6.300 Signal Processing

Week 12, Lecture A:
2D Fourier Representation

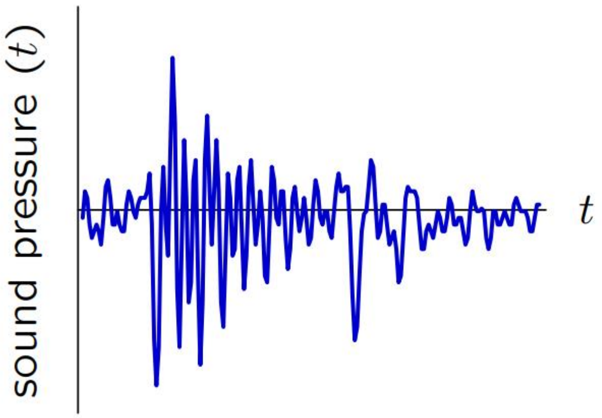
- Introduction to 2D Signal Processing
- 2D Fourier Representations

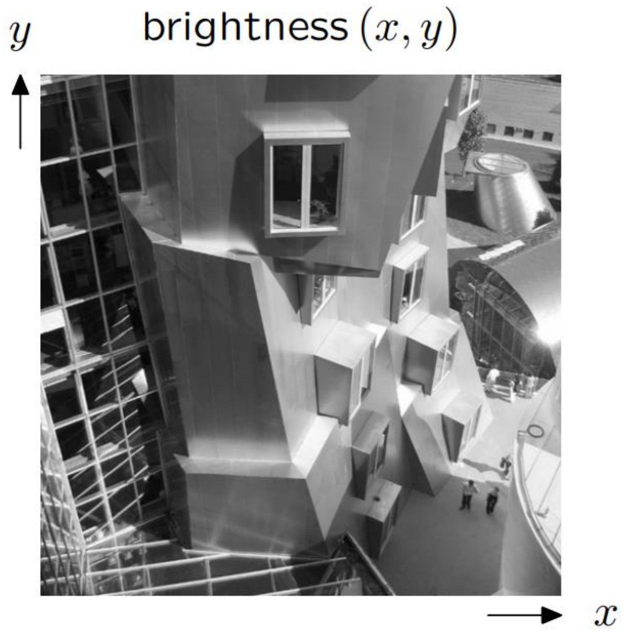
Lecture slides are available on CATSOOP:

https://sigproc.mit.edu/fall24

Signals: Functions Used to Convey Information

- Signals may have 1 or 2 or 3 or even more independent variables.

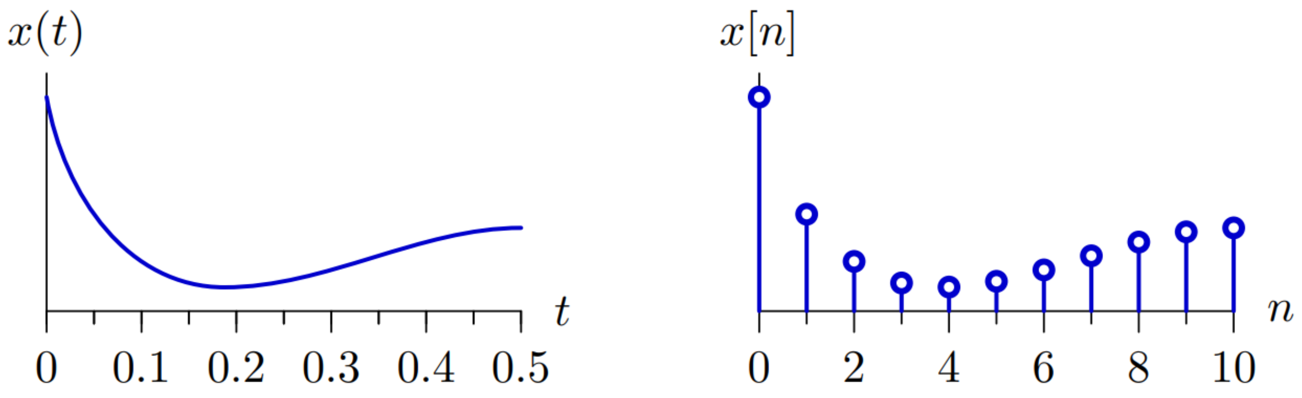




A 1D signal has a one-dimensional domain.
We usually think of it as time t or discrete time n.

A 2D signal has a two-dimensional domain.
We usually think of the domains as x and y or n_x and n_y (or r and c).

Signals: Continuous vs. Discrete



Signals from physical systems are often of continuous domain:

- continuous time – measured in seconds, etc: f(t)
- continuous spatial coordinates – measured in meters, cm, etc : f(x, y)

Computations usually manipulate functions of discrete domain:

- discrete time – measured in samples: f[n]
- discrete spatial coordinates – measured in pixels: f[r, c]

Fourier Representations

From "Continuous Time" to "Continuous Space."

1D Continuous-Time Fourier Transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\, dt$$

Analysis equation

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot e^{j\omega t}\, d\omega$$

Synthesis equation

Two dimensional CTFT:

$$F\left(\omega_x, \omega_y\right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \cdot e^{-j(\omega_x x + \omega_y y)}\, dxdy$$

x and y are continuous spatial variables (units: cm, m, etc.)

$$f(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F\left(\omega_x, \omega_y\right) \cdot e^{j(\omega_x x + \omega_y y)}\, d\omega_x d\omega_y$$

ω_x and ω_y are spatial frequencies (units: radians / length)

- integrals → double integrals;
- sum of x and y exponents in kernal function.

Fourier Representations

From "Discrete Time" to "Discrete Space."

1D Discrete-Time Fourier Transform

$$F(\Omega) = \sum_{n=-\infty}^{\infty} f[n] \cdot e^{-j\Omega n}$$

Analysis equation

$$f[n] = \frac{1}{2\pi} \int_{2\pi} f(\Omega) \cdot e^{j\Omega n} \, d\Omega$$

Synthesis equation

Two dimensional DTFT:

$$F(\Omega_r, \Omega_c) = \sum_{r=-\infty}^{\infty} \sum_{c=-\infty}^{\infty} f[r, c] \cdot e^{-j(\Omega_r r + \Omega_c c)}$$

r and c are discrete spatial variables (units: pixels)

$$f[r, c] = \frac{1}{4\pi^2} \int_{2\pi} \int_{2\pi} F(\Omega_r, \Omega_c) \cdot e^{j(\Omega_r r + \Omega_c c)} \, d\Omega_r d\Omega_c$$

Ω_r and Ω_c are spatial frequencies (units: radians / pixel)

- sum → double sums; integral → double integrals;
- sum of r and c exponents in kernal function.

Fourier Representations

1D DFT to 2D DFT

1D Discrete Fourier Transform

$$F[k] = \frac{1}{N}\sum_{n=0}^{N-1} f[n] \cdot e^{-j\frac{2\pi k}{N}n}$$

Analysis equation

$$f[n] = \sum_{k=0}^{N-1} f[n] \cdot e^{j\frac{2\pi k}{N}n}$$

Synthesis equation

Two dimensional DFT:

$$F[k_r, k_c] = \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1} f[r,c] \cdot e^{-j\left(\frac{2\pi k_r}{R}r + \frac{2\pi k_c}{C}c\right)}$$

r and c are discrete spatial variables (units: pixels)

$$f[r,c] = \sum_{r=0}^{R-1}\sum_{c=0}^{C-1} F[k_r, k_c] \cdot e^{-j\left(\frac{2\pi k_r}{R}r + \frac{2\pi k_c}{C}c\right)}$$

k_r and k_c are integers representing frequencies

Orthogonality

DFT basis functions are orthogonal to each other in 1D and 2D.

1D DFT basis functions: $\phi_k[n] = e^{-j\frac{2\pi k}{N}n}$

"Inner product" of 1D basis functions:

See slide #11 of Lec 03B.

$$\sum_{n=0}^{N-1} \phi_k^*[n]\phi_l[n] = \sum_{n=0}^{N-1} e^{j\frac{2\pi k}{N}n} \cdot e^{-j\frac{2\pi l}{N}n} = \sum_{n=0}^{N-1} e^{j\frac{2\pi(k-l)}{N}n} = \begin{cases} N & if\ k = l \\ 0 & otherwise \end{cases} \quad (0 \le k, l < N) \quad = N \cdot \delta[k-l]$$

or: $= N \cdot \delta[(k-l)\ mod\ N]$

2D DFT basis functions: $\phi_{k_r,k_c}[r,c] = e^{-j\frac{2\pi k_r}{R}r}\ e^{-j\frac{2\pi k_c}{C}c}$

"Inner product" of 2D basis functions:

$$\sum_{r=0}^{R-1}\sum_{c=0}^{C-1} \phi_{k_r,k_c}^*[r,c]\phi_{l_r,l_c}[r,c] = \sum_{r=0}^{R-1}\sum_{c=0}^{C-1} (e^{j\frac{2\pi k_r}{R}r} \cdot e^{j\frac{2\pi k_c}{C}c}) \cdot (e^{-j\frac{2\pi l_r}{R}r} \cdot e^{-j\frac{2\pi l_c}{C}c})$$

$$= \sum_{r=0}^{R-1} e^{j\frac{2\pi(k_r-l_r)}{R}r} \sum_{c=0}^{C-1} e^{j\frac{2\pi(k_c-l_c)}{C}c} = \begin{cases} RC & if\, k_r = l_r\ and\ k_c = l_c \\ 0 & otherwise \end{cases} = RC \cdot \delta[k_r - l_r] \cdot \delta[k_c - l_c]$$

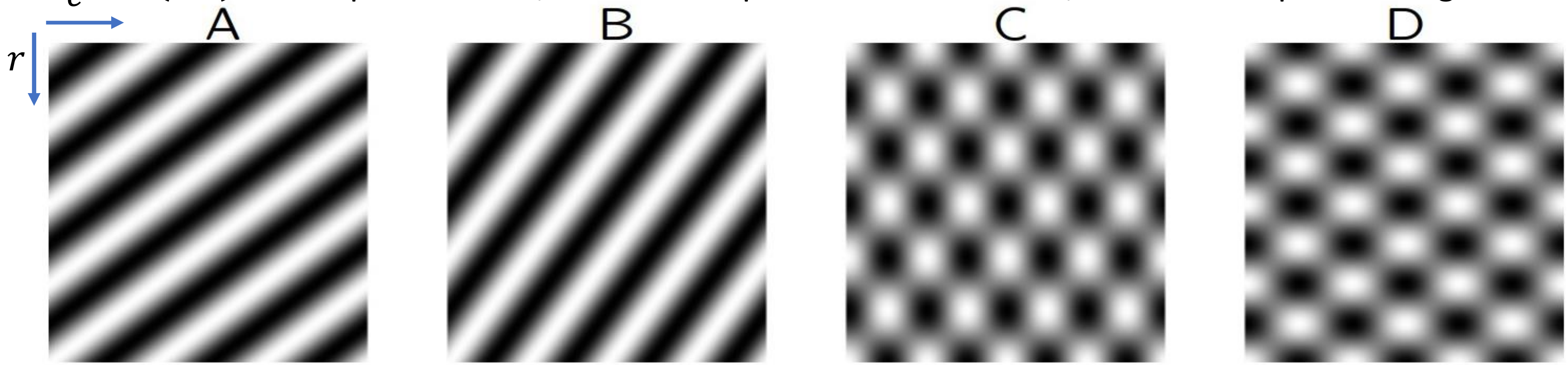
$(0 \le k_r, l_r < R, 0 \le k_c, l_c < C)$

Check yourself!

The 2D DFT basis functions have the form $\phi_{k_r,k_c}[r,c] = e^{-j\frac{2\pi k_r}{R}r}\, e^{-j\frac{2\pi k_c}{C}c}$

Which (if any) of the following images show the real part of one of the basis functions $\phi_{k_r,k_c}[r,c]$?

$(0,0)$ is at top left corner, black correspond to lowest value, white correspond to highest value.



What values of k_r and k_c correspond to each basis function?

Check yourself!

The 2D DFT basis functions have the form:

$$\phi_{k_r,k_c}[r,c] = e^{-j\frac{2\pi k_r}{R}r}\; e^{-j\frac{2\pi k_c}{C}c}$$

$$= cos\left(\frac{2\pi k_r}{R}r + \frac{2\pi k_c}{C}c\right) - jsin\left(\frac{2\pi k_r}{R}r + \frac{2\pi k_c}{C}c\right)$$

If $\frac{2\pi k_r}{R}r + \frac{2\pi k_c}{C}c$ is constant, the real and imaginary parts will be constant.

Example: Let k_r =3 and k_c = − 4 when R = C = 128.

Then the exponent is $\frac{2\pi 3}{128}r - \frac{2\pi 4}{128}c$. This exponent is zero if 3 r = 4 c.

If the exponent is zero, then cosine is at its peak value of 1.

Thus the real part of the 2D basis function is 1 along the line $r = \frac{4}{3}c$.

Therefore the real part of the 2D basis function will be 1 along the lines $r = -\frac{k_c}{k_r}c$.

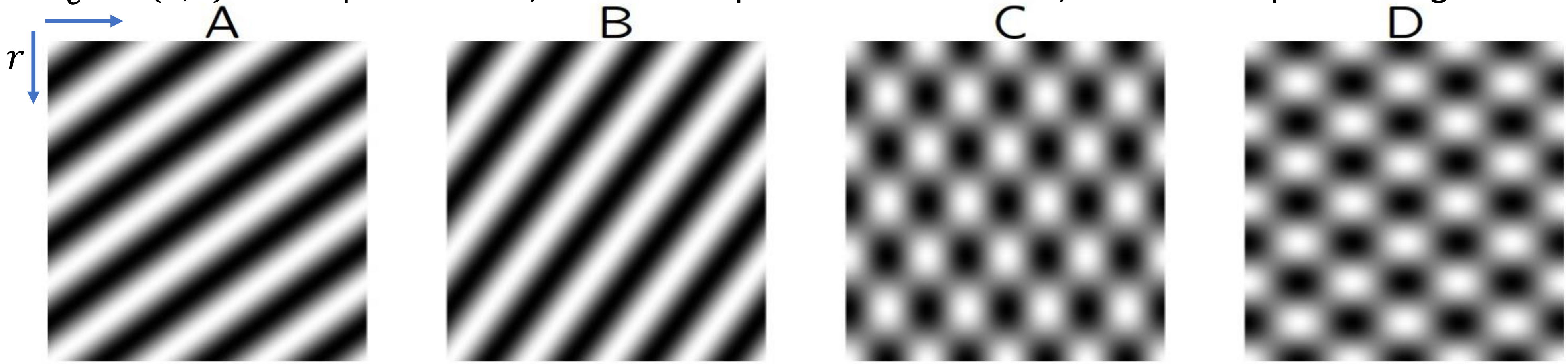
Check yourself!

$$Re(\phi_{k_r,k_c}[r,c]) = cos\left(\frac{2\pi k_r}{R}r + \frac{2\pi k_c}{C}c\right)$$

The 2D DFT basis functions have the form $\phi_{k_r,k_c}[r,c] = e^{-j\frac{2\pi k_r}{R}r}\, e^{-j\frac{2\pi k_c}{C}c}$

Which (if any) of the following images show the real part of one of the basis functions $\phi_{k_r,k_c}[r,c]$? A and B

c $(0,0)$ is at top left corner, black correspond to lowest value, white correspond to highest value.



What values of k_r and k_c correspond to each basis function?

A: (4, -3) or (-4, 3) B: (3, -4) or (-3, 4) C: none; D: none

Fourier Transform Pairs

In 1D, we found that it was useful to know how the transforms of simple shapes looked (for example delta → constant), in part because it was often possible to use that understanding to simplify thinking about bigger problems.

The same will be true in 2D!

We can look at some 2D Fourier analysis of simple shapes in the following.

2D Discrete Fourier Transform

Example: Find the DFT of a 2D unit sample:

$$f[r,c] = \delta[r]\delta[c] = \begin{cases} 1, & r = 0 \; and \; c = 0 \\ 0, & otherwise \end{cases}$$

$$F[k_r,k_c] = \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1}\delta[r]\delta[c]\cdot e^{-j\left(\frac{2\pi k_r}{R}r+\frac{2\pi k_c}{C}c\right)}$$

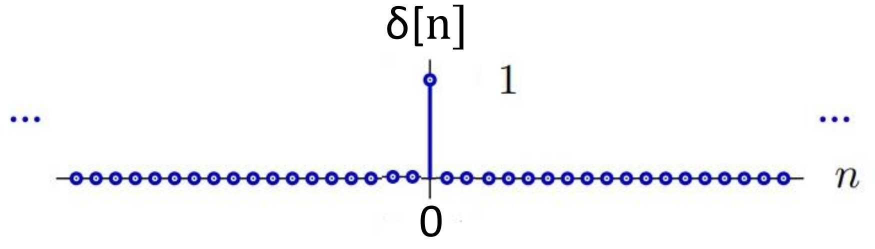
$$= \frac{1}{RC}e^{-j\left(\frac{2\pi k_r}{R}0+\frac{2\pi k_c}{C}0\right)}$$

$$= \frac{1}{RC}$$

$$\delta[r]\delta[c] \stackrel{DFT}{\Longrightarrow} \frac{1}{RC}$$

1D unit sample

$$f[n] = \delta[n] = \begin{cases} 1, & n = 0 \\ 0, & otherwise \end{cases}$$



$$F[k] = \frac{1}{N}\sum_{n=0}^{N-1}\delta[n]\cdot e^{-j\frac{2\pi k}{N}n} = \frac{1}{N}$$

2D Discrete Fourier Transform

Generally, implement a 2D DFT as a sequence of 1D DFTs:

$$F[k_r, k_c] = \frac{1}{RC} \sum_{r=0}^{R-1} \sum_{c=0}^{C-1} f[r,c] \cdot e^{-j(\frac{2\pi k_r}{R} r + \frac{2\pi k_c}{C} c)}$$

$$= \underbrace{\frac{1}{R} \sum_{r=0}^{R-1} \underbrace{\left(\frac{1}{C} \sum_{c=0}^{C-1} f[r,c] \cdot e^{-j\frac{2\pi k_c}{C} c} \right)}_{\text{first, obtain the DFT for each column}} \cdot e^{-j\frac{2\pi k_r}{R} r}}_{\text{then, take the DFT of each resulting rows}}$$

Alternatively, we can start with rows and then do columns just as well.

