6.300 Signal Processing

Week 11, Lecture A:
Communication Systems

- Matching Signals to Communications Media
- Amplitude Modulation (AM)
- Frequency-Division Multiplexing

Lecture slides are available on CATSOOP:
https://sigproc.mit.edu/fall24

Communications Systems

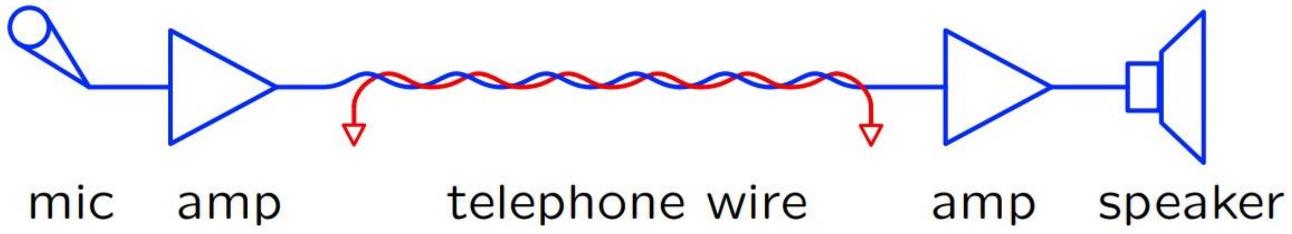
Beginning with commercial radio (1900s), communications technologies continue to be among the fastest growing applications of signal processing.

Examples:

- cellular communications
- wifi
- bluetooth
- GPS (Global Positioning System)
- IOT (Internet of Things)
  - smart house / smart appliances
  - smart car
  - medical devices
- cable
- private networks: fire departments, police
- radar and navigation systems

Telephone

Popular thirst for communications has been evident since the early days of telephony.

