sin(\phi)$ are both periodic functions of ϕ, with a period of 2π.

[2] This kind of diagram, which shows complex numbers as vectors in the complex plane, is often referred to as an *Argand diagram*.

Basic Operations

Next, let's look at arithmetic and geometric interpretations of various operations on complex numbers. As we'll see, there is no universal right answer for the question of "which representation is best?" Rather, the operation we perform determines the form we want to use.

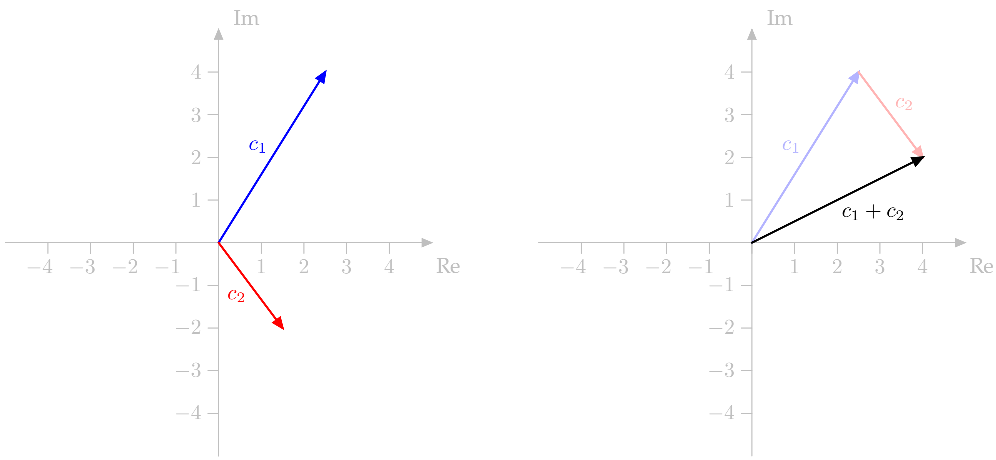
Addition

Let's start by considering the addition of two complex numbers, c_1 and c_2. For addition, rectangular form tends to be the easiest form, since the real and imaginary parts of these numbers add independently. That is, for imaginary numbers $c_1 = a_1 + b_1 j$ and $c_2 = a_2 + b_2 j$, we have:

$$c_1 + c_2 = a_1 + b_1 j + a_2 + b_2 j = (a_1 + a_2) + (b_1 + b_2)j$$

The result is a new complex number, whose real part is $a_1 + a_2$, and whose imaginary part is $b_1 + b_2$.

Geometrically, this means that, when viewing complex numbers as vectors in the complex plane, they add like regular vectors. For example, consider the following depiction of adding two complex numbers. On the left, we show vectors representing two numbers c_1 and c_2. On the right, we show those vectors stacked "end-to-end," as well as their sum:



Subtraction works similarly in that the real parts and imaginary parts subtract independently:

$$c_1 - c_2 = (a_1 - a_2) + (b_1 - b_2)j$$

Multiplication

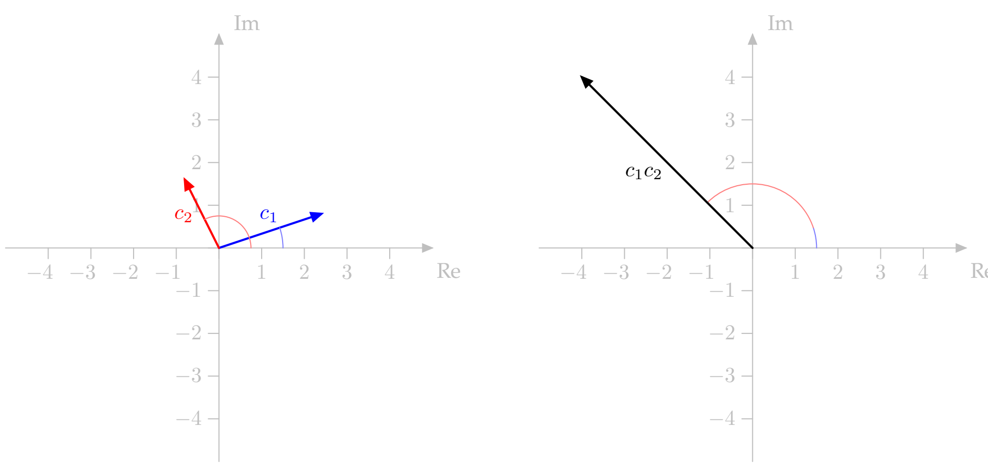
Now let's consider the multiplication of two complex numbers, c_1 and c_2. We *could* approach this by multiplying the rectangular forms together and applying the "FOIL" rule to compute the product:

$$c_1c_2 = (a_1 + b_1j)(a_2 + b_2j) = (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)j$$

However, this process is a bit of a pain. It's already tedious with only two numbers, and it gets even more painful when when we are considering the product of more than just two numbers. However, thinking about the product becomes a bit easier if we represent both numbers in their polar forms ($c_1 = r_1e^{j\phi_1}$ and $c_2 = r_2e^{j\phi_2}$):

$$c_1c_2 = (r_1e^{j\phi_1})(r_2e^{j\phi_2}) = (r_1r_2)e^{j(\phi_1+\phi_2)}$$

This is a new complex number with magnitude r_1r_2, and with angle $\phi_1 + \phi_2$: the magnitudes multiply, and the angles sum. It's worth noting that while complex numbers add like vectors, they do not follow the normal rules for vector multiplication. Rather, multiplication of complex numbers has a different interesting geometric interpretation that involves *rotating* through the complex plane.