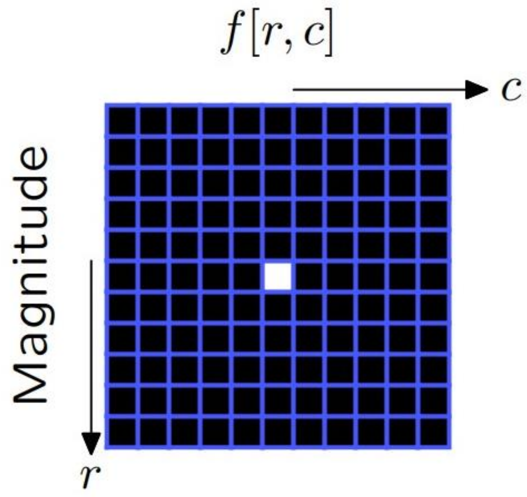
Note: the above equation is a special case of the general form:

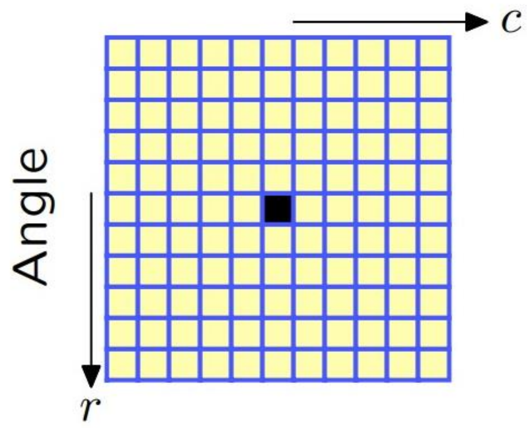
$$F[k_r, k_c] = \frac{1}{RC} \sum_{r=<R>} \sum_{c=<C>} f_p[r,c] \cdot e^{-j(\frac{2\pi k_r}{R} r + \frac{2\pi k_c}{C} c)}$$

where $f_p[r,c]$ is the periodically extended version of $f[r,c]$

2D Discrete Fourier Transform

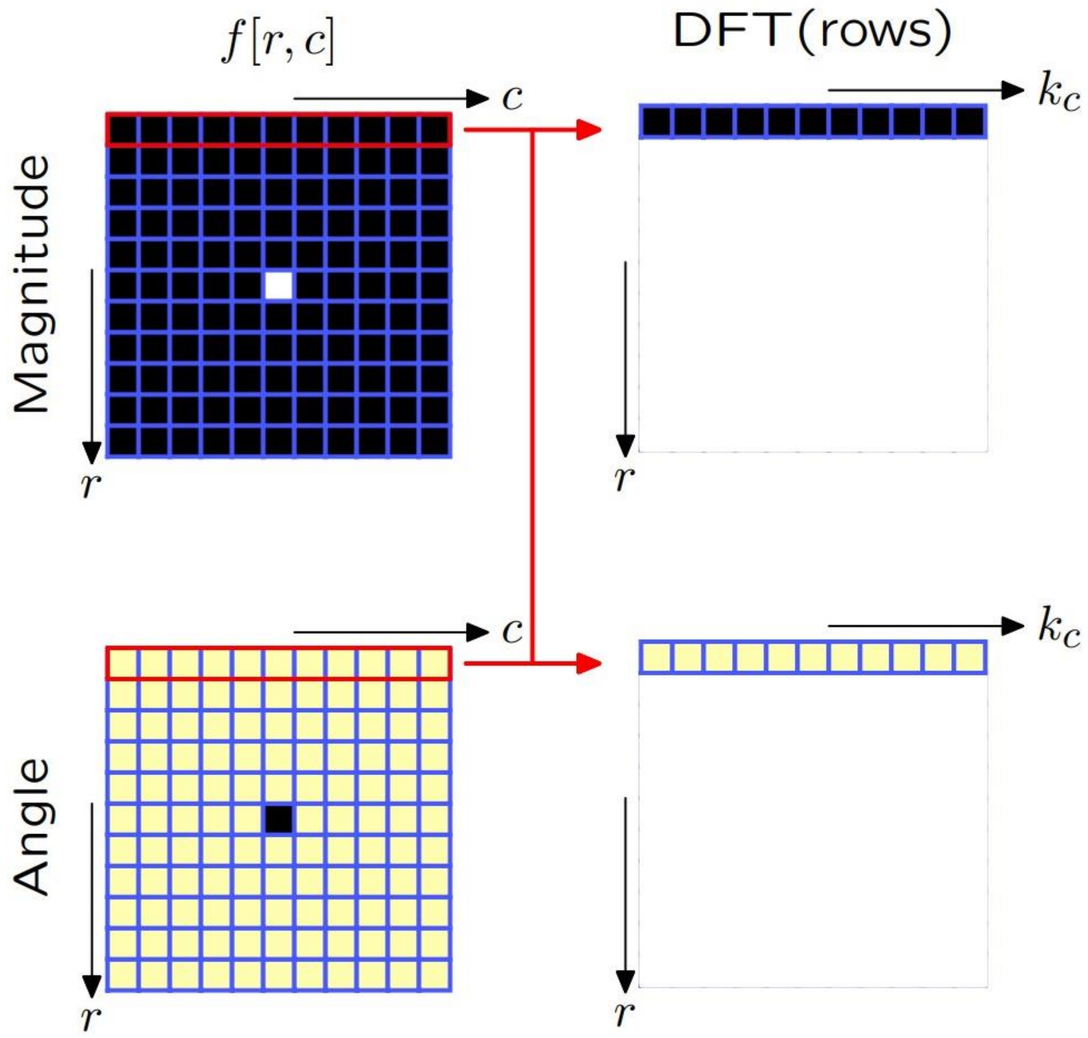
Example: Find the DFT of a 2D unit sample: (The image is centered at $r = 0, c = 0$)





2D Discrete Fourier Transform

Example: Find the DFT of a 2D unit sample:



2D Discrete Fourier Transform

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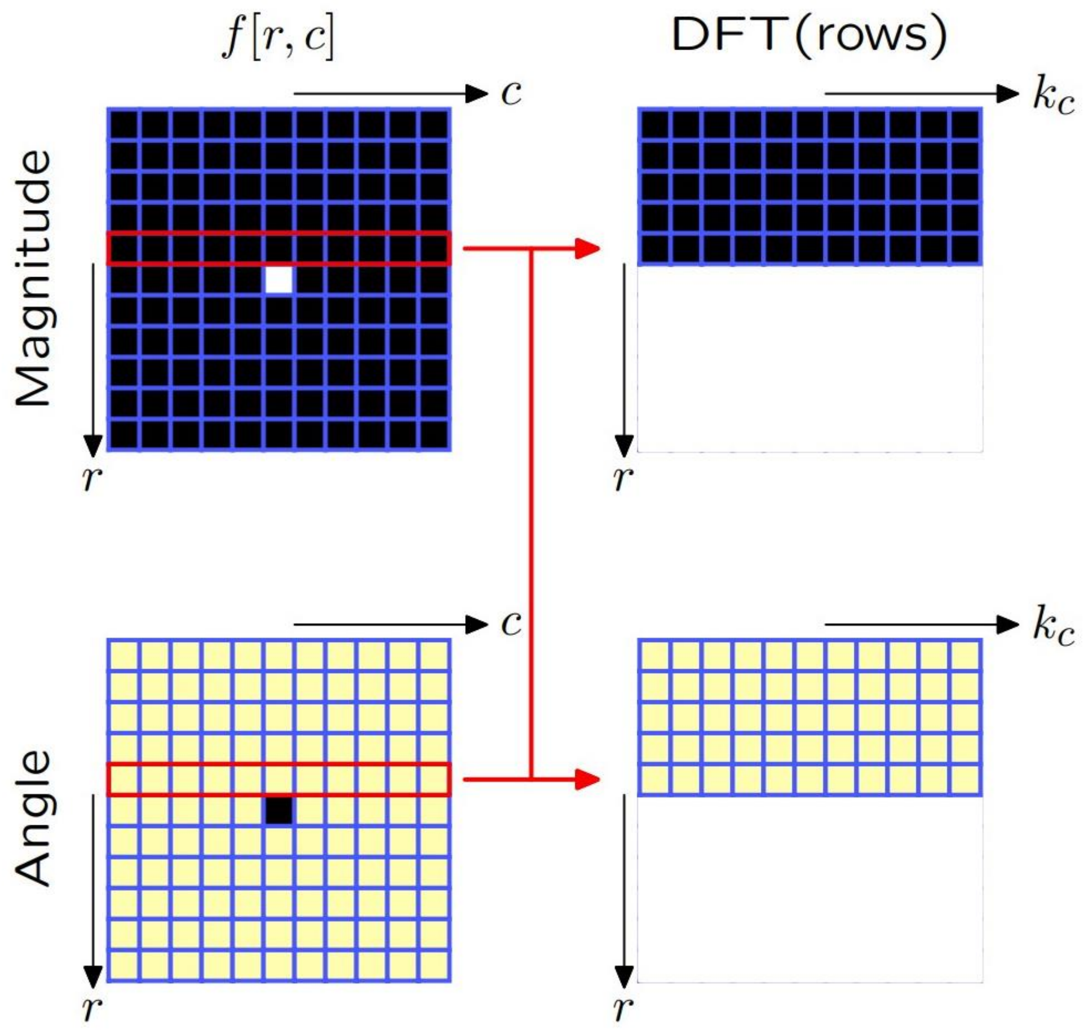

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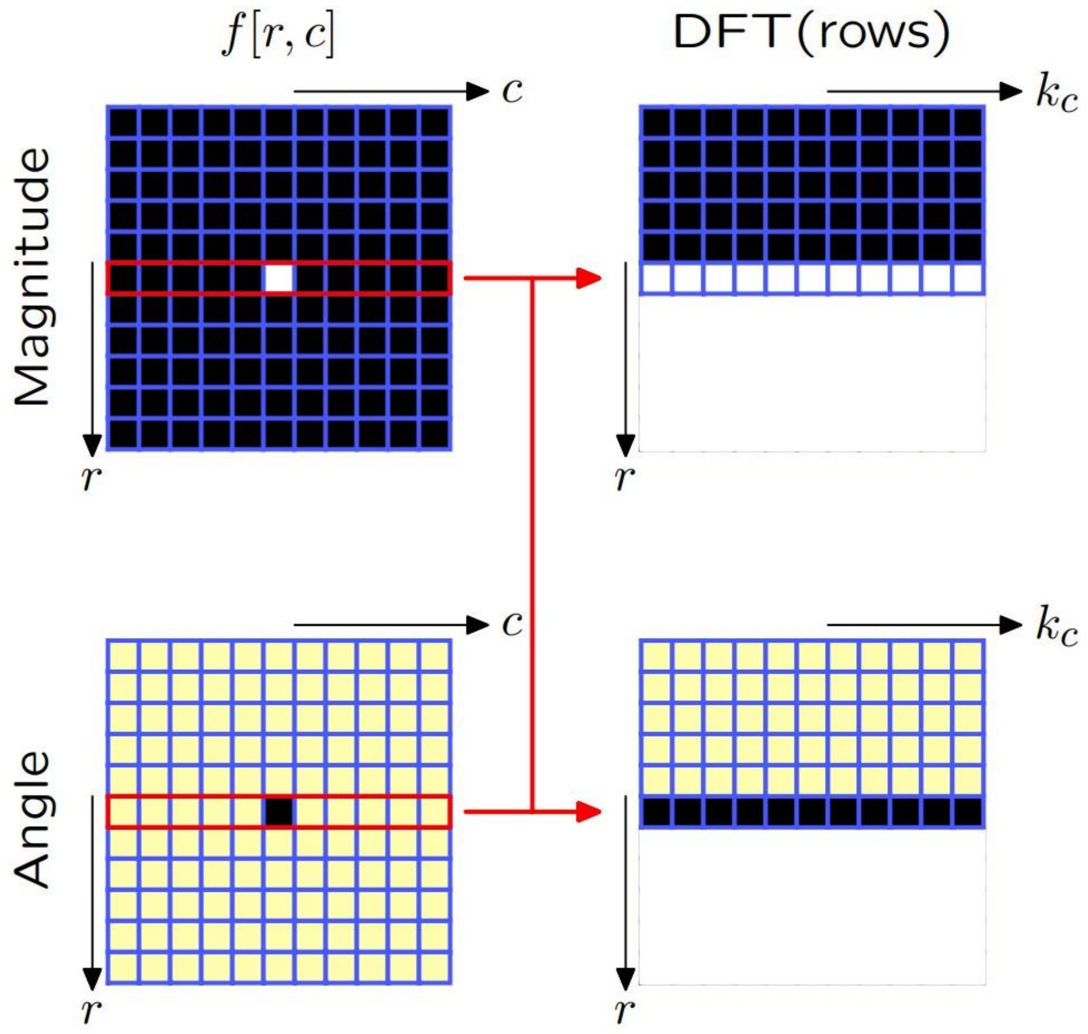

2D Discrete Fourier Transform

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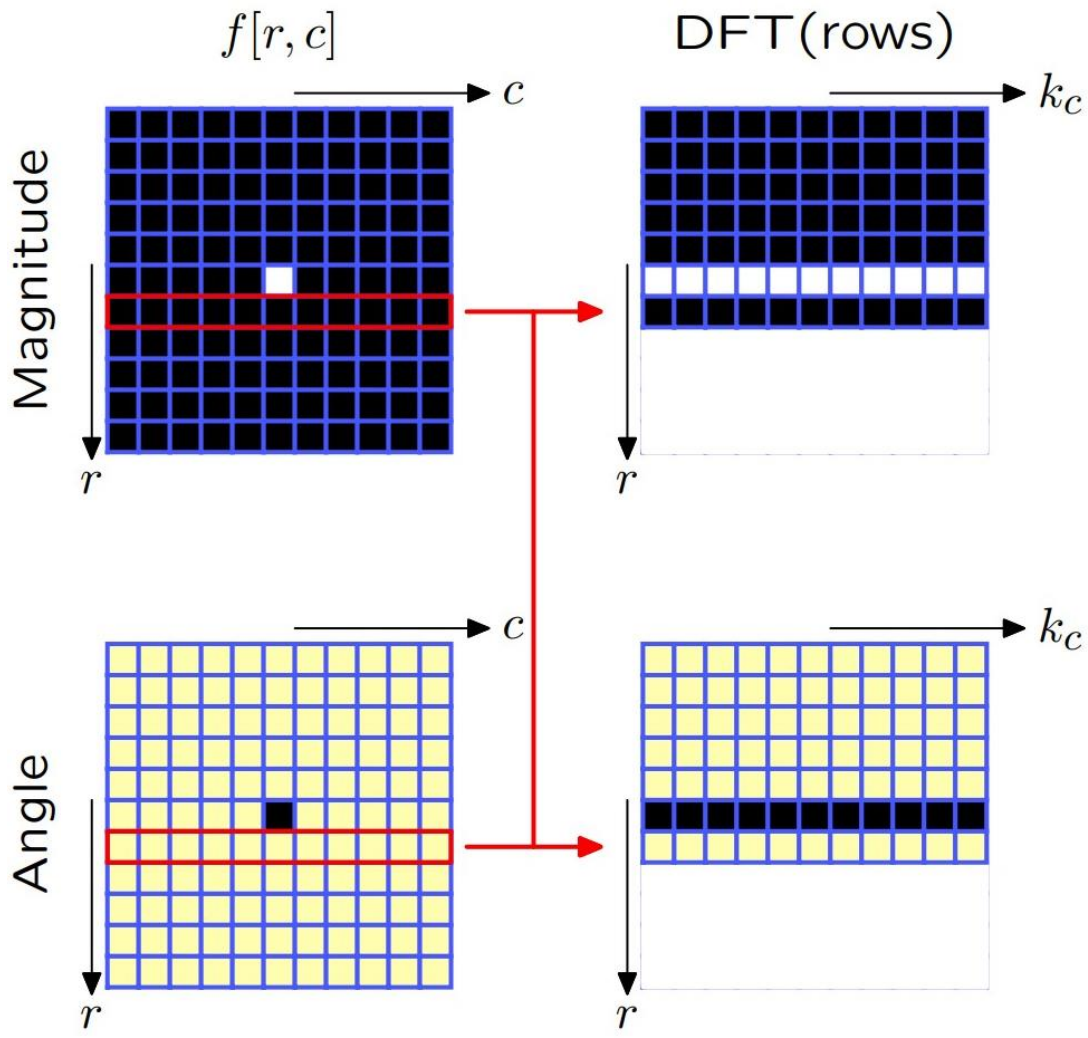

2D Discrete Fourier Transform

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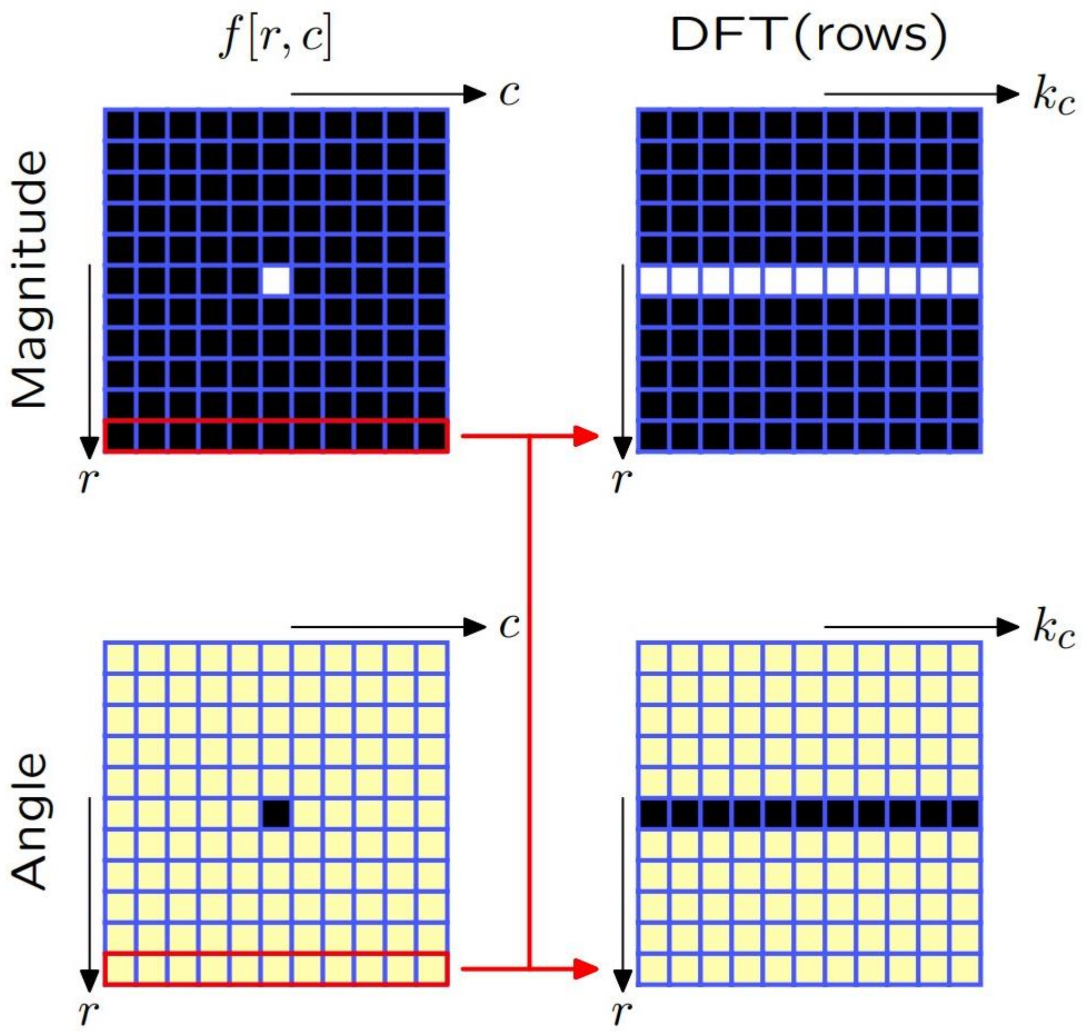

2D Discrete Fourier Transform

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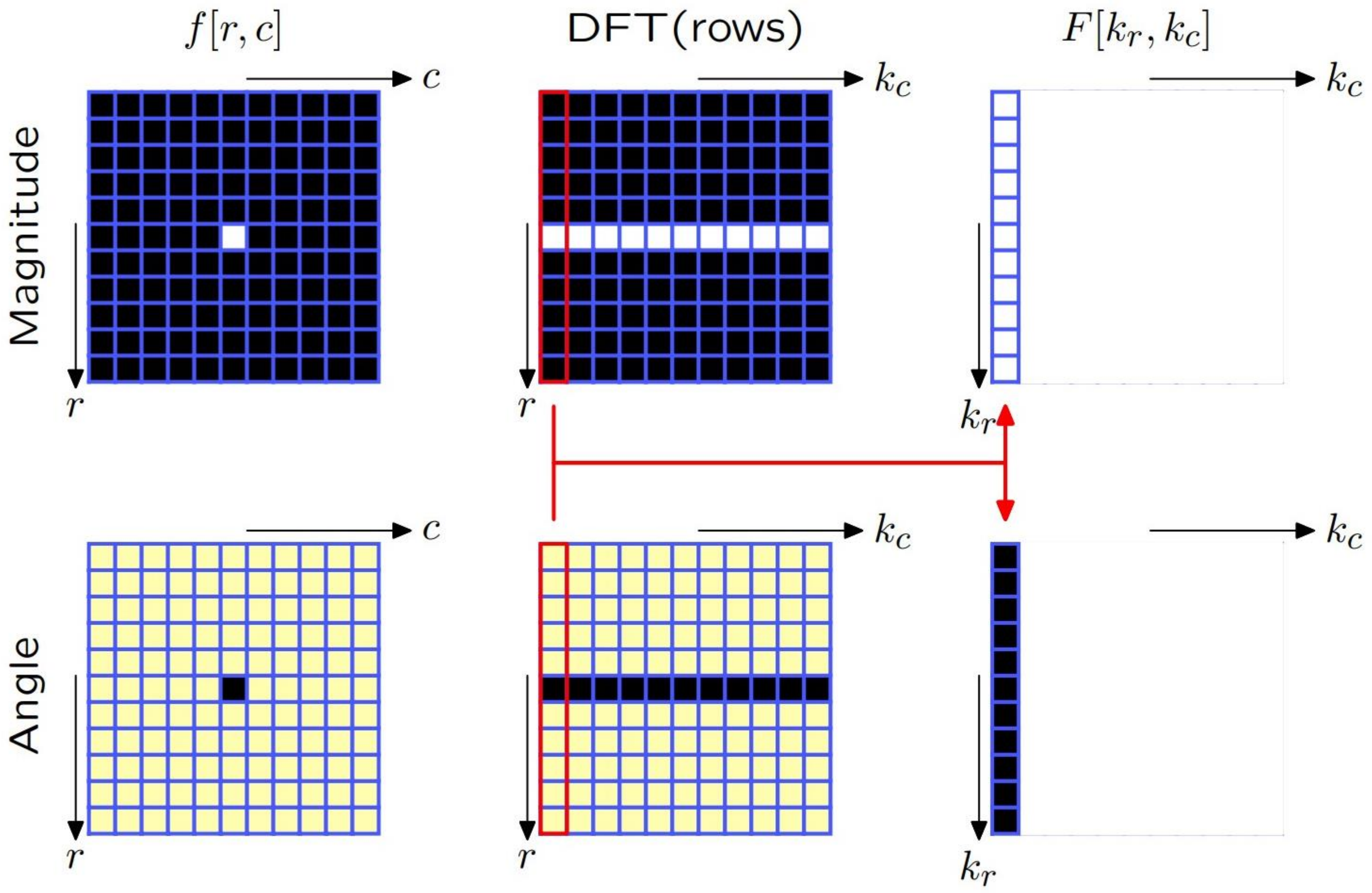