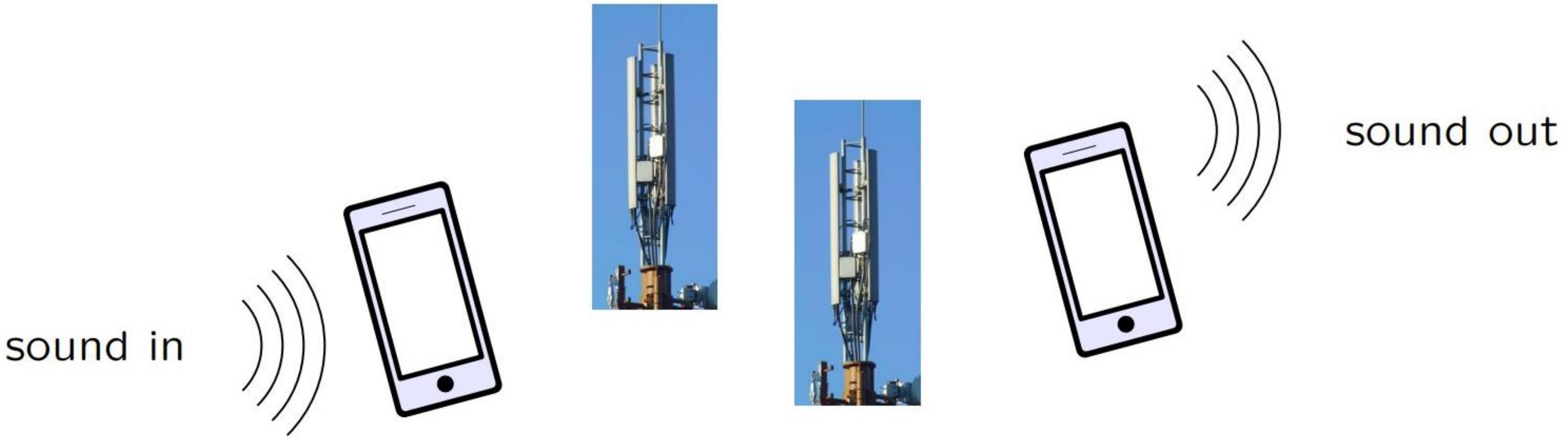


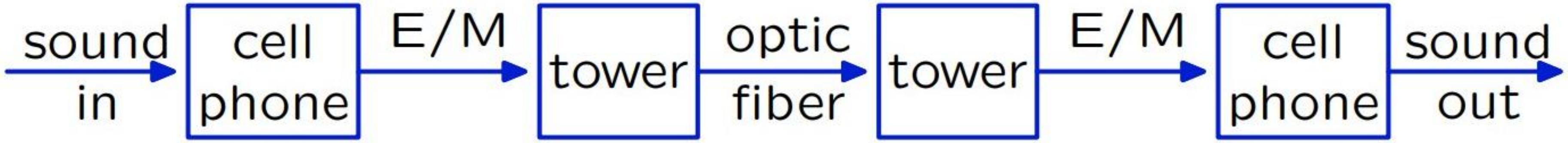
Patented by Alexander Graham Bell (1876), this technology flourished first as a network of copper wires and later as optical fibers ("long-distance" network) connecting virtually every household in the US by the 1980s.

Bell Labs became a premier research facility, developing information theory and a host of wired and wireless communications technologies that built on that theory, as well has hardware innovations such as the transistor and the laser.

Cellular Communication

First demonstrated by Motorola (1973), cellular communications quickly revolutionized the field. There are now more cell phones than people in the world.

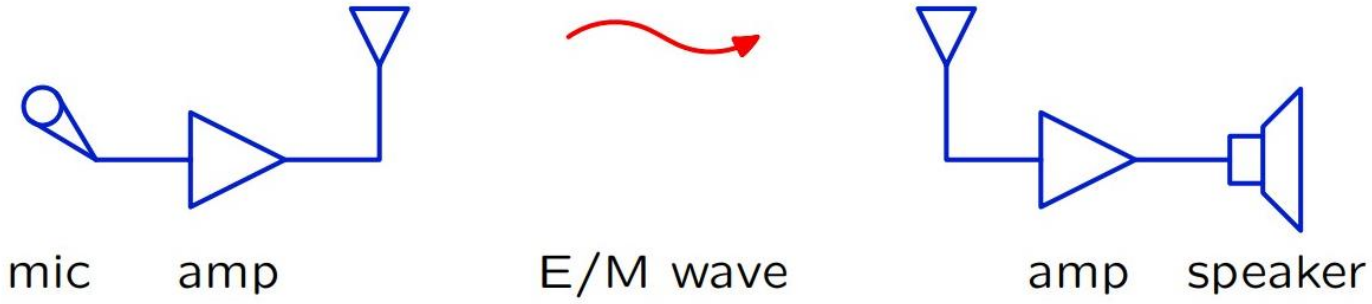




Much of the popularity and convenience of cellular communications is that the communication is wireless (at least to the local tower).

Wireless Communication

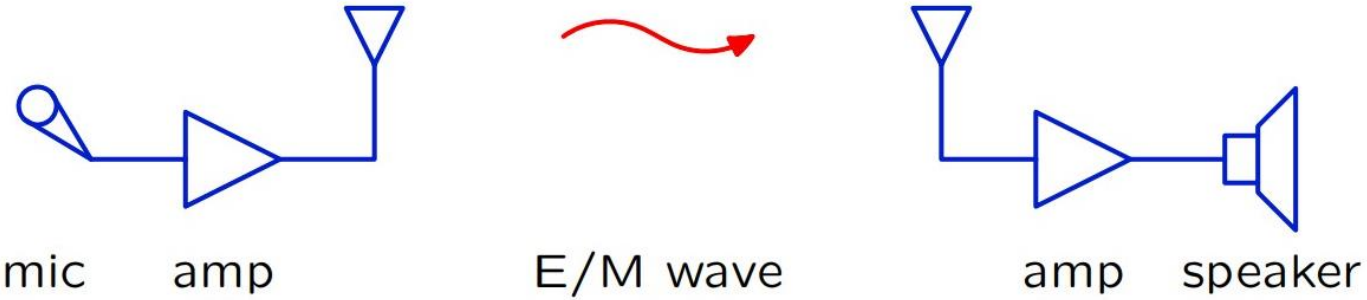
Wireless signals are transmitted via electromagnetic (E/M) waves.



How well are E/M waves matched to speech?