Division works in a similar way, producing a new complex number whose magnitude is a ratio of the input magnitudes and whose angle is the difference of the input angles:

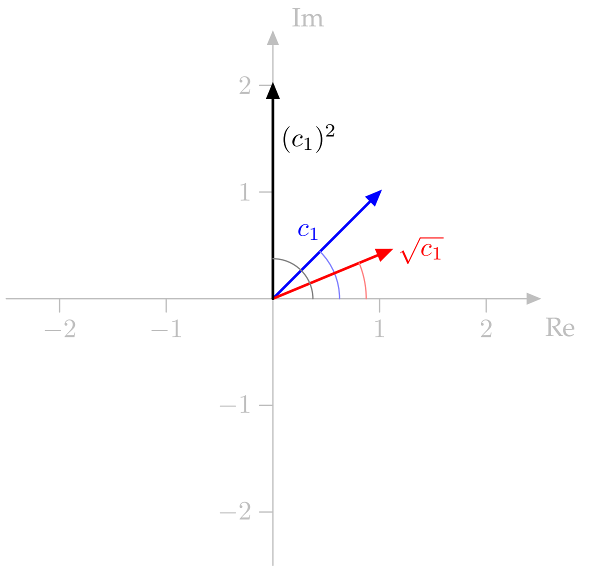
$$\frac{c_1}{c_2} = \frac{r_1e^{j\phi_1}}{r_2e^{j\phi_2}} = \left(\frac{r_1}{r_2}\right)e^{j(\phi_1-\phi_2)}$$

Exponentiation

Finally (for now), let's consider raising a complex number c_1 to a power. As with multiplication, we *could* do this using rectangular form, but given the relationship between exponentiation and multiplication (and the fact that multiplication was way easier in polar form than rectangular form), polar form seems like a better bet. If we represent c_1 as

$$c_1{}^x = \left(r_1 e^{j\phi_1}\right)^x = \left(r_1{}^x\right) e^{j\phi_1 x}$$

Note that this result is a new complex number whose magnitude is r_1^x and whose angle is $\phi_1 \cdot x$. For example:



In this example, $c_1 = 1 + j$. $|c_1| = \sqrt{2}$ and $\angle c_1 = \frac{\pi}{4}$, so it can be represented as $c_1 = 1 + j = \sqrt{2}e^{j\frac{\pi}{4}}$, which is shown in blue above. We can also see two results: squaring c_1 gives us a new number whose angle is $2\angle c_1 = \frac{\pi}{2}$ and whose magnitude is $|c_1|^2 = 2$. This number is $2e^{j\frac{\pi}{2}}$, or $2j$, which is shown in black above. Similarly, we can find the square root of c_1 by raising it to the $\frac{1}{2}$ power, which gives us a new number whose magnitude is $\sqrt{|c_1|} = \sqrt[4]{2}$ and whose angle is $\frac{\angle c_1}{2} = \frac{\pi}{8}$. This number is shown in red above.

Operations: Summary

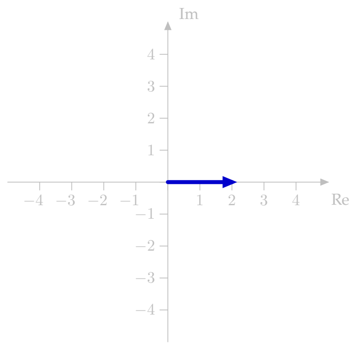
Developing facility with moving between these representations is a crucial skill to develop as we work through 6.300, in part because different representations lend themselves to performing different operations. As we move forward with the class, we'll spend a lot more time talking about these ideas. But for now, we hope that what you take away from these notes is an exposure to complex numbers (particularly in polar form and a geometric interpretation of those numbers; as well as an idea about *when* each representation is useful. Specifically,

- For addition and subtraction, rectangular form tends to be easiest.
- For multiplication, division, and exponentiation, polar form tends to be easiest.

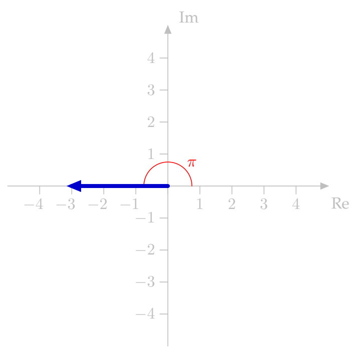
Real Numbers in Rectangular and Polar Form

It is perhaps worth mentioning, at this point, that we can use any of the above representations to represent a purely real number as well. If we have a positive, real number n, that's already represented in rectangular form, where the imaginary part just happens to be 0. And so we can bring all of the nice geometric interpretation from above to bear on real numbers as well (those interpretations aren't limited to numbers with a nonzero imaginary part!).

In polar form, we can gain some insight by plotting that number as a vector on the complex plane, where we can see that this number has a magnitude of n, and it has an angle of 0, so it can be represented as $n = ne^{j0}$:



In the case of a negative, real-valued number (let's call it $-n$), we see something slightly different. Here, the magnitude is n, but the angle is π, so we can represent it as $-n = ne^{j\pi}$:



If you ever wondered about Euler's identity, $e^{j\pi} = -1$, now we can make sense of it! All that that's saying is that negative one is a number whose magnitude is 1 and whose angle in the complex plane is π!