2D Discrete Fourier Transform

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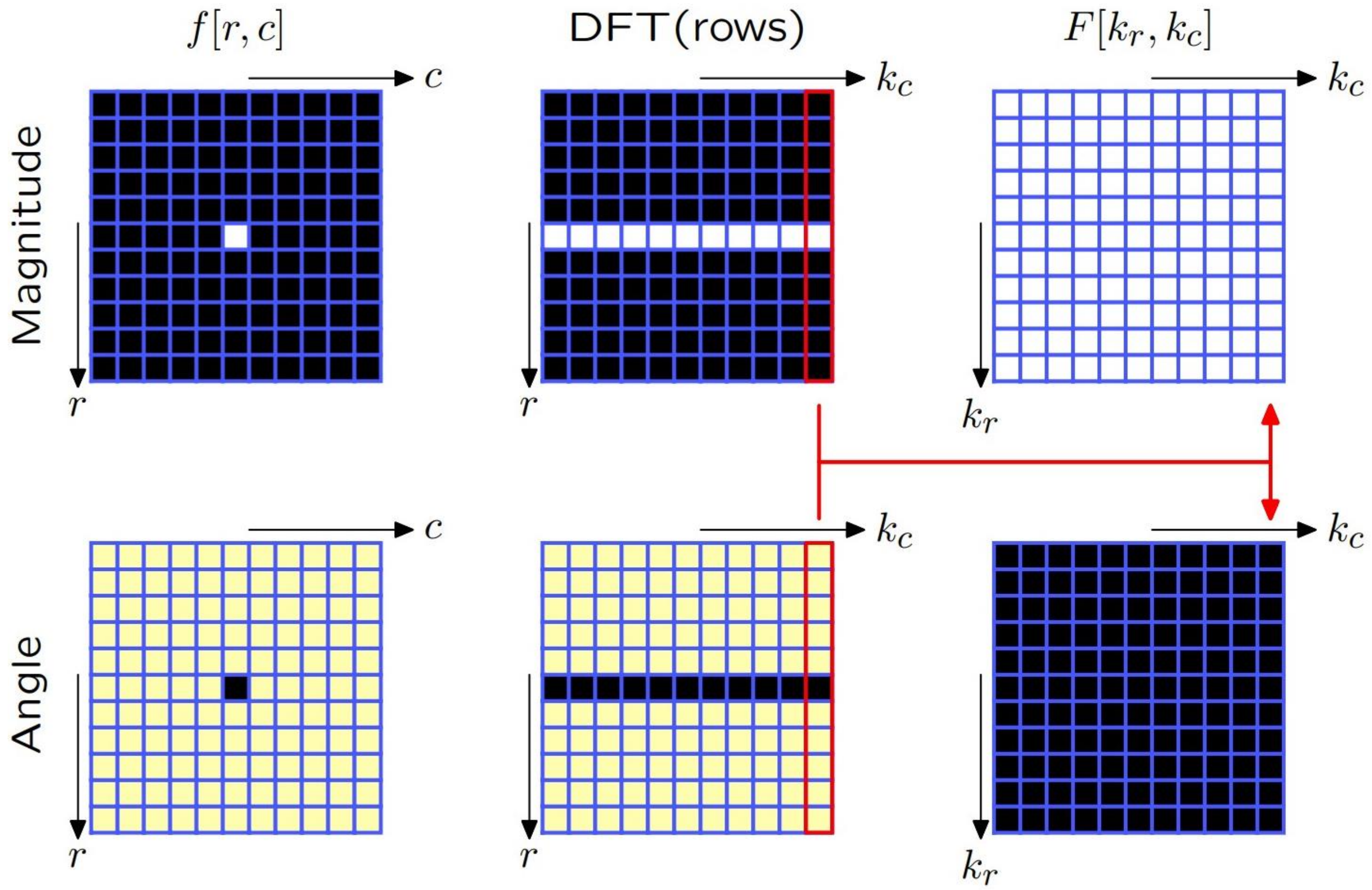
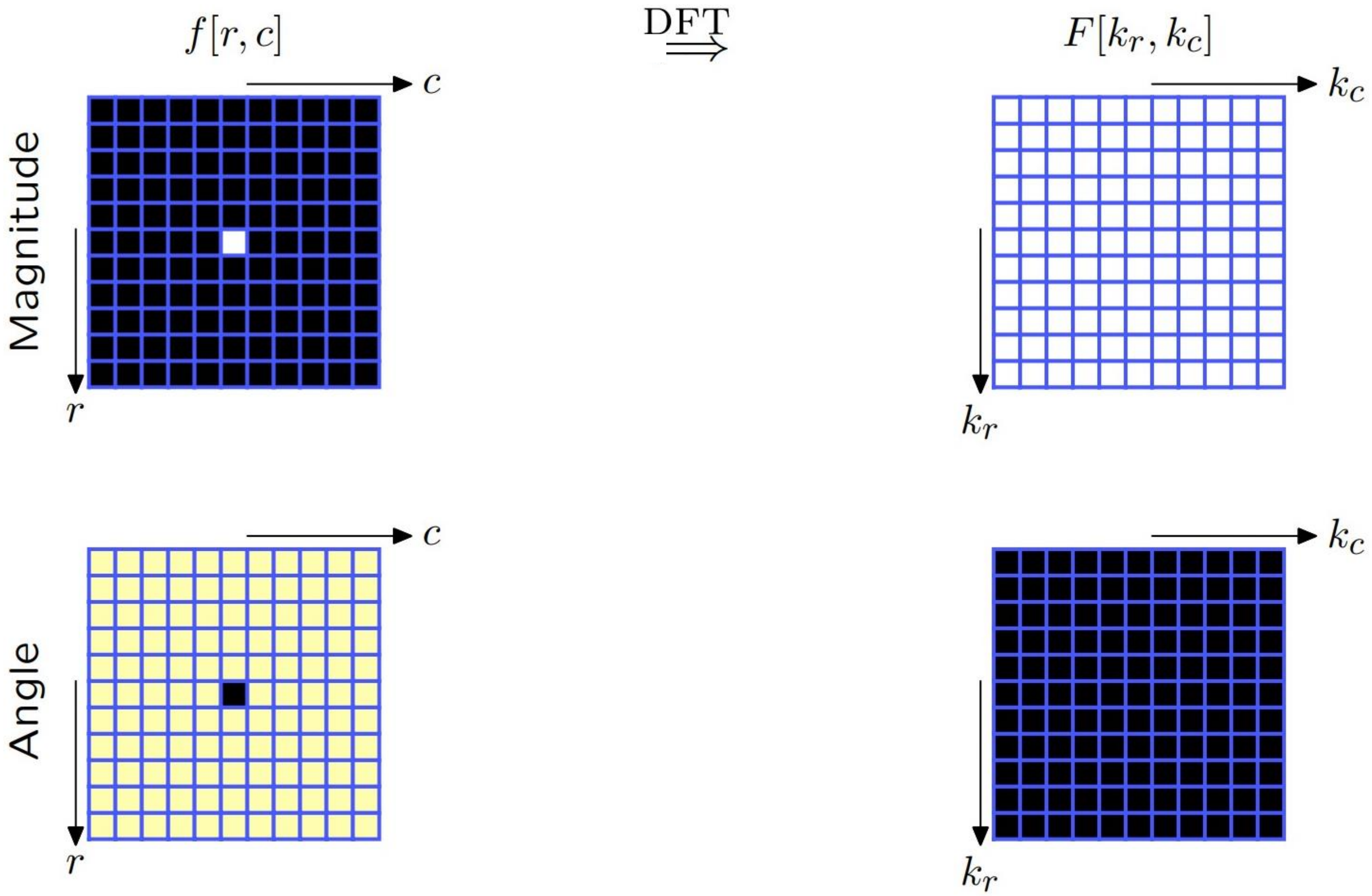


2D Discrete Fourier Transform

Example: Find the DFT of a 2D unit sample:



2D Discrete Fourier Transform

Example: Find the DFT of a 2D constant.

$$f[r,c] = 1$$

$$F[k_r,k_c] = \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1} 1 \cdot e^{-j(\frac{2\pi k_r}{R}r+\frac{2\pi k_c}{C}c)} = \frac{1}{RC}\sum_{r=0}^{R-1} e^{-j\frac{2\pi k_r}{R}r}\sum_{c=0}^{C-1} e^{-j\frac{2\pi k_c}{C}c}$$

And we know:

$$\sum_{c=0}^{C-1} e^{-j\frac{2\pi k_c}{C}c} = \begin{cases} C, & k_c = 0 \\ 0, & otherwise \end{cases} \qquad \sum_{r=0}^{R-1} e^{-j\frac{2\pi k_r}{R}r} = \begin{cases} R, & k_r = 0 \\ 0, & otherwise \end{cases}$$

$$F[k_r,k_c] = \frac{1}{RC} \cdot R \cdot \delta[k_r] \cdot C \cdot \delta[k_c] = \delta[k_c]\delta[k_r]$$

$$1 \overset{DFT}{\Longrightarrow} \delta[k_r]\delta[k_c]$$

1D constant

$$f[n] = 1$$

$$F[k] = \frac{1}{N}\sum_{n=0}^{N-1} f[n] \cdot e^{-j\frac{2\pi k}{N}n}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} e^{-j\frac{2\pi k}{N}n}$$

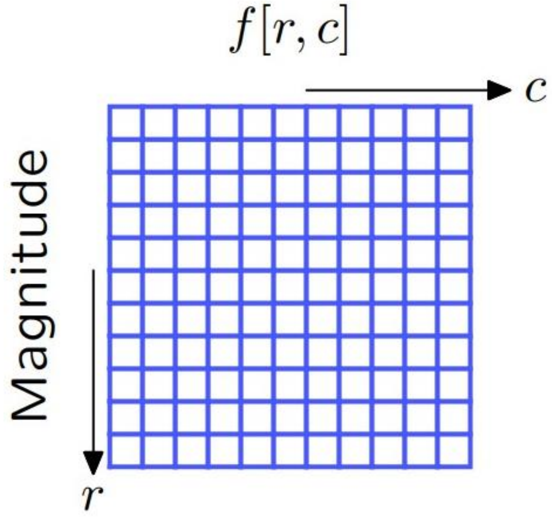
$$= \begin{cases} 1, & k = 0 \\ 0, & otherwise \end{cases}$$

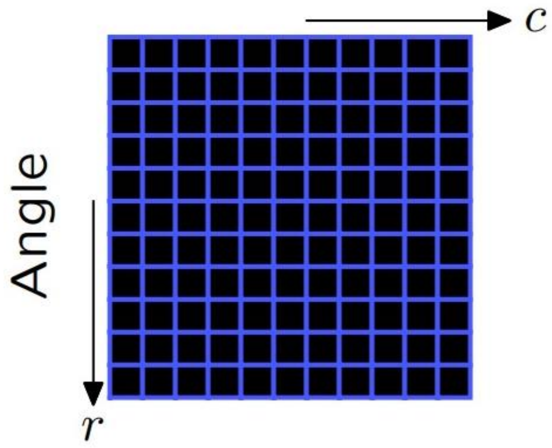
$$= \delta[k]$$

See slide #11 of Lec 03B.

2D Discrete Fourier Transform

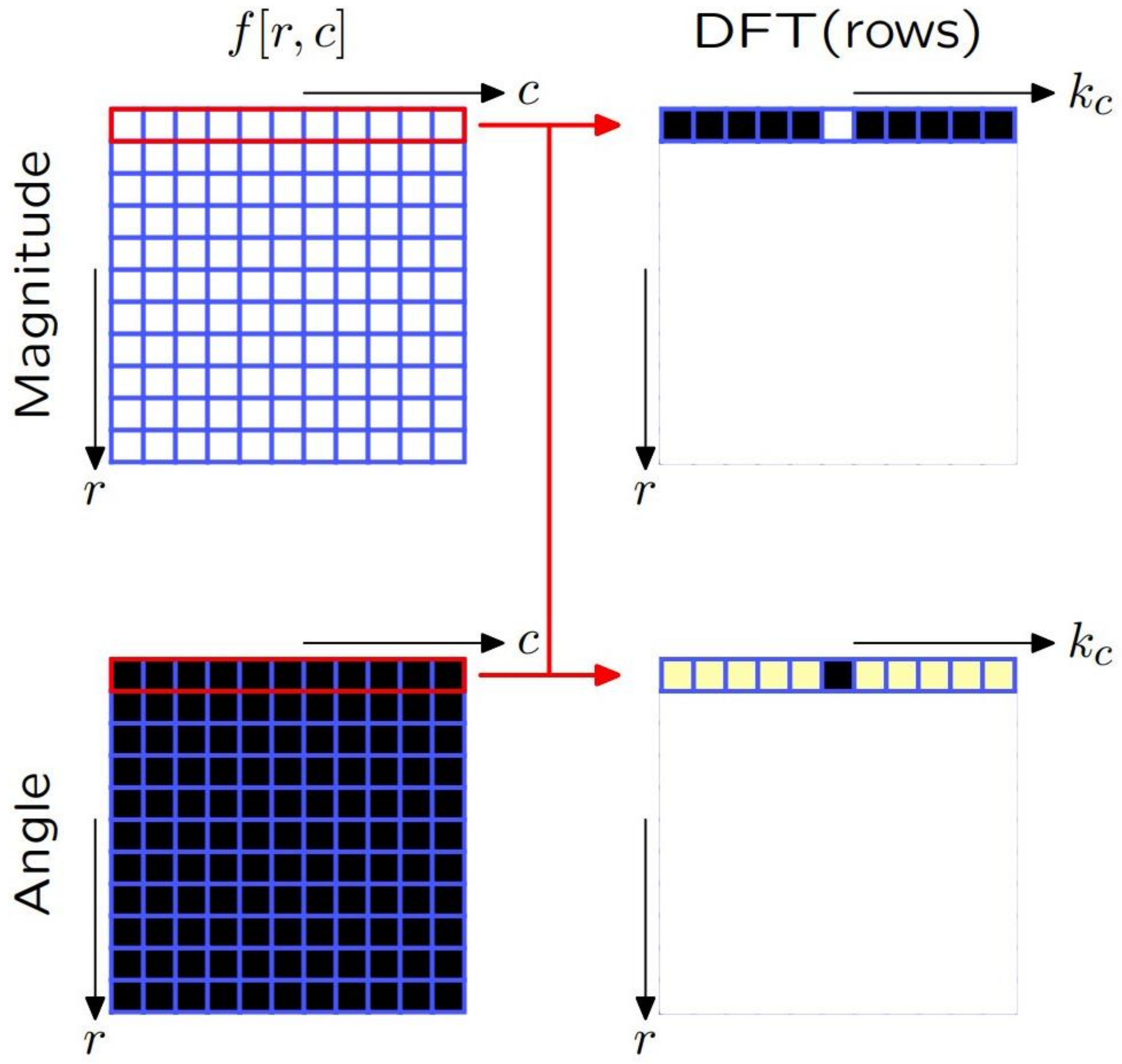
Example: Find the DFT of a constant. (The image is centered at $r = 0, c = 0$)





2D Discrete Fourier Transform

Example: Find the DFT of a constant.



2D Discrete Fourier Transform

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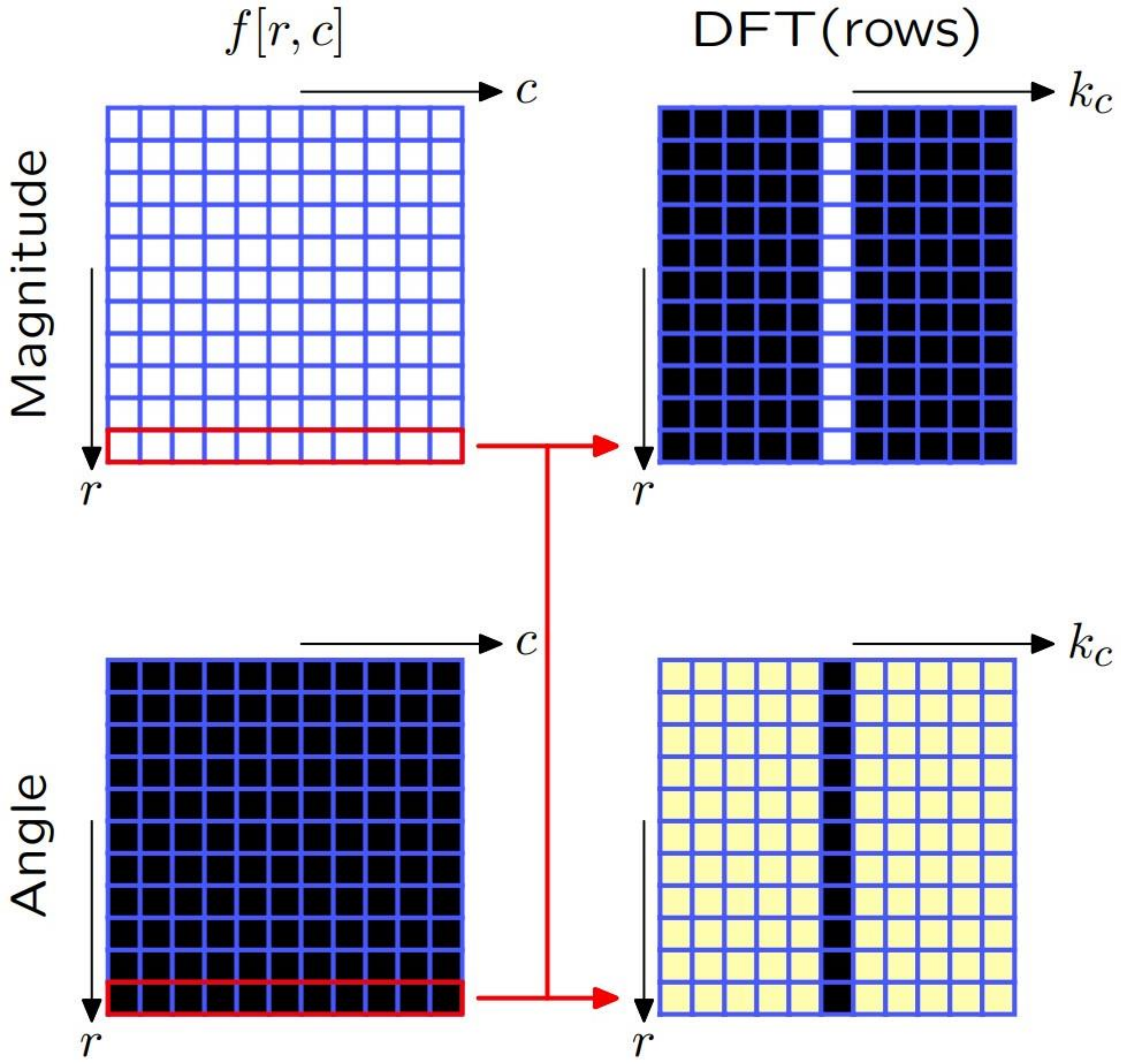

2D Discrete Fourier Transform

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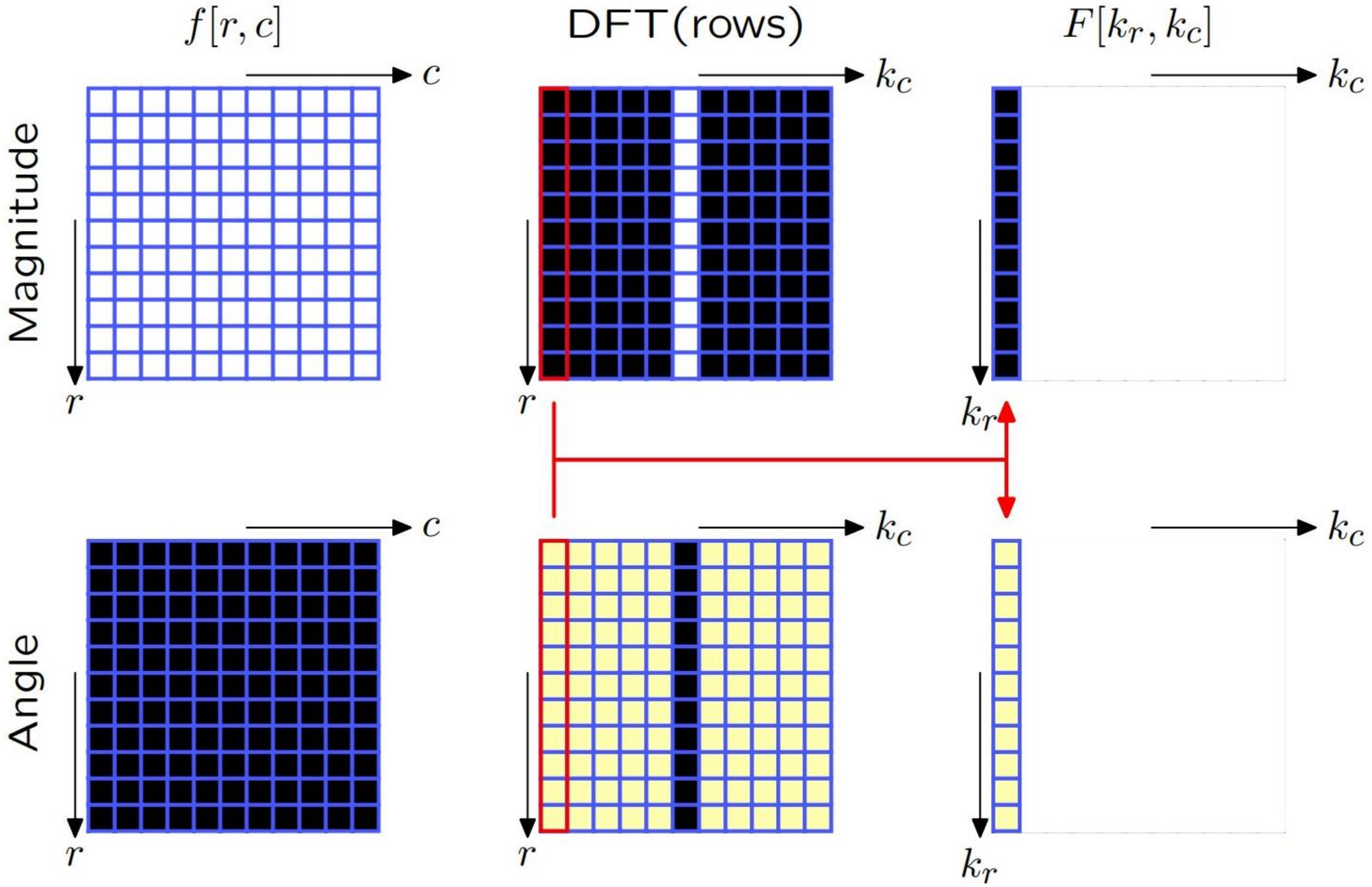