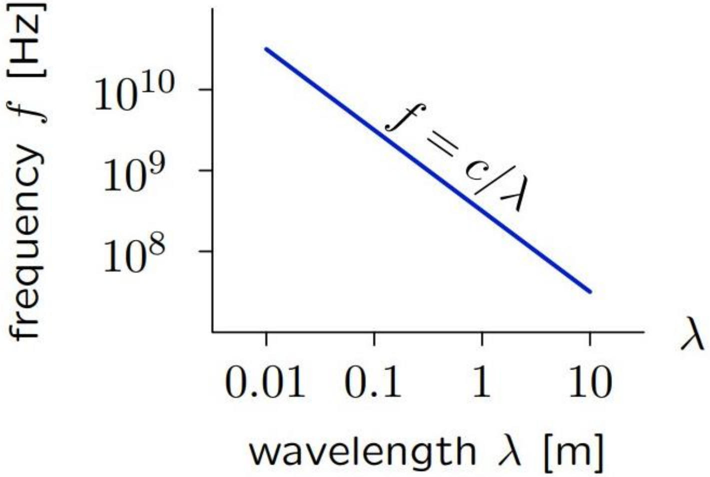
Wireless Communication

Wireless signals are transmitted via electromagnetic (E/M) waves.



For energy-efficient transmission and reception, the dimensions of the antenna should be on the order of the wavelength.

But the wavelength of an electromagnetic wave depends on frequency.



Matching Signals to Communications Media

A key problem in the design of any communications system is matching characteristics of the signal to those of the media.

Telephone-quality speech contains frequencies from 200 Hz to 3000 Hz.

How large must the antenna be for efficient transmission and reception of E/M waves?

Check yourself!

For energy-efficient transmission and reception, the length of the antenna should be on the order of the wavelength.

Telephone-quality speech contains frequencies between 200 Hz and 3000 Hz.

How long should the antenna be?

1. < 1 mm
2. $\sim$ cm
3. $\sim$ m
4. $\sim$ km
5. > 100 km

Check Yourself

Wavelength is $\lambda = c/f$.

The lowest frequencies (200 Hz) produce the longest wavelengths:

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8\, m/s}{200 \text{ Hz}} = 1.5 \times 10^6\, m = 1500\, km$$

and highest frequencies (3000 Hz) produce the shortest wavelengths

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8\, m/s}{3000 \text{ Hz}} = 10^5\, m = 100\, km$$

The size of the antenna should be in the range of 100-1500 km (~60 to 900 miles)!

Check yourself!

For energy-efficient transmission and reception, the length of the antenna should be on the order of the wavelength.

Telephone-quality speech contains frequencies between 200 Hz and 3000 Hz.

How long should the antenna be? 5

1. < 1 mm
2. $\sim$ cm
3. $\sim$ m
4. $\sim$ km
5. > 100 km

Check yourself!

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)?

1. < 100 kHz
2. 1 MHz
3. 10 MHz
4. 100 MHz
5. > 1 GHz

A wavelength of 10 cm corresponds to a frequency of

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8\ m/s}{10cm} = 3 \times 10^9 \text{ Hz} = 3\text{GHz}$$

4G phones use frequencies near 2.5 GHz.

5G phones use a mix of frequencies ... up to 50 GHz.

Matching Signals to Communications Media

Speech is not well matched to the wireless medium.

Matching the message to the medium is important in all communications systems.

Example media:
- space
- cable (coaxial wires)
- Optical fibers

Today we will introduce simple matching strategies based on modulation, which underlies virtually all matching schemes.

Matching Signals to Communications Media

How can we use frequencies in one band to communicate messages in a different band?

One simple method is based on the frequency-shift property of Fourier transforms.