2D Discrete Fourier Transform

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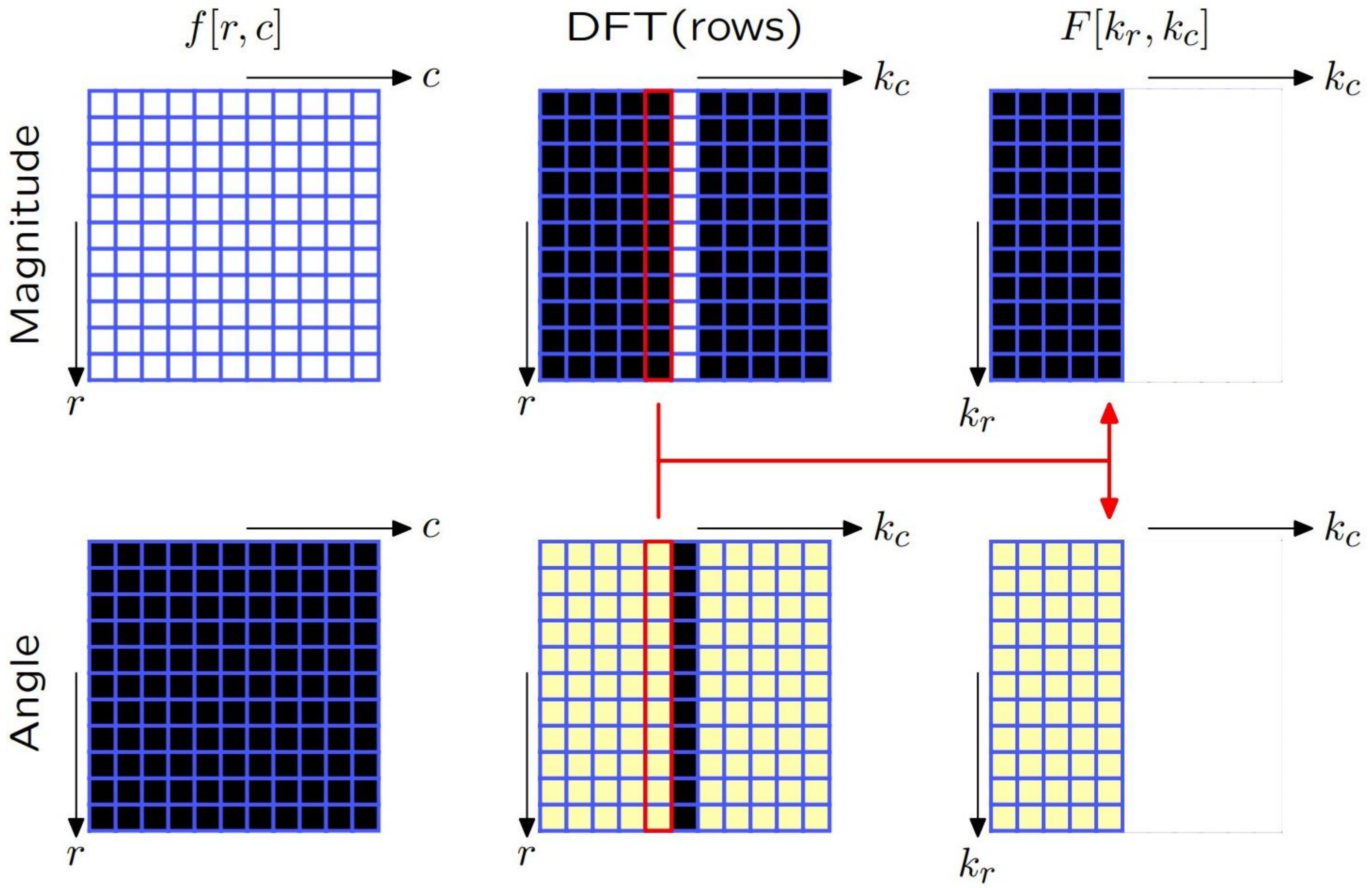

2D Discrete Fourier Transform

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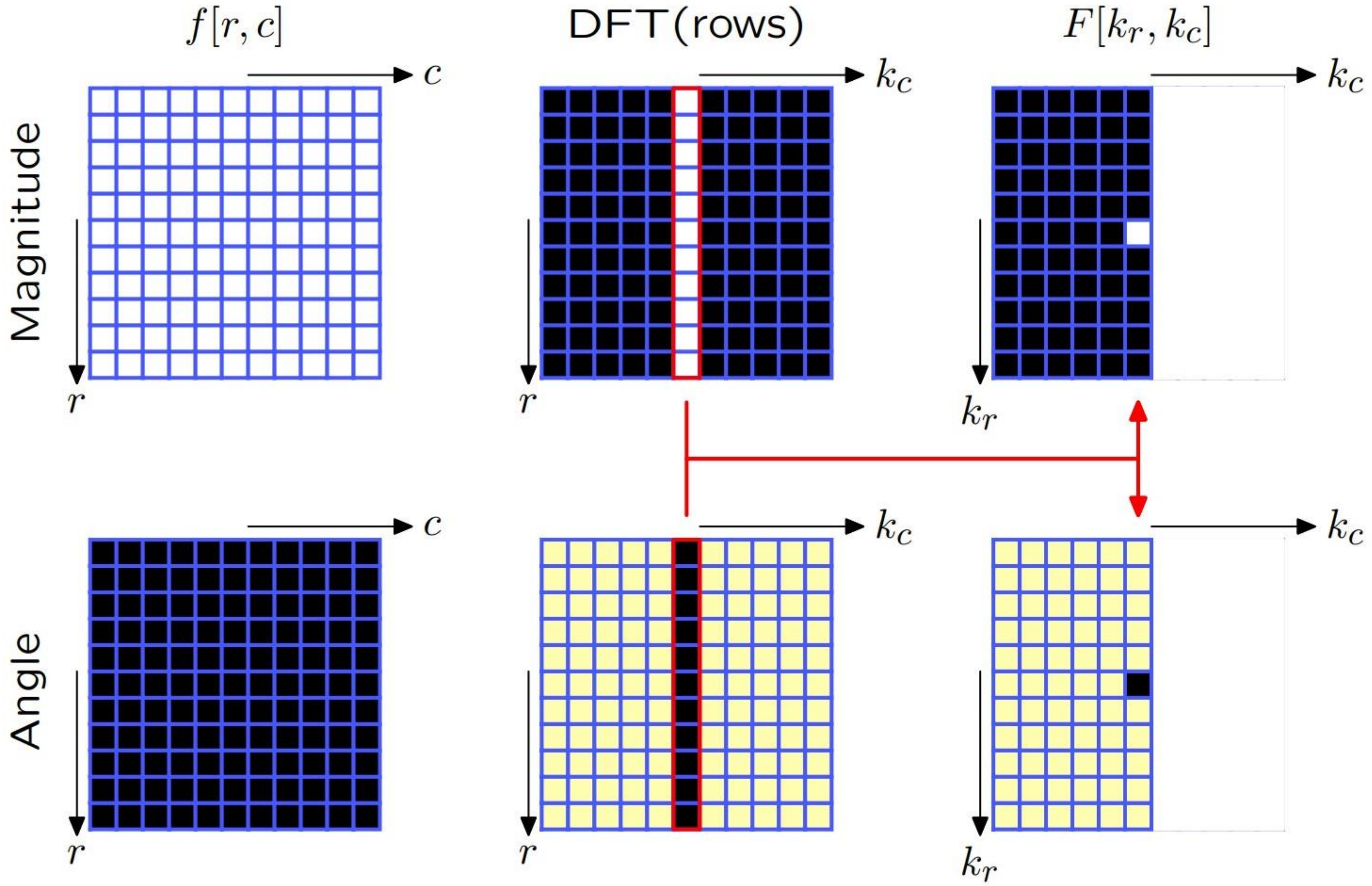

2D Discrete Fourier Transform

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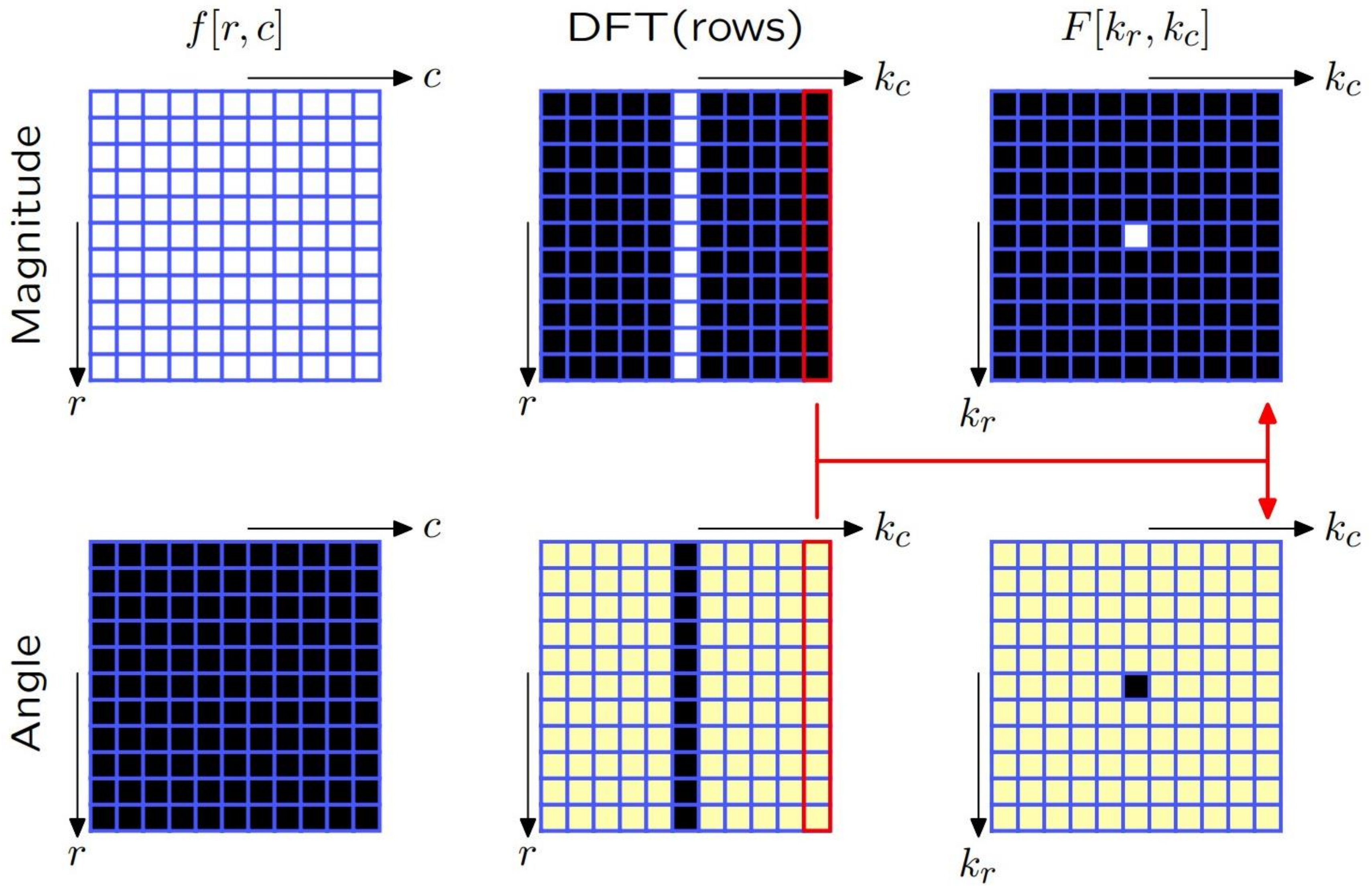
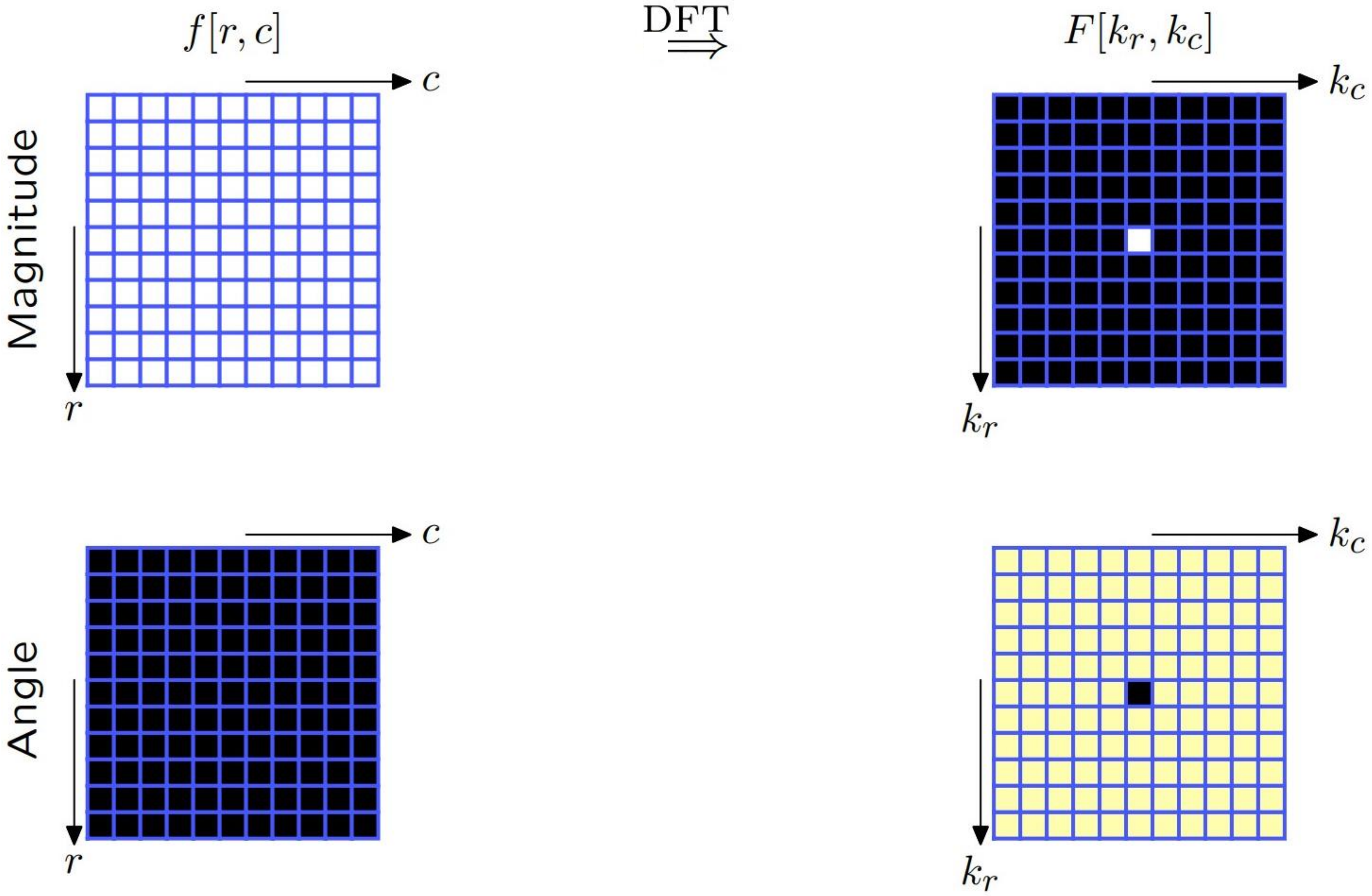


2D Discrete Fourier Transform

Example: Find the DFT of a constant.



2D Discrete Fourier Transform

Example: Find the DFT of a vertical line.

Participation question for Lecture

$$f[r,c] = \delta[c] = \begin{cases} 1, & c = 0 \\ 0, & otherwise \end{cases}$$

$$F[k_r,k_c] = \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1}\delta[c]\cdot e^{-j(\frac{2\pi k_r}{R}r+\frac{2\pi k_c}{C}c)} = \frac{1}{RC}\sum_{r=0}^{R-1} e^{-j(\frac{2\pi k_r}{R}r+\frac{2\pi k_c}{C}0)} = \frac{1}{RC}\sum_{r=0}^{R-1} e^{-j(\frac{2\pi k_r}{R}r)}$$

And: $$\sum_{r=0}^{R-1} e^{-j\frac{2\pi k_r}{R}r} = \begin{cases} R, & k_r = 0 \\ 0, & otherwise \end{cases}$$

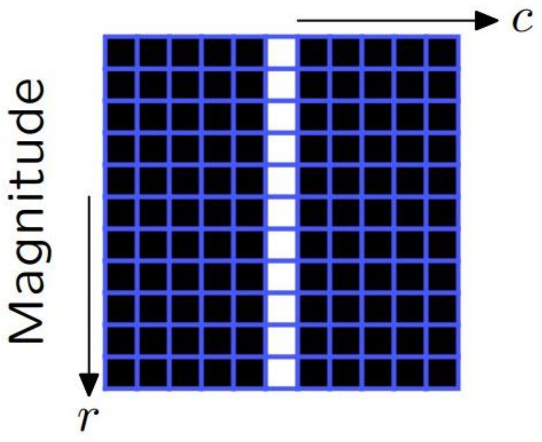
$$F[k_r,k_c] = \frac{1}{RC}\cdot R\cdot\delta[k_r] = \frac{1}{C}\delta[k_r]$$

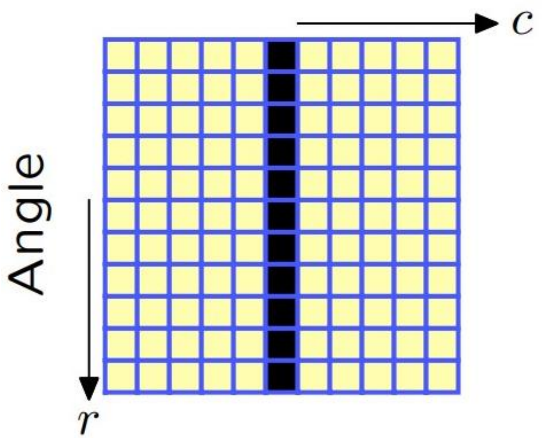
$$\boxed{\delta[c] \overset{DFT}{\Longrightarrow} \frac{1}{C}\delta[k_r]}$$

2D Discrete Fourier Transform

Example: Find the DFT of a vertical line: (The image is centered at $r = 0, c = 0$)

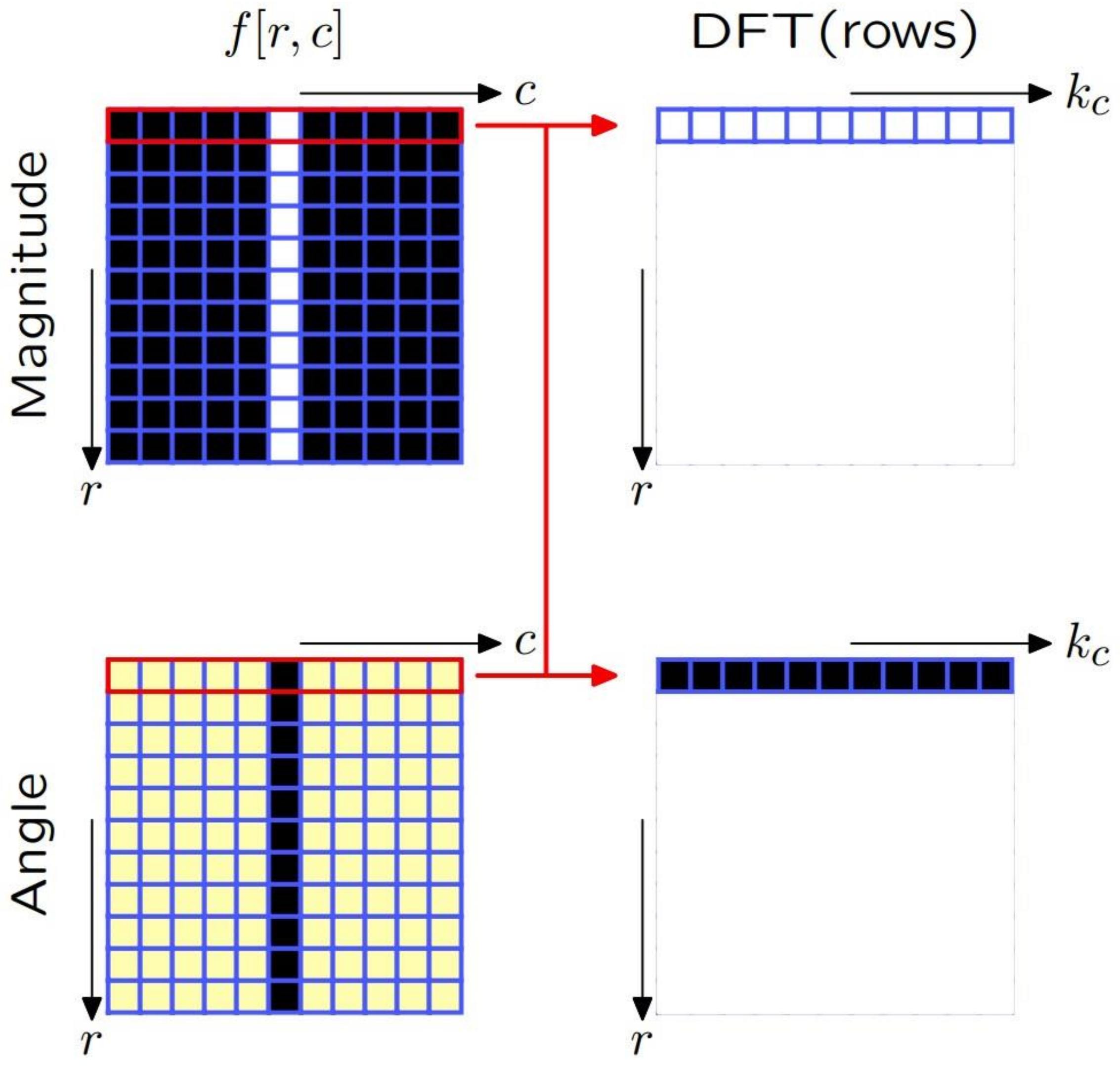
$f[r, c]$





2D Discrete Fourier Transform

Example: Find the DFT of a vertical line:



2D Discrete Fourier Transform

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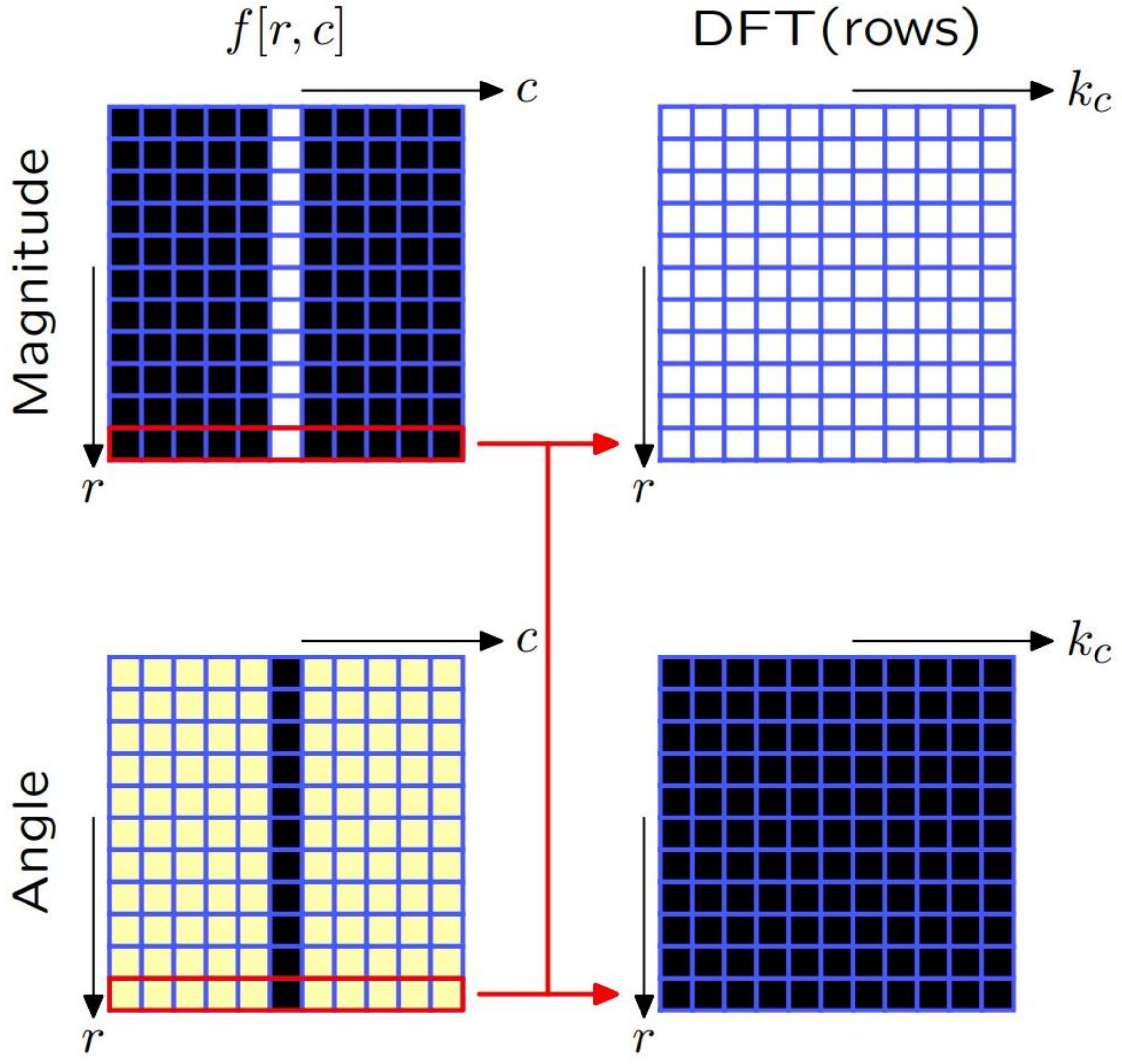

2D Discrete Fourier Transform

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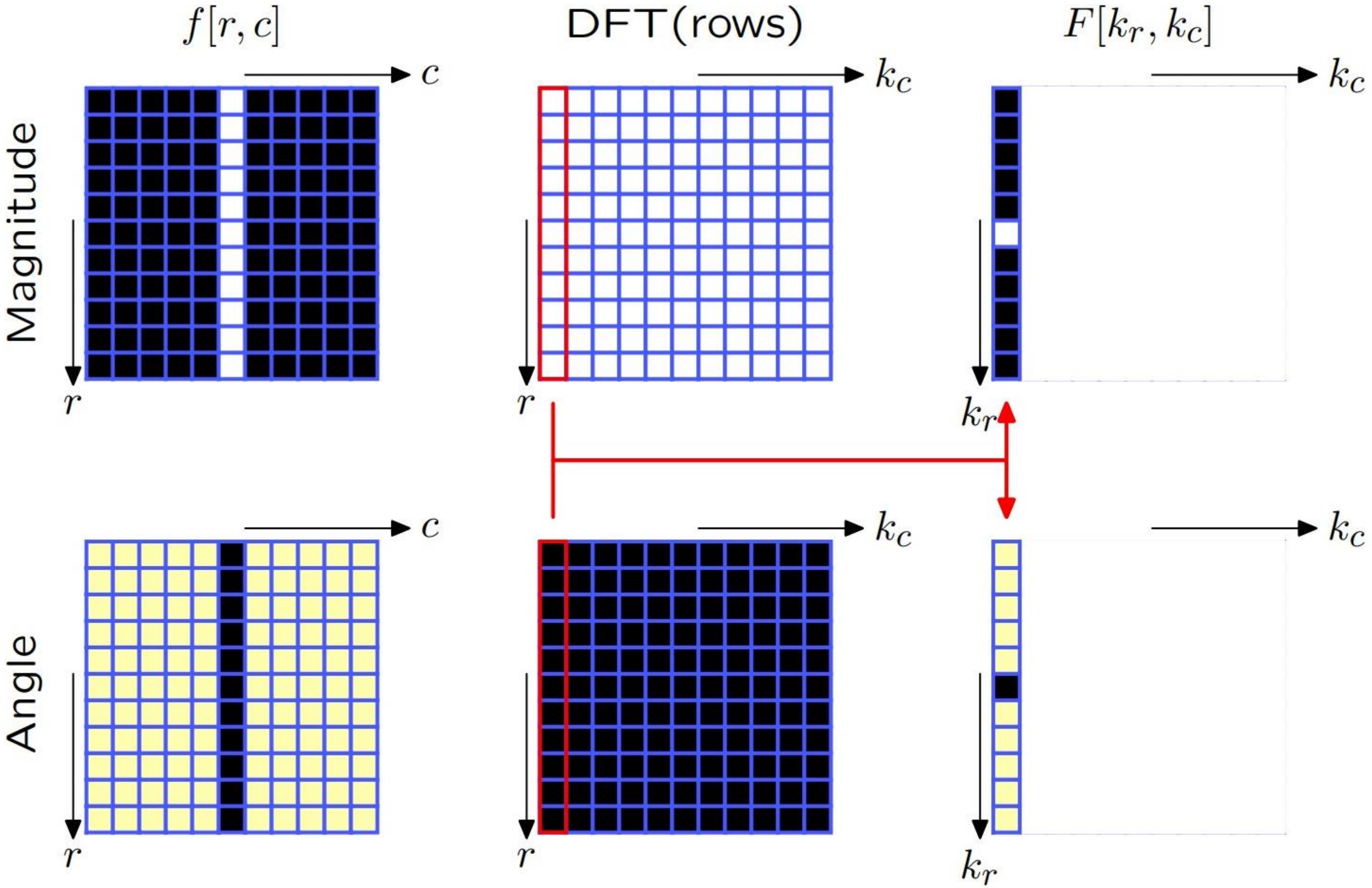
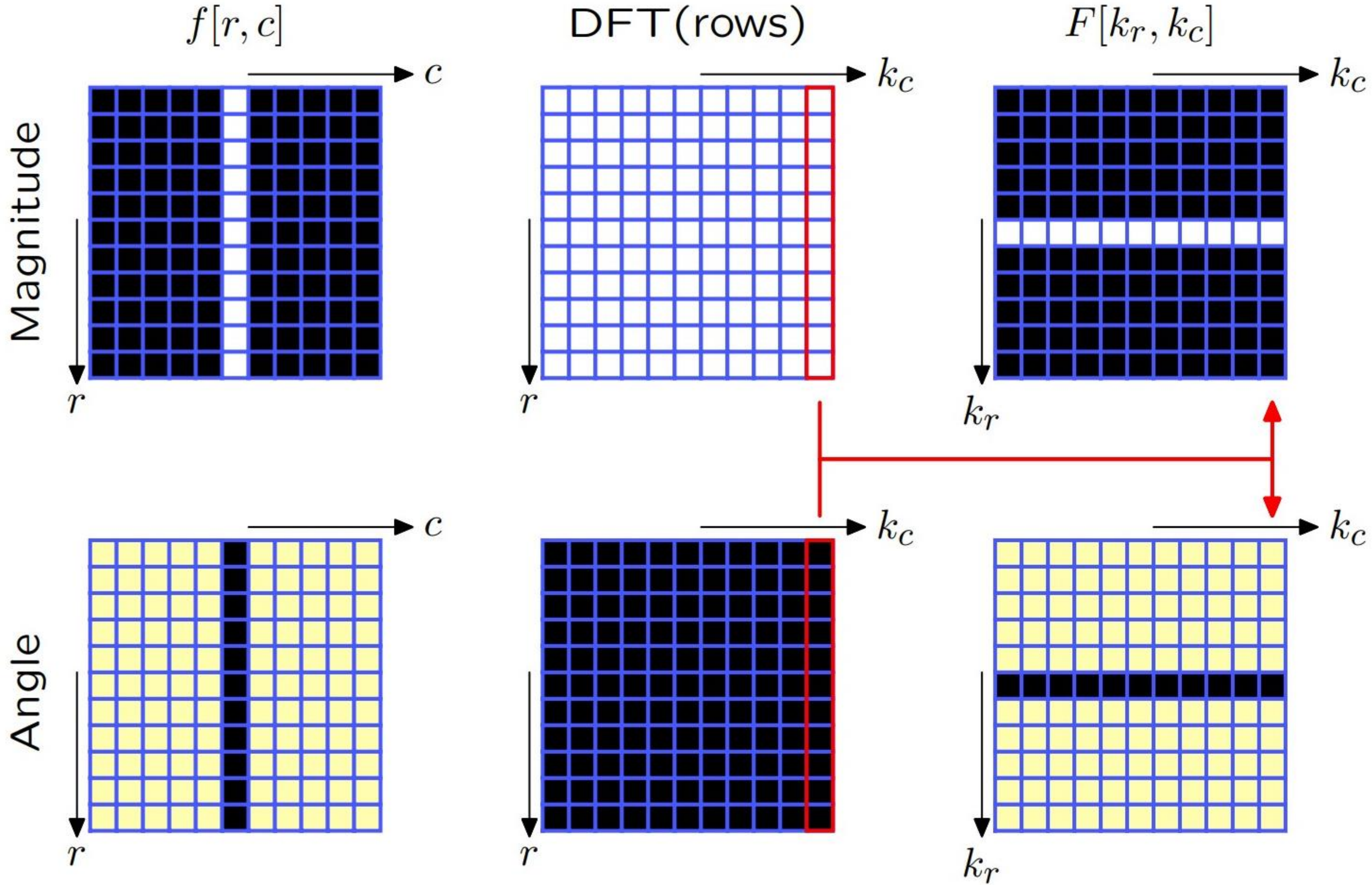


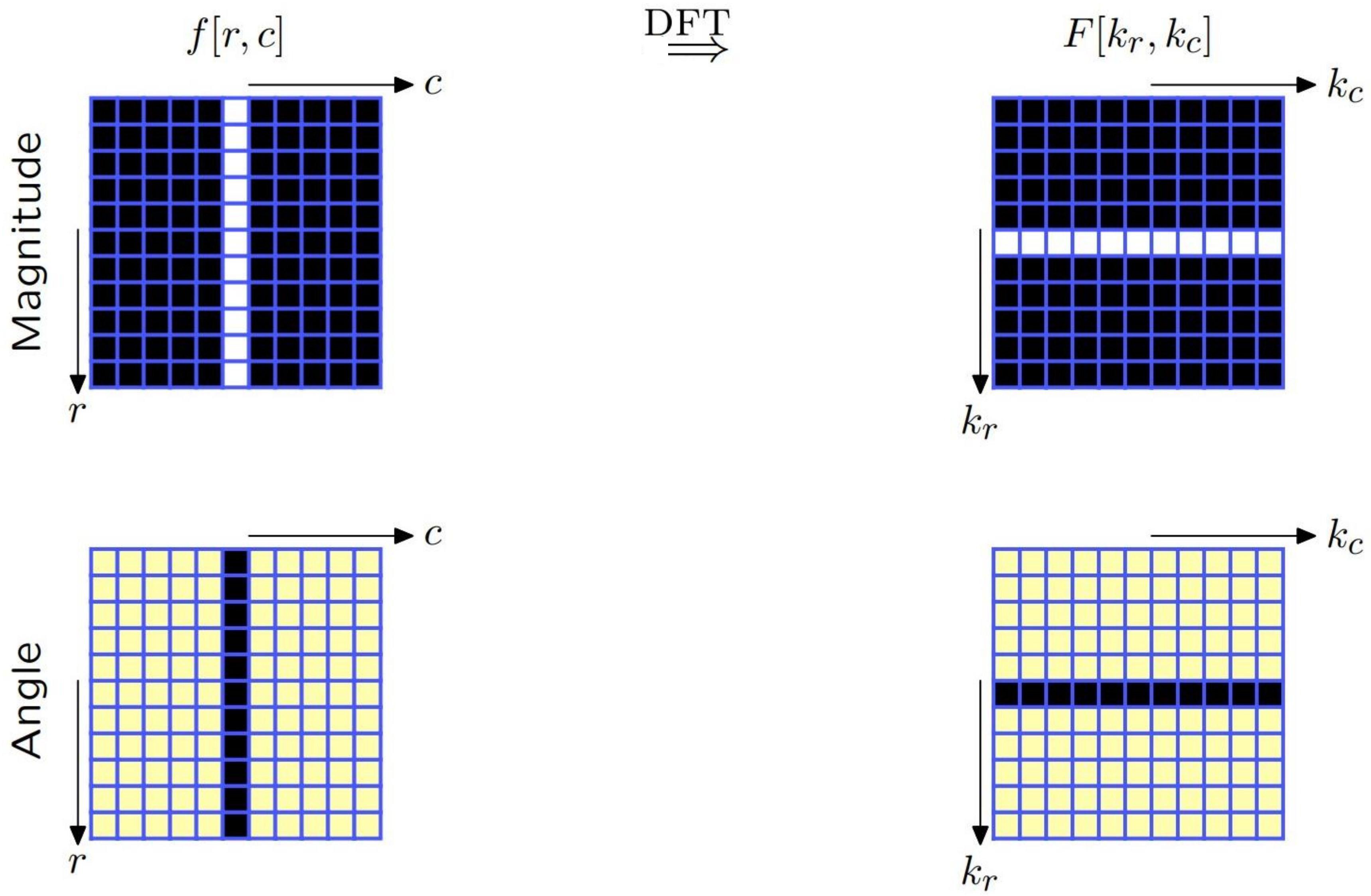
2D Discrete Fourier Transform

Example: Find the DFT of a vertical line:



2D Discrete Fourier Transform

Example: Find the DFT of a vertical line:



2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:

$$f[r,c] = \delta[r] = \begin{cases} 1, & r = 0 \\ 0, & otherwise \end{cases}$$

$$F[k_r,k_c] = \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1}\delta[r]\cdot e^{-j\left(\frac{2\pi k_r}{R}r+\frac{2\pi k_c}{C}c\right)} = \frac{1}{RC}\sum_{c=0}^{C-1}e^{-j\left(\frac{2\pi k_r}{R}0+\frac{2\pi k_c}{C}c\right)} = \frac{1}{RC}\sum_{c=0}^{C-1}e^{-j\left(\frac{2\pi k_c}{C}c\right)}$$

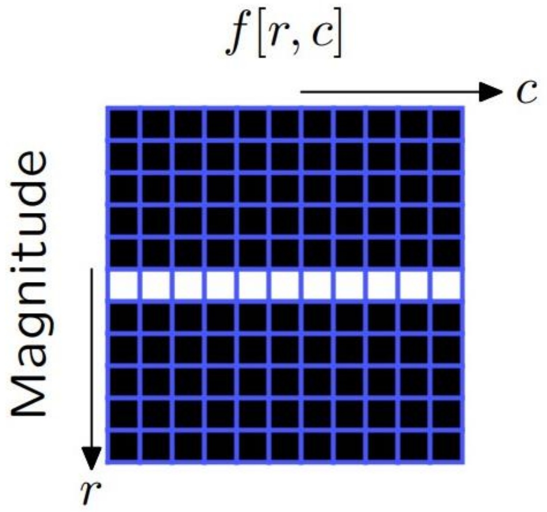
Since: $$\sum_{c=0}^{C-1}e^{-j\frac{2\pi k_c}{C}c} = \begin{cases} C, & k_c = 0 \\ 0, & otherwise \end{cases}$$

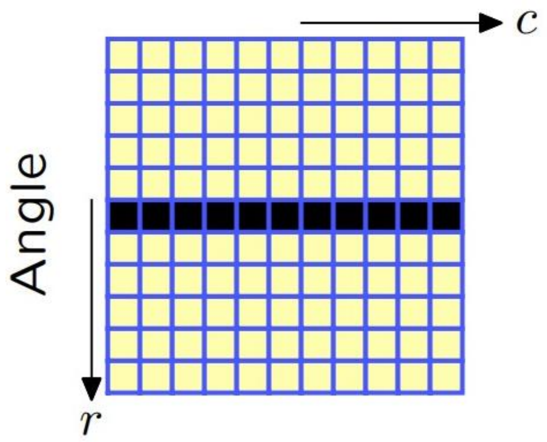
$$F[k_r,k_c] = \frac{1}{RC}\cdot C\cdot\delta[k_c] = \frac{1}{R}\delta[k_c]$$

$$\boxed{\delta[r] \overset{DFT}{\Longrightarrow} \frac{1}{R}\delta[k_c]}$$

2D Discrete Fourier Transform

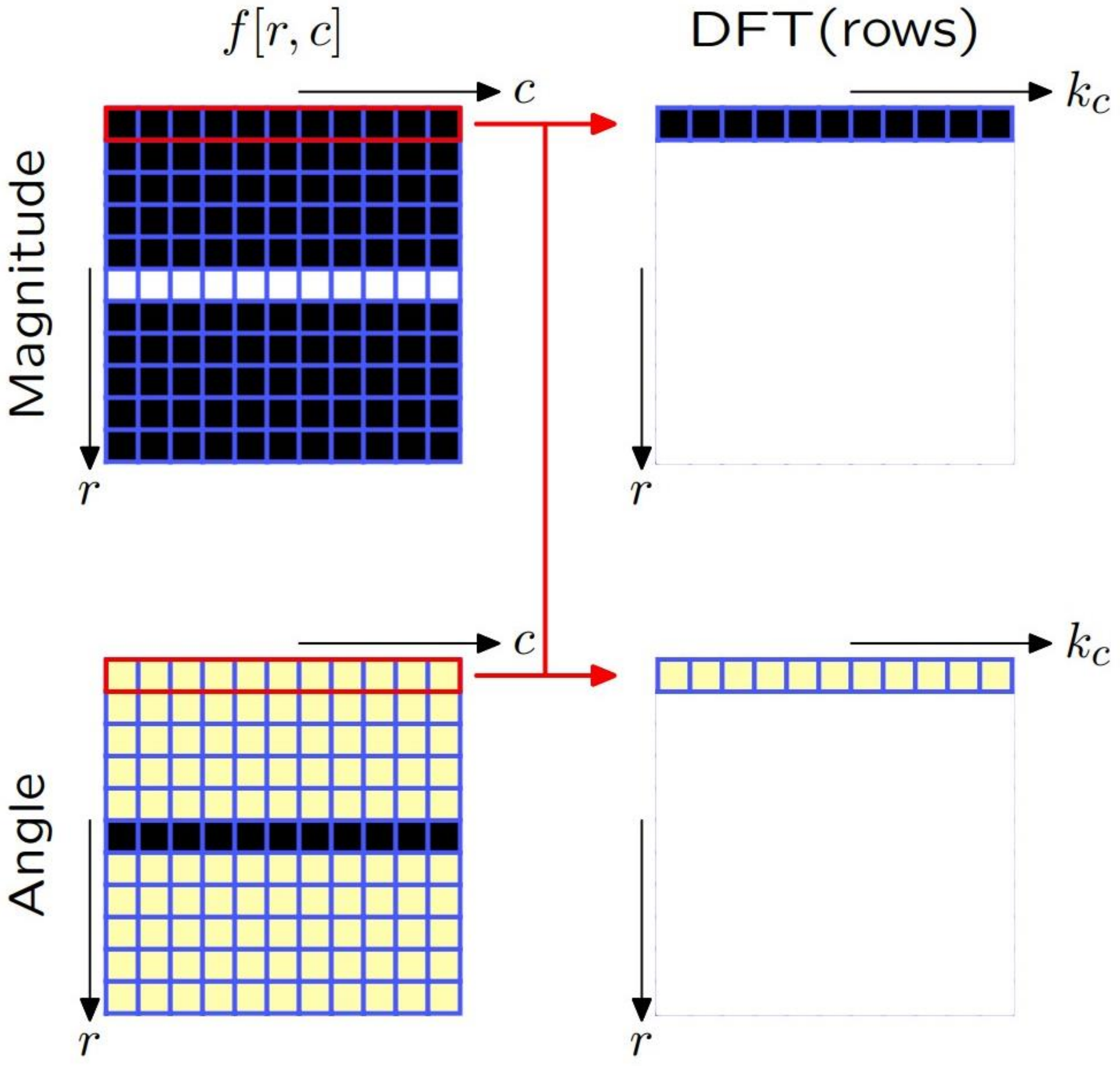
Example: Find the DFT of a horizontal line: (The image is centered at $r = 0, c = 0$)