If

$$x(t) \overset{CTFT}{\Longleftrightarrow} X(\omega)$$

then

$$y(t) = e^{j\omega_c t} \cdot x(t) \overset{CTFT}{\Longleftrightarrow} \quad Y(\omega) = X(\omega - \omega_c)$$

$$Y(\omega) = \int_{-\infty}^{\infty} y(t) \cdot e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} (e^{j\omega_c t} \cdot x(t)) \cdot e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} x(t) \cdot e^{-j(\omega-\omega_c)t}\, dt$$

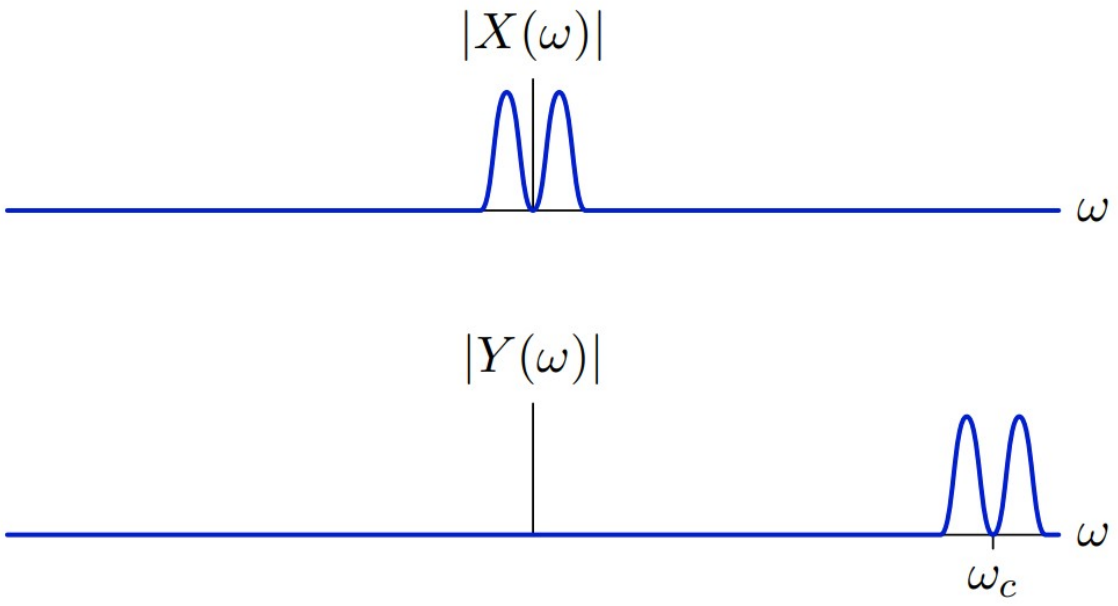
Let

$$\lambda = \omega - \omega_c, \qquad \omega = \lambda + \omega_c$$

$$Y(\lambda + \omega_c) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\lambda t}\, dt = X(\lambda) \quad \Rightarrow \quad Y(\omega) = X(\omega - \omega_c)$$

Matching Signals to Communications Media

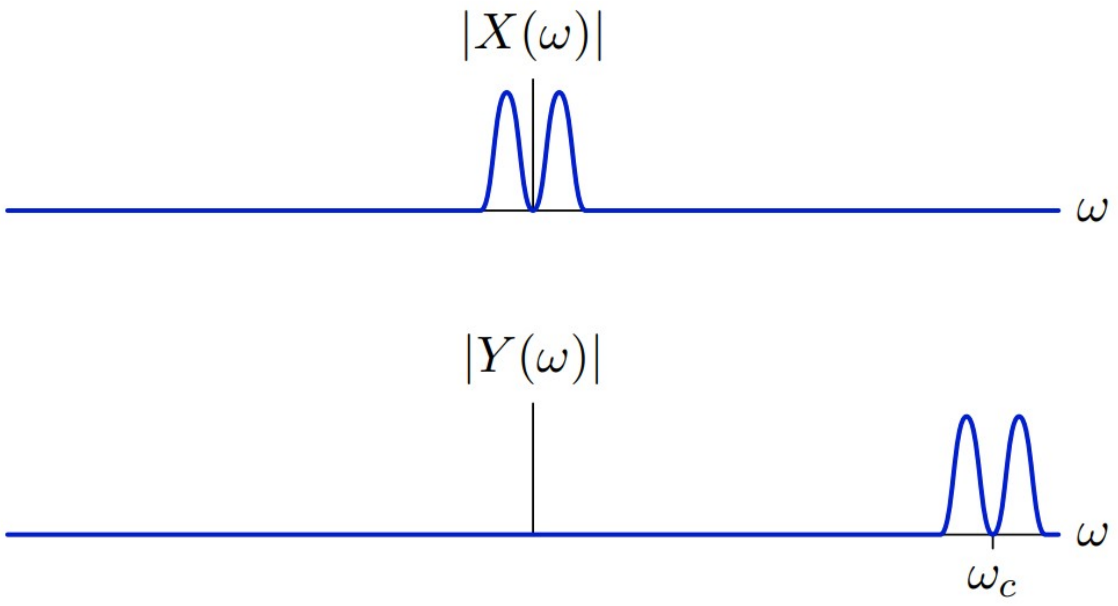
Multiplying a signal $x(t)$ by $e^{j\omega_c t}$ shifts the frequency content of $x(t)$ upward in frequency by ω_c.