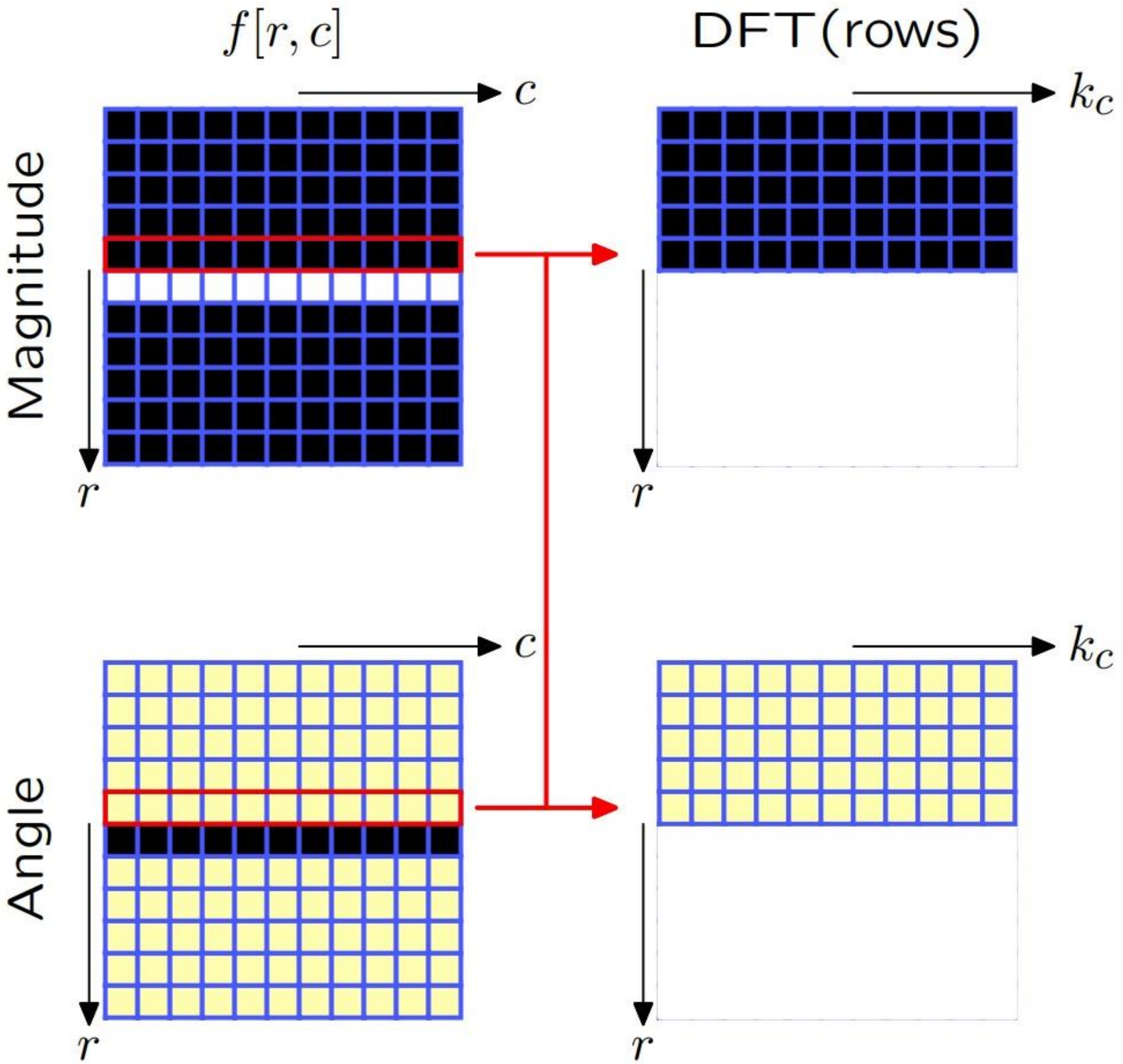
2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



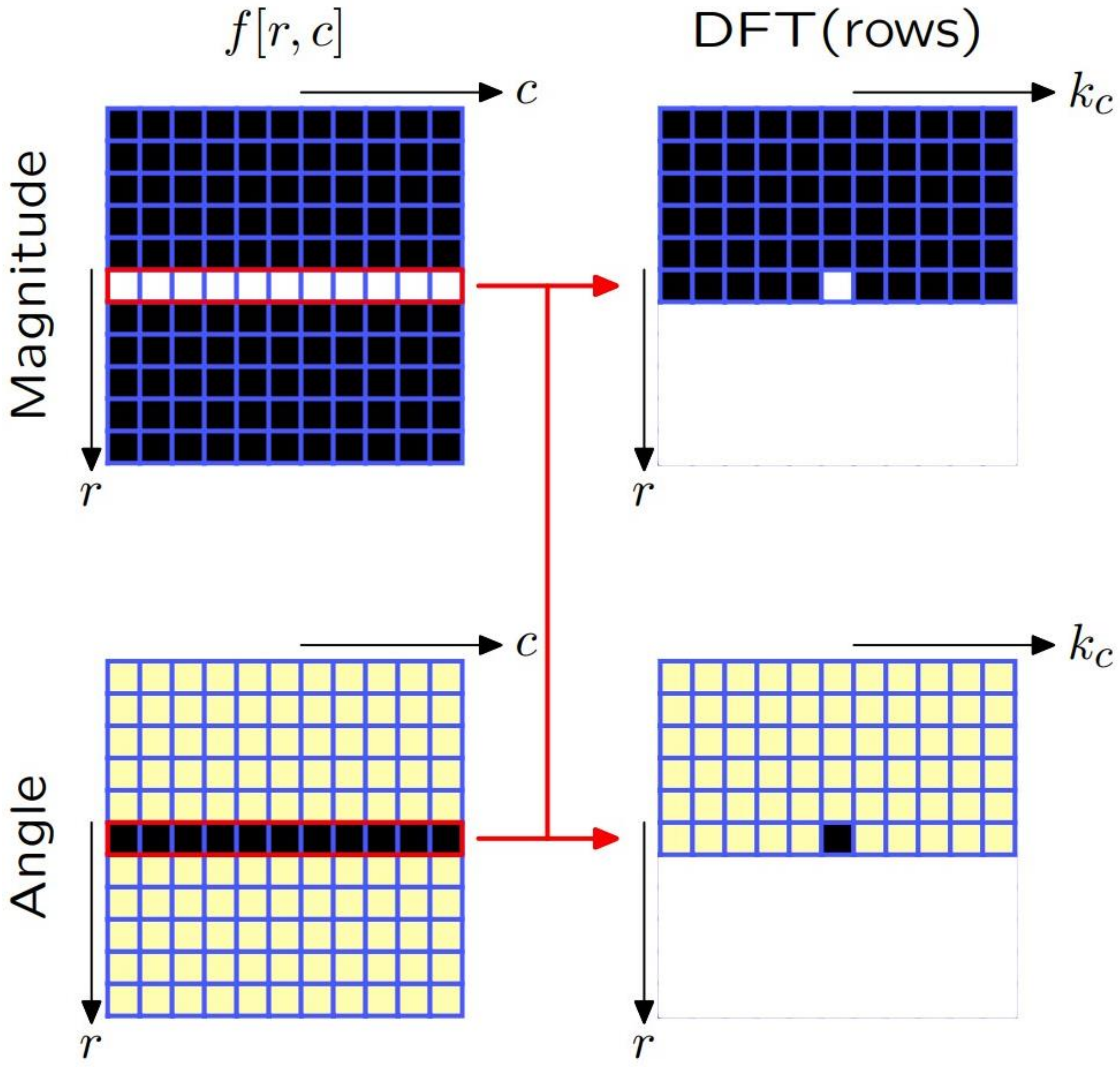
2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



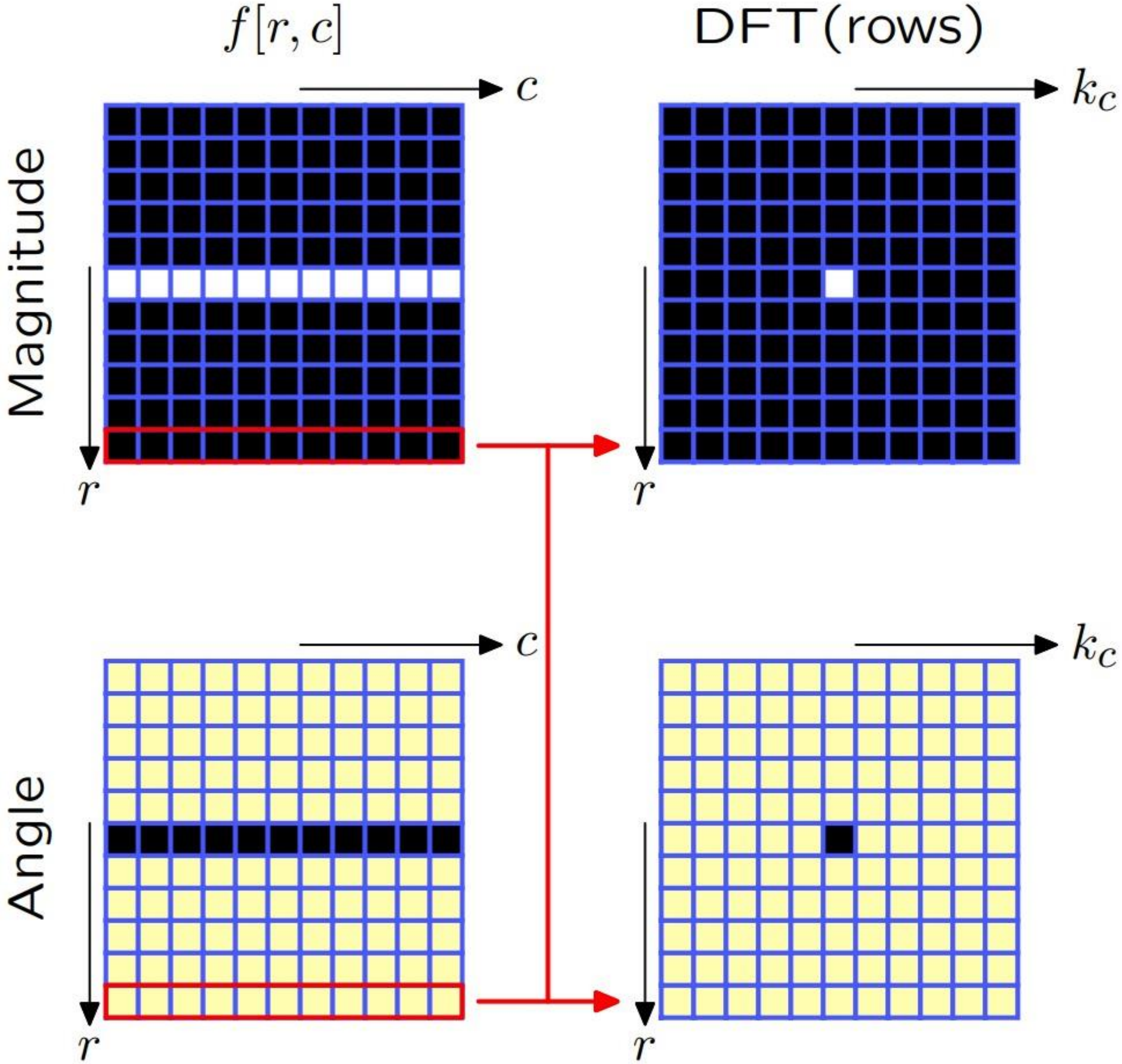
2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



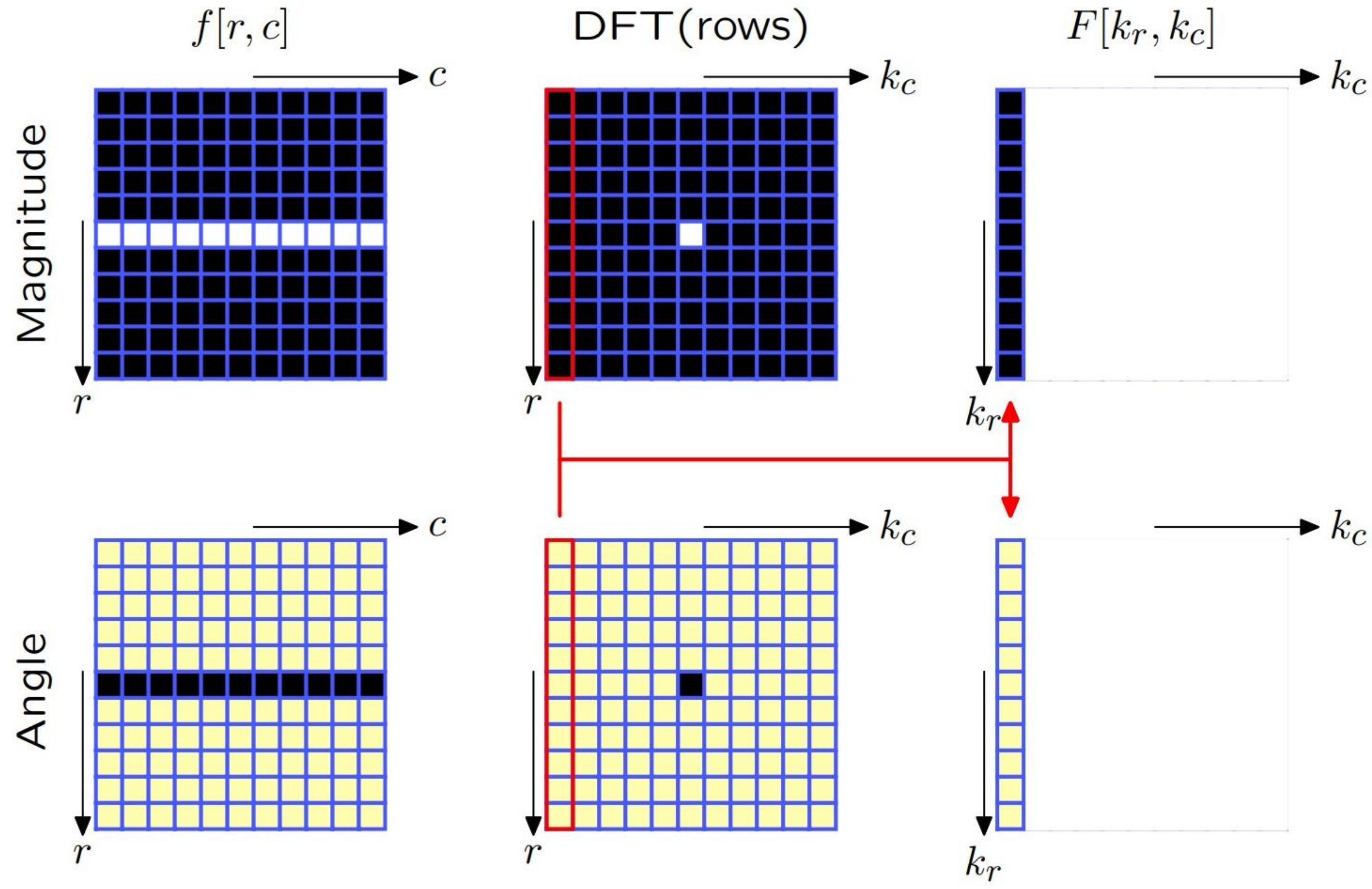
2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



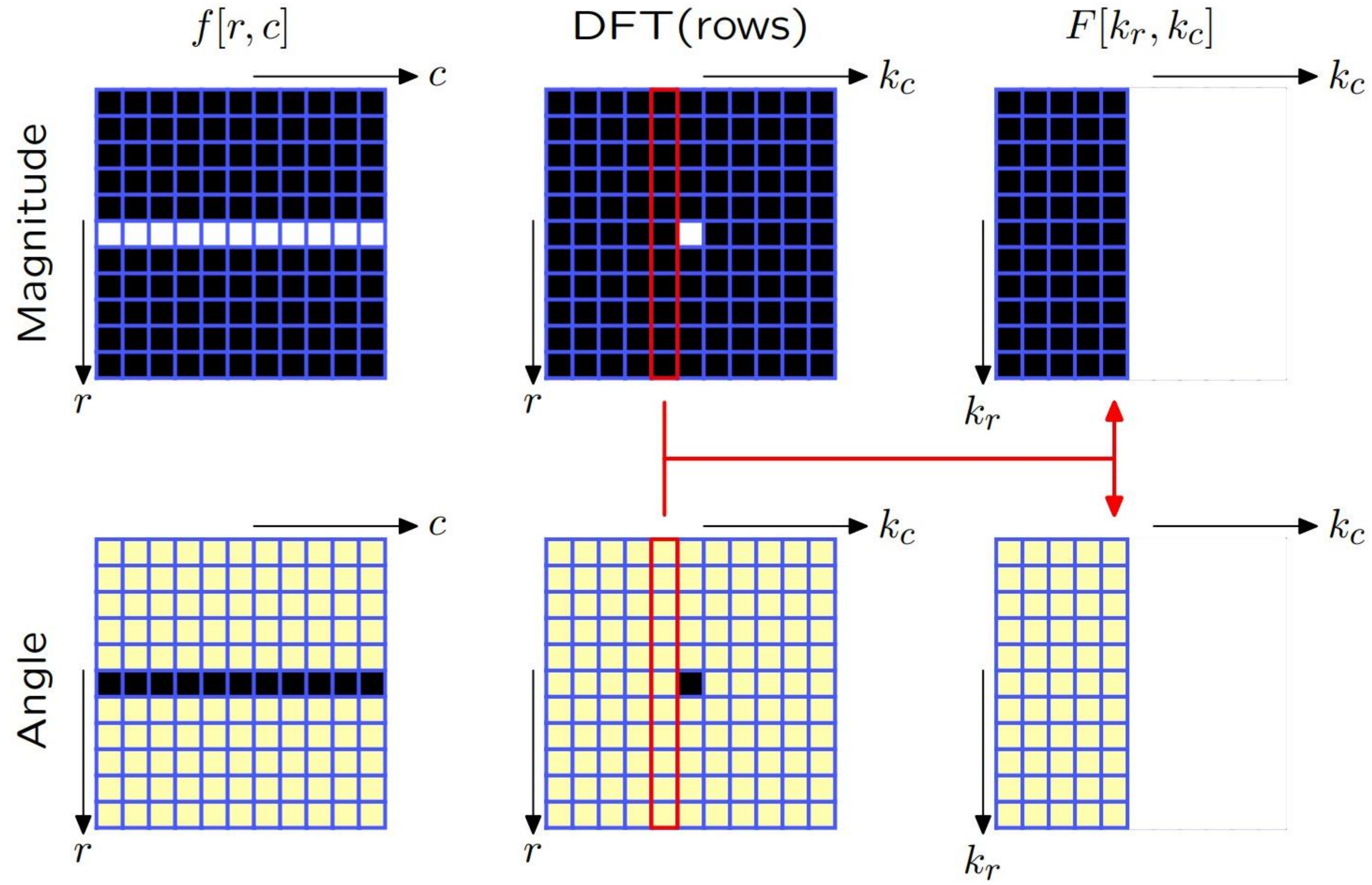
2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