Any problems with this scheme?

Matching Signals to Communications Media

Multiplying a signal $x(t)$ by $e^{j\omega_c t}$ shifts the frequency content of $x(t)$ upward in frequency by ω_c.



The problem with this idea is that the signal $e^{j\omega_c t}x(t)$ has both real and imaginary parts – it is hard to be implemented physically.

Matching Signals to Communications Media

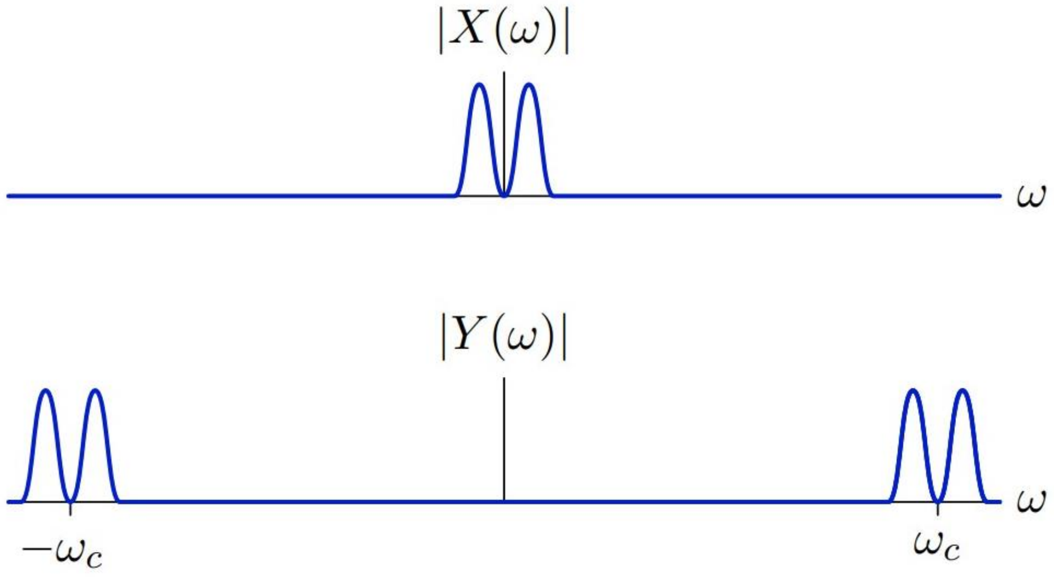
A purely real-valued signal can be obtained if we shift a copy of $X(\omega)$ upward in frequency by ω_c and a second copy downward by the same amount.

If

$$x(t) \overset{CTFT}{\Longleftrightarrow} X(\omega)$$

then

$$e^{j\omega_c t} \cdot x(t) + e^{-j\omega_c t} \cdot x(t) = 2\cos(\omega_c t) \cdot x(t) \overset{CTFT}{\Longleftrightarrow} X(\omega - \omega_c) + X(\omega + \omega_c)$$

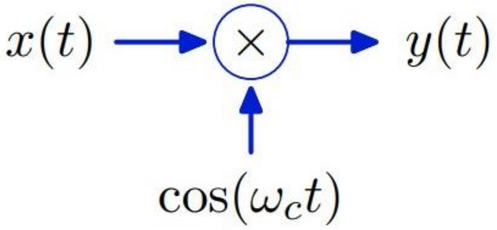