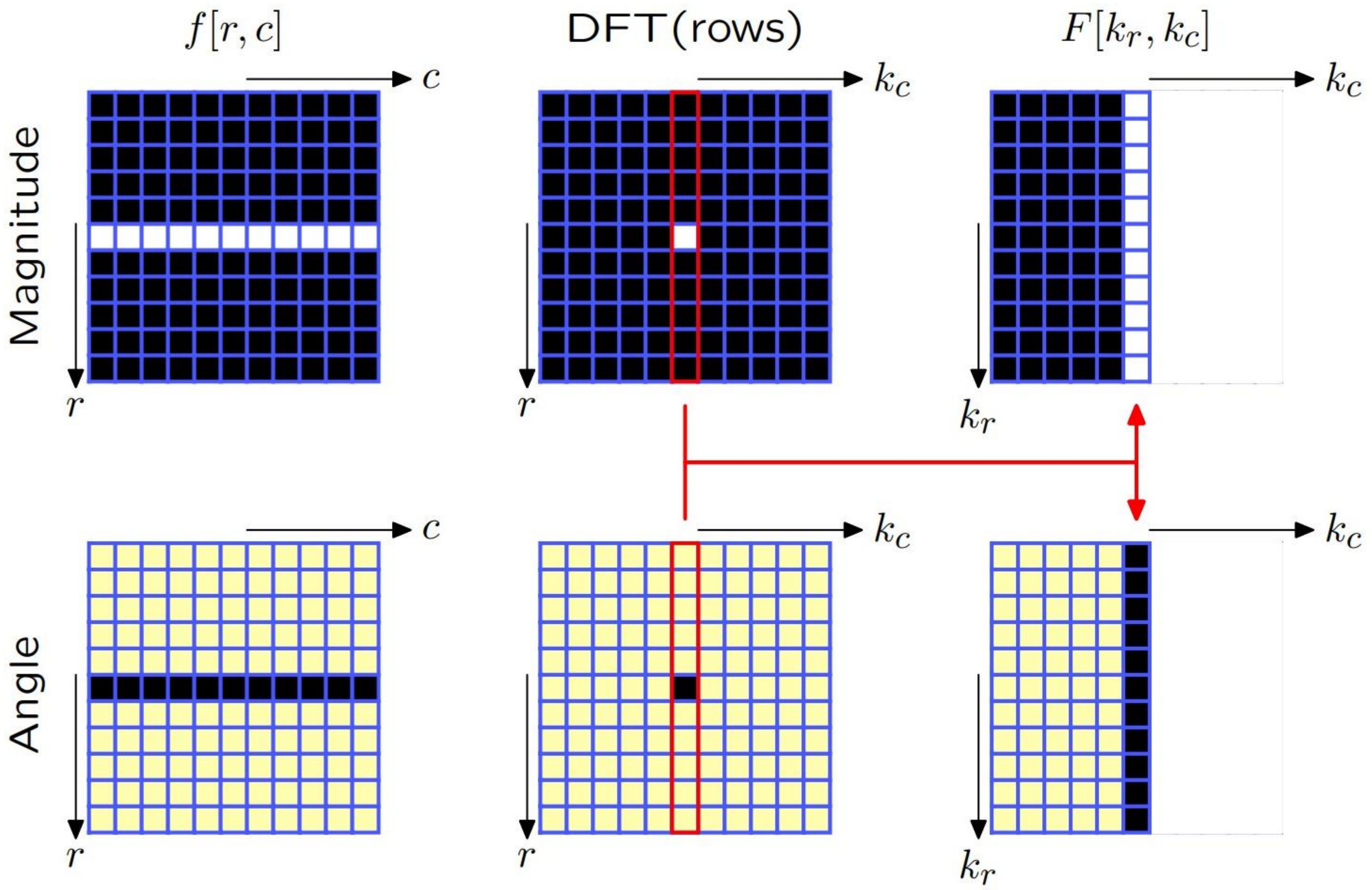
2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



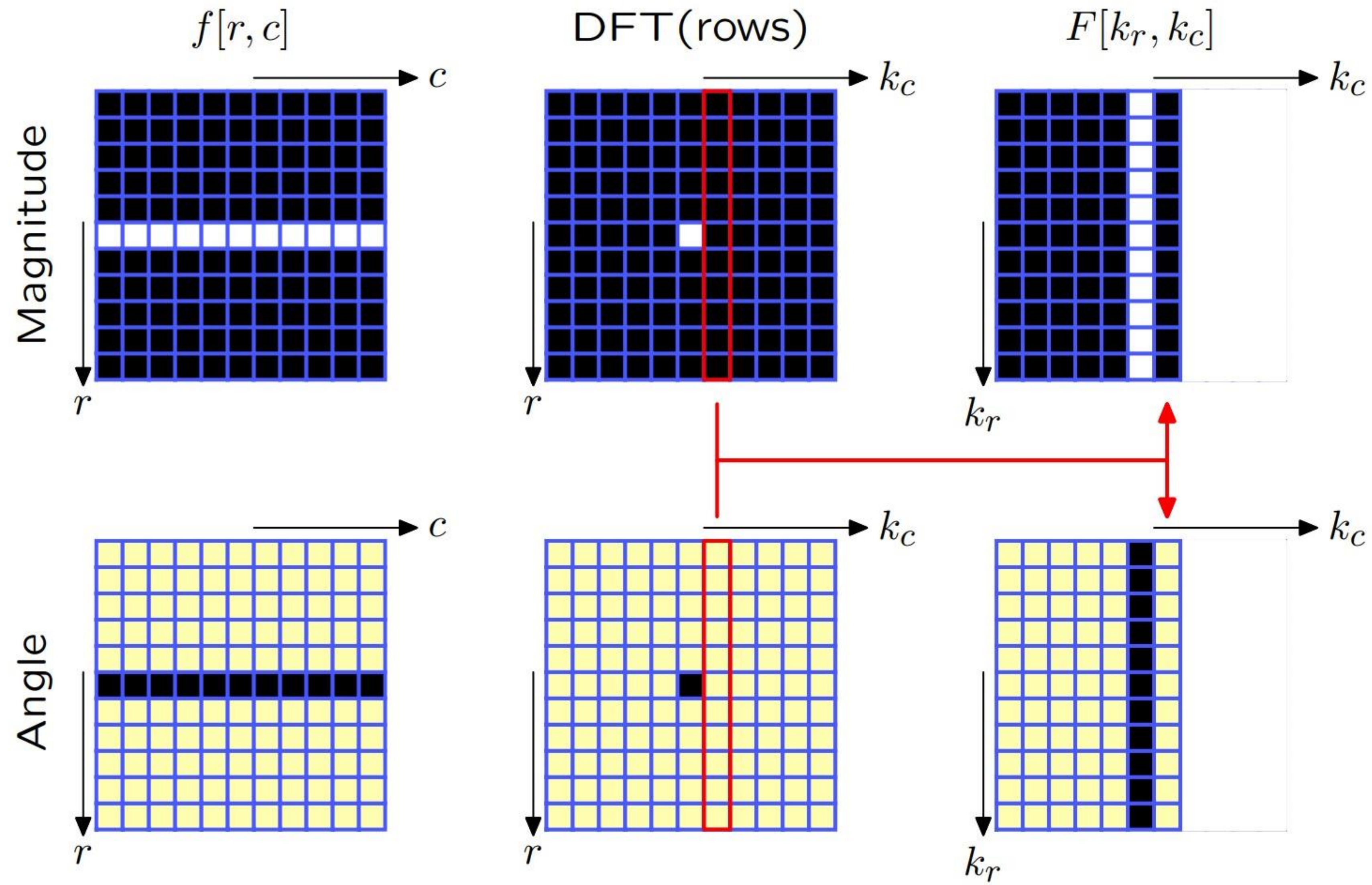
2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



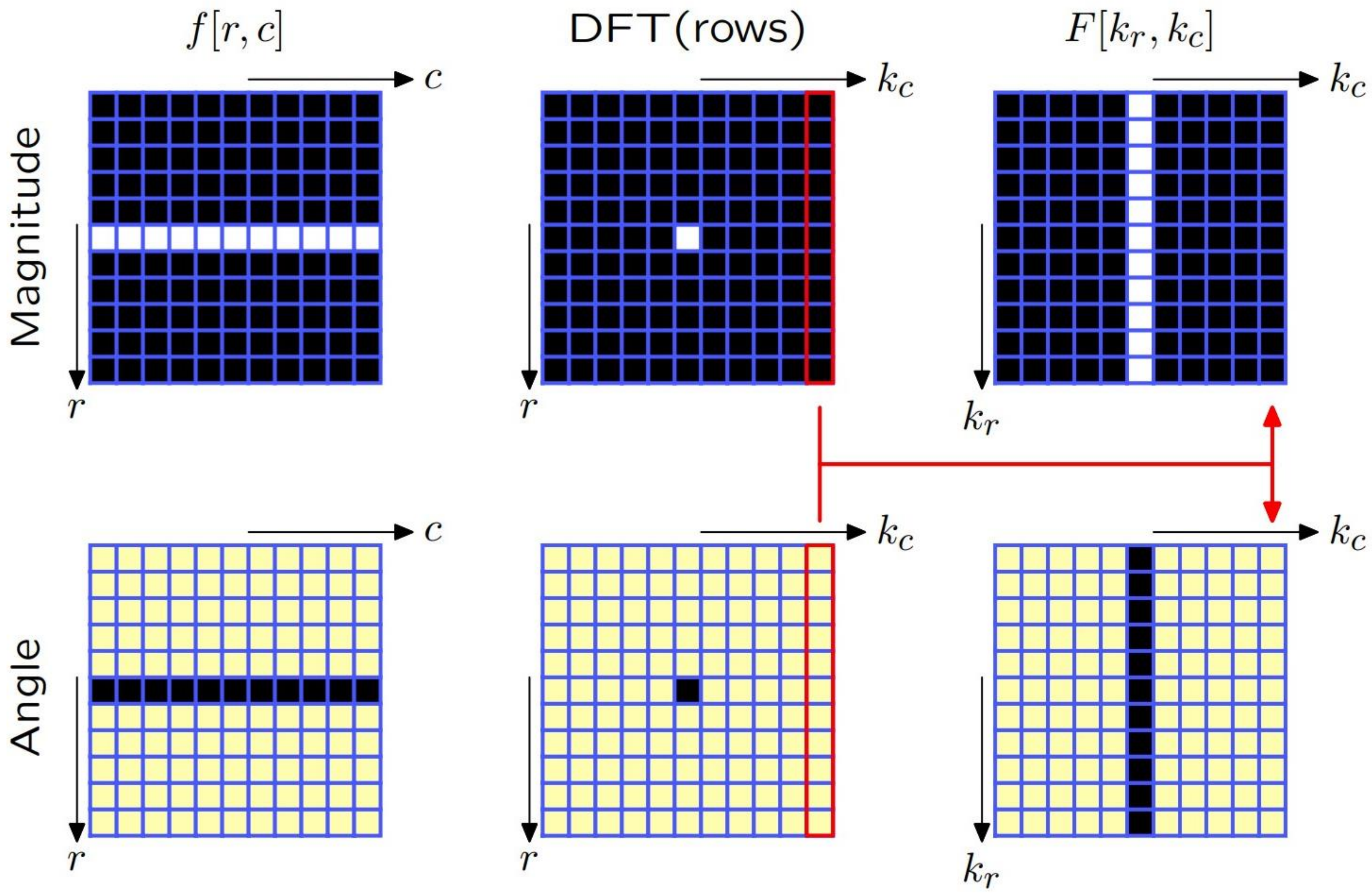
2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



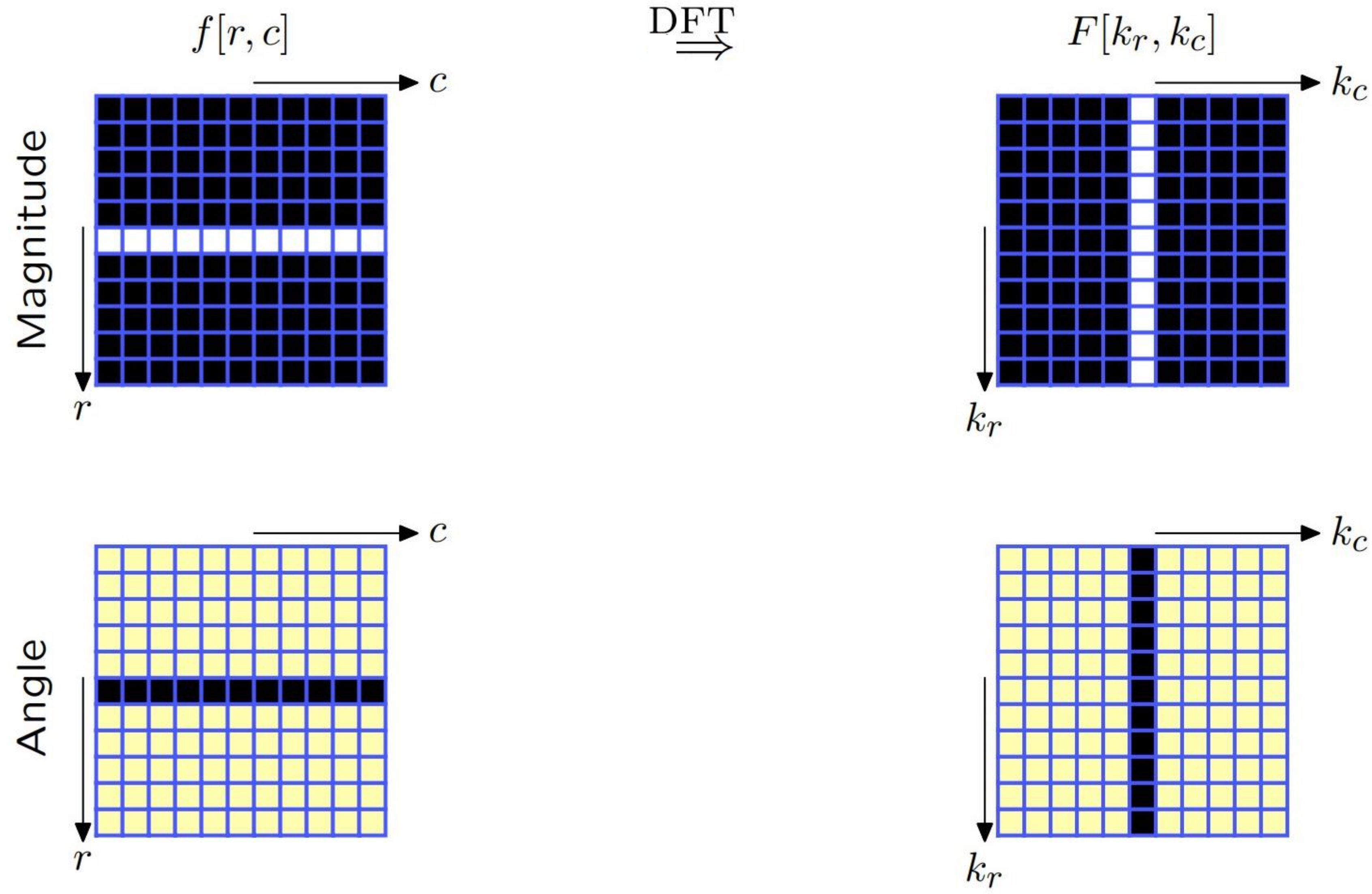
2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



2D Discrete Fourier Transform

Example: Find the DFT of a horizontal line:



Translating/Shifting an Image

Effect of image translation/shifting on its Fourier transform.

Assume that $f_0[r,c] \overset{DFT}{\Longrightarrow} F_0[k_r,k_c]$

Find the 2D DFT of $f_1[r,c] = f_0[r-r_0, c-c_0\ ]$

$$F_1[k_r,k_c] = \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1} f_1[r,c]\cdot e^{-j\left(\frac{2\pi k_r}{R}r+\frac{2\pi k_c}{C}c\right)}$$

$$= \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1} f_0[r-r_0,c-c_0]\cdot e^{-j\left(\frac{2\pi k_r}{R}r+\frac{2\pi k_c}{C}c\right)}$$

Let: $l_r = r - r_0$, $l_c = c - c_0$. Then:

$$F_1[k_r,k_c] = \frac{1}{RC}\sum_{l_r=<R>}\sum_{l_c=<C>} f_0[l_r,l_c]\cdot e^{-j\left(\frac{2\pi k_r}{R}((l_r+r_0))+\frac{2\pi k_c}{C}((l_c+c_0))\right)}$$

$$= e^{-j\frac{2\pi k_r}{R}r_0}\cdot e^{-j\frac{2\pi k_c}{C}c_0}\,\frac{1}{RC}\sum_{l_r=<R>}\sum_{l_c=<C>} f_0[l_r,l_c]\cdot e^{-j\left(\frac{2\pi k_r}{R}l_r+\frac{2\pi k_c}{C}l_c\right)} = e^{-j\frac{2\pi k_r}{R}r_0}\cdot e^{-j\frac{2\pi k_c}{C}c_0}\cdot F_0[k_r,k_c]$$

Translating an image adds linear phase to its transform.

2D Discrete Fourier Transform

Example: Find the DFT of a 2D unit sample:

$$f[r,c] = \delta[r]\delta[c] \overset{DFT}{\Longrightarrow} F[k_r,k_c] = \frac{1}{RC}$$

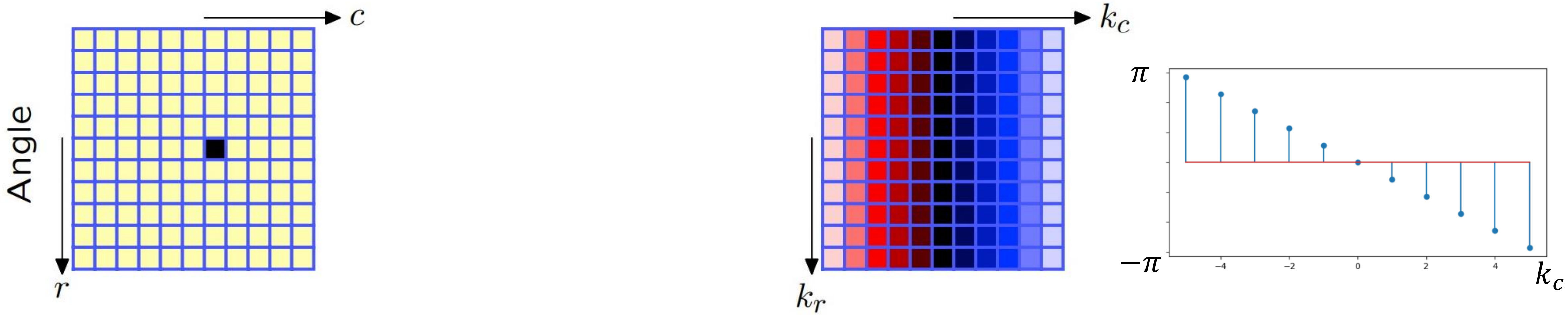
(The image is centered at $r = 0$, $c = 0$)

2D Discrete Fourier Transform

Example: Find the DFT of a shifted 2D unit sample.

$$f[r,c] = \delta[r]\delta[c-1] \overset{DFT}{\Longrightarrow} F[k_r,k_c] = e^{-j\frac{2\pi k_c}{C}\cdot 1}\frac{1}{RC}$$

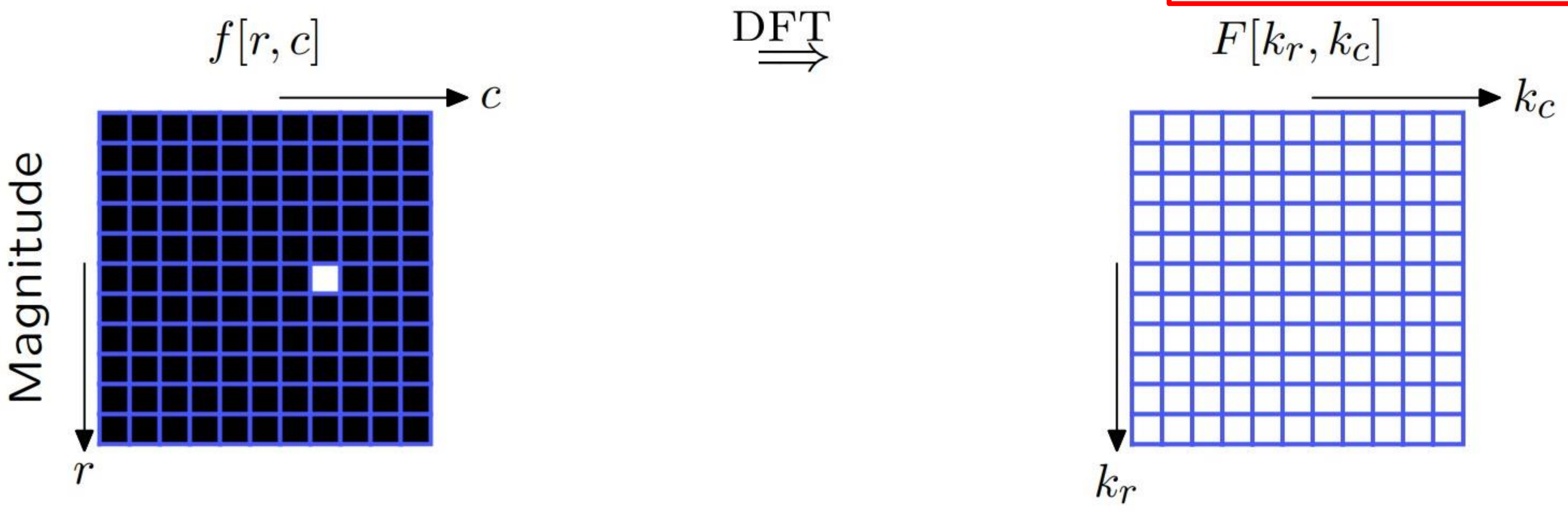
(The image is centered at $r=0$, $c=0$)



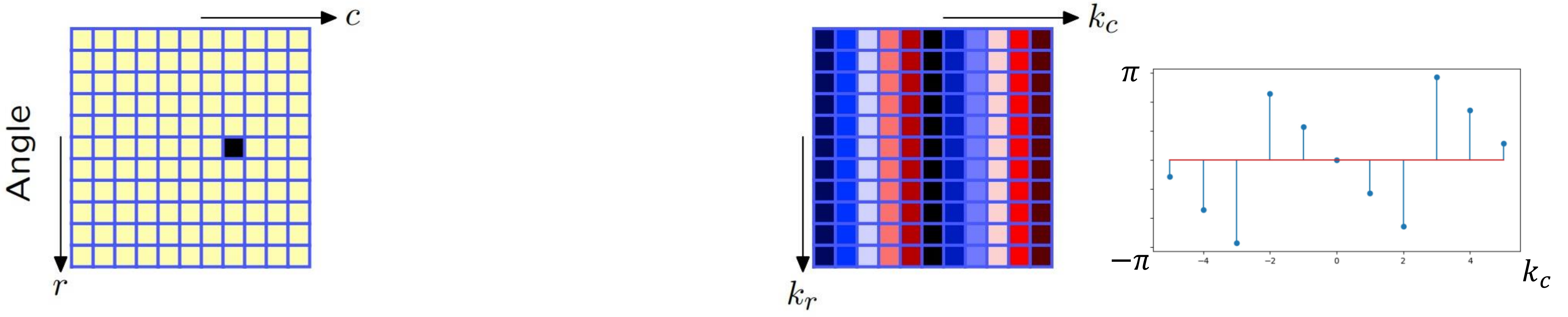
2D Discrete Fourier Transform

Example: Find the DFT of a shifted 2D unit sample.

$$f[r,c] = \delta[r]\delta[c-2] \overset{DFT}{\Longrightarrow} F[k_r,k_c] = e^{-j\frac{2\pi k_c}{C}\cdot 2}\frac{1}{RC}$$



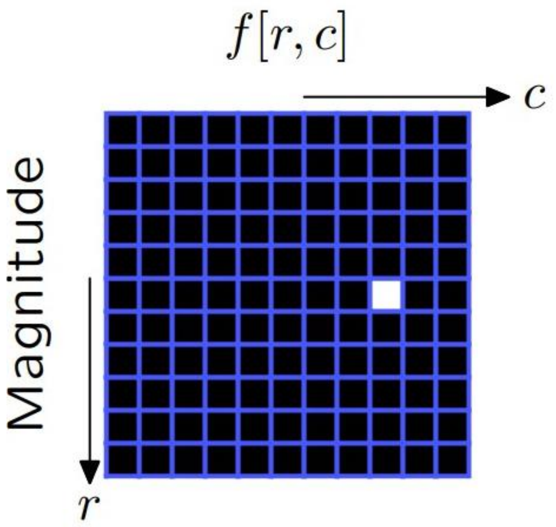
(The image is centered at $r = 0, c = 0$)



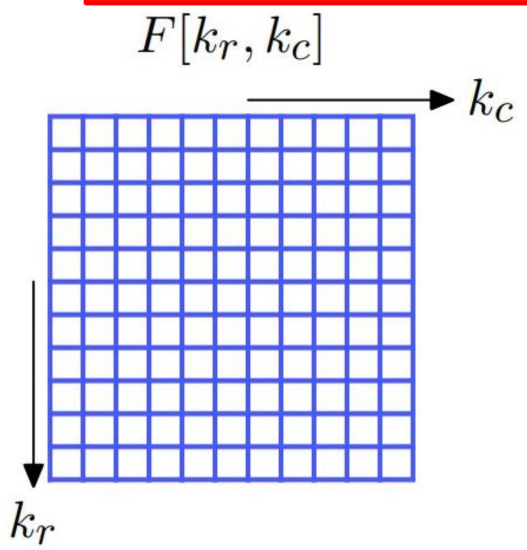
2D Discrete Fourier Transform

Example: Find the DFT of a shifted 2D unit sample.

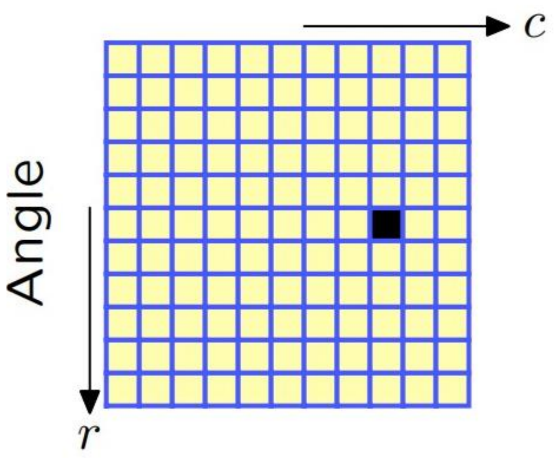
$$f[r,c] = \delta[r]\delta[c-3] \overset{DFT}{\Longrightarrow} F[k_r,k_c] = e^{-j\frac{2\pi k_c}{C}\cdot 3}\frac{1}{RC}$$

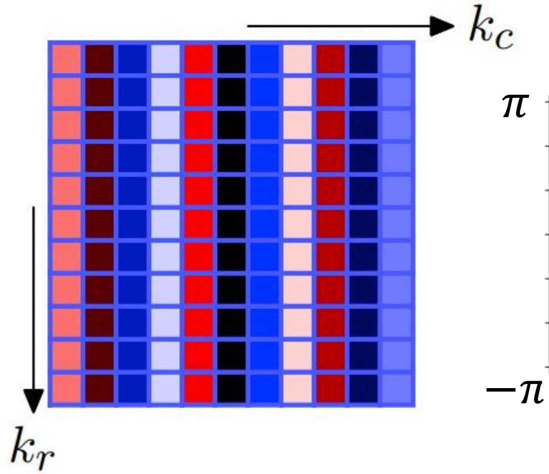


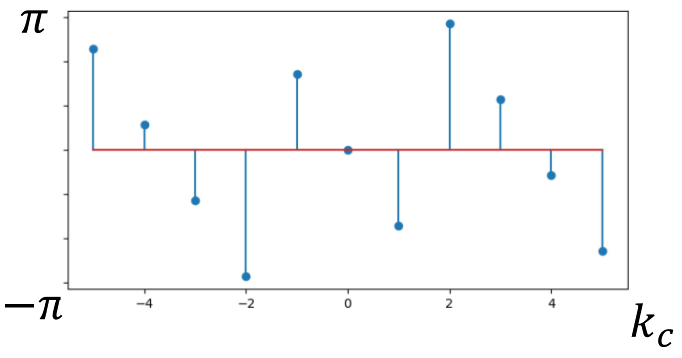
$\overset{DFT}{\Longrightarrow}$



(The image is centered at $r = 0$, $c = 0$)



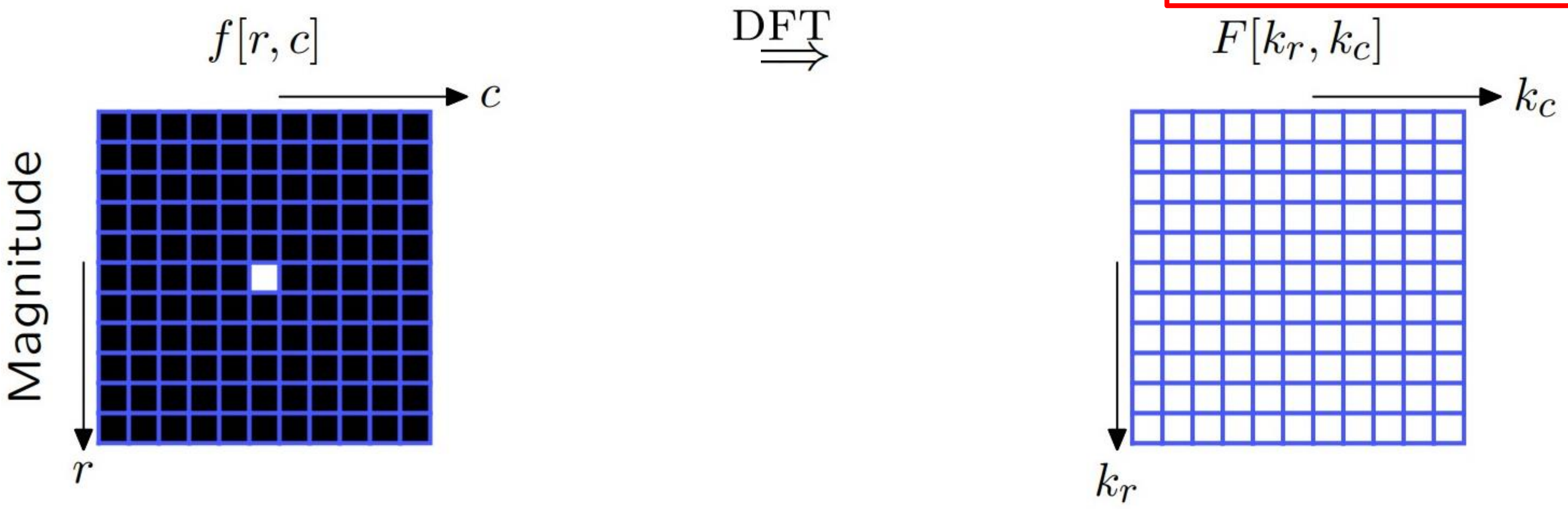




2D Discrete Fourier Transform

Example: Find the DFT of a 2D unit sample:

$$f[r,c] = \delta[r]\delta[c] \overset{DFT}{\Longrightarrow} F[k_r, k_c] = \frac{1}{RC}$$

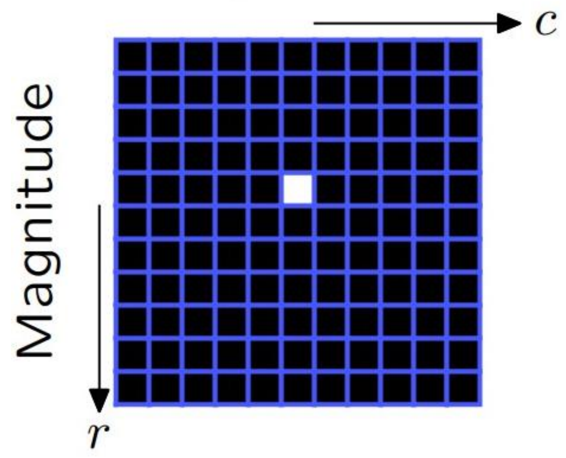


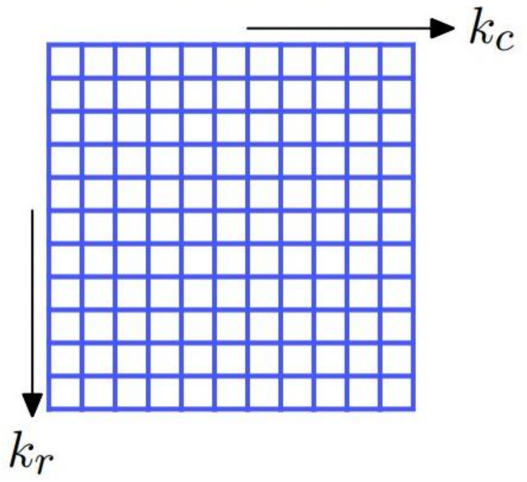
(The image is centered at $r = 0$, $c = 0$)

2D Discrete Fourier Transform

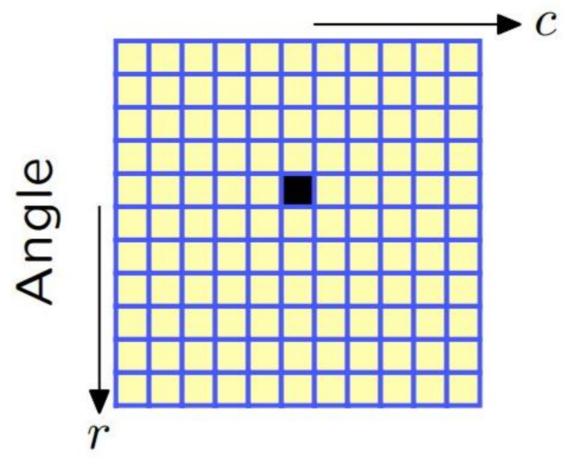
Example: Find the DFT of a shifted2D unit sample: $f[r,c] = \delta[r+1]\delta[c] \overset{DFT}{\Longrightarrow} F[k_r,k_c] = e^{-j\frac{2\pi k_r}{R}\cdot(-1)}\frac{1}{RC}$

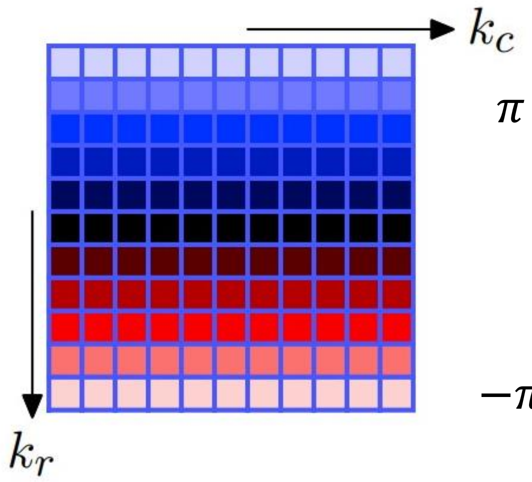
$f[r,c]$ $\overset{\text{DFT}}{\Longrightarrow}$ $F[k_r,k_c]$

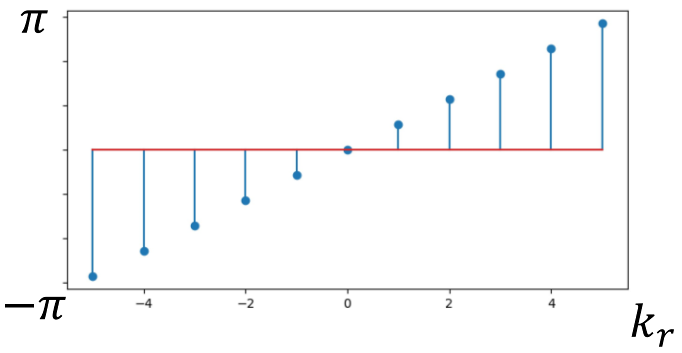




(The image is centered at $r=0$, $c=0$)



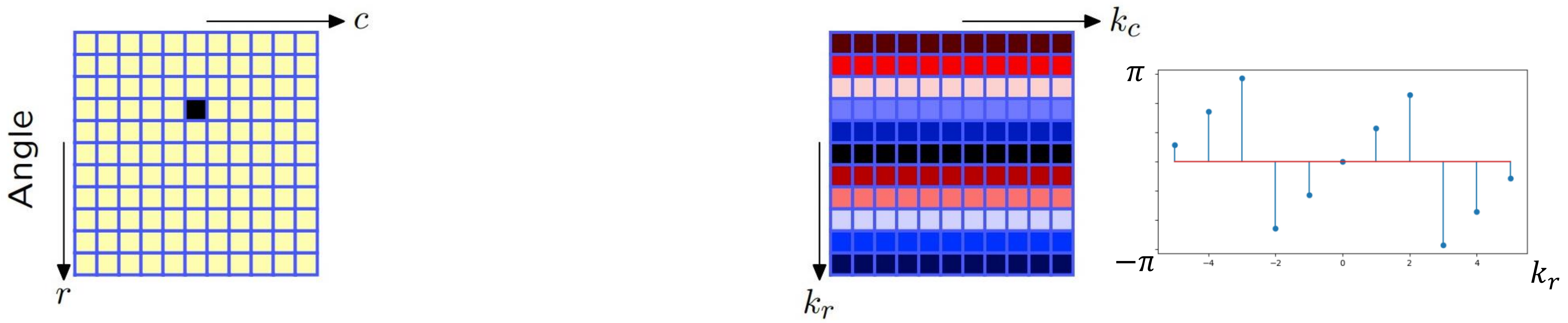




2D Discrete Fourier Transform

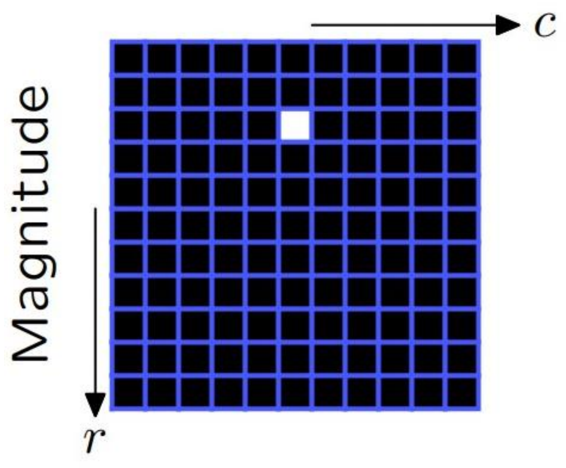
Example: Find the DFT of a shifted 2D unit sample: $f[r,c] = \delta[r+2]\delta[c] \overset{DFT}{\Longrightarrow} F[k_r,k_c] = e^{-j\frac{2\pi k_r}{R}\cdot(-2)} \frac{1}{RC}$

(The image is centered at $r = 0, c = 0$)

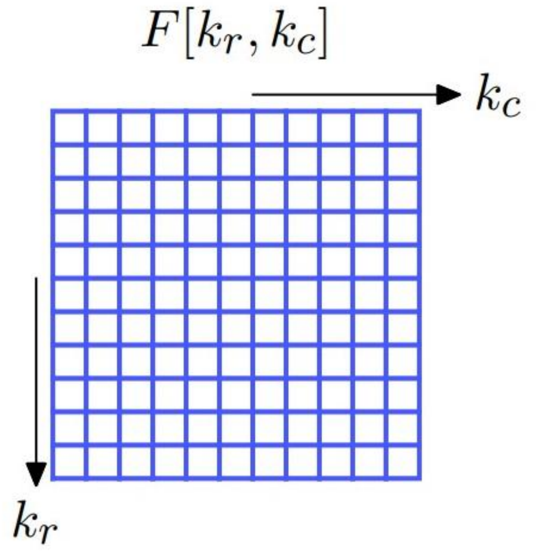


2D Discrete Fourier Transform

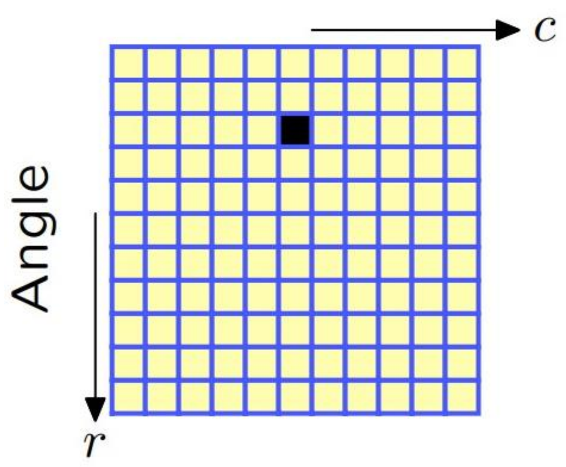
Example: Find the DFT of a shifted 2D unit sample: $f[r,c] = \delta[r+3]\delta[c] \overset{DFT}{\Longrightarrow} F[k_r, k_c] = e^{-j\frac{2\pi k_r}{R}\cdot(-3)} \frac{1}{RC}$

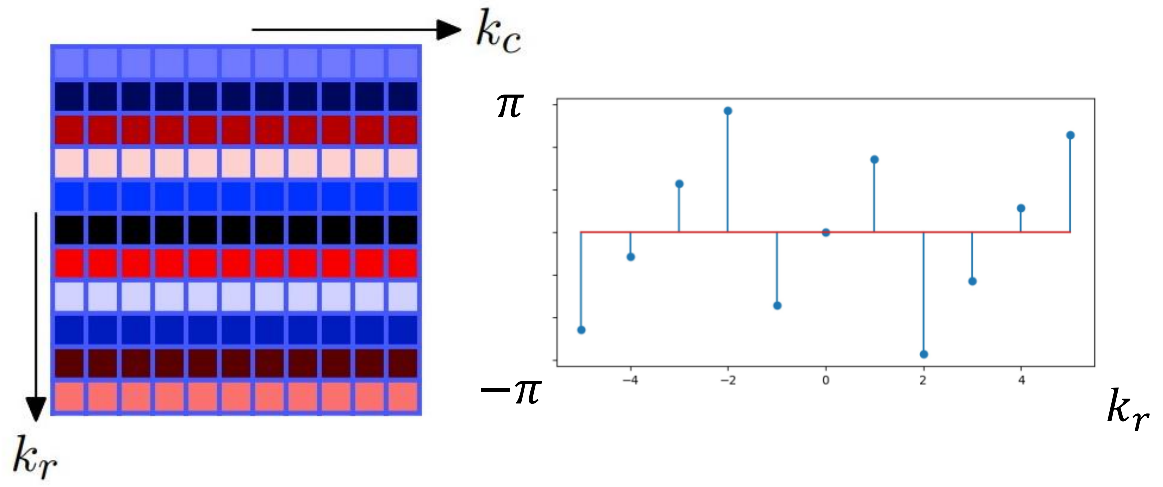


$\overset{\text{DFT}}{\Longrightarrow}$



(The image is centered at $r = 0, c = 0$)





Summary

We will now go to 4-370 for recitation & common hour

Introduced 2D signal processing:
- Mostly simple extensions of 1D ideas
- Some small differences

Introduced 2D Fourier representations:
- Fourier kernal comprises the sum of an x part and a y part (or r, c)
- Basis functions look like sinusoids turned at angles determined by the ratio of k_c to k_r.

Multiple 2D Fourier Transform pairs:
- 2D unit sample => 2D constant
- 2D constant => 2D unit sample
- vertical line => horizontal line
- horizontal line => vertical line

Translating an image does not change the magnitude but adds linear phase to its transform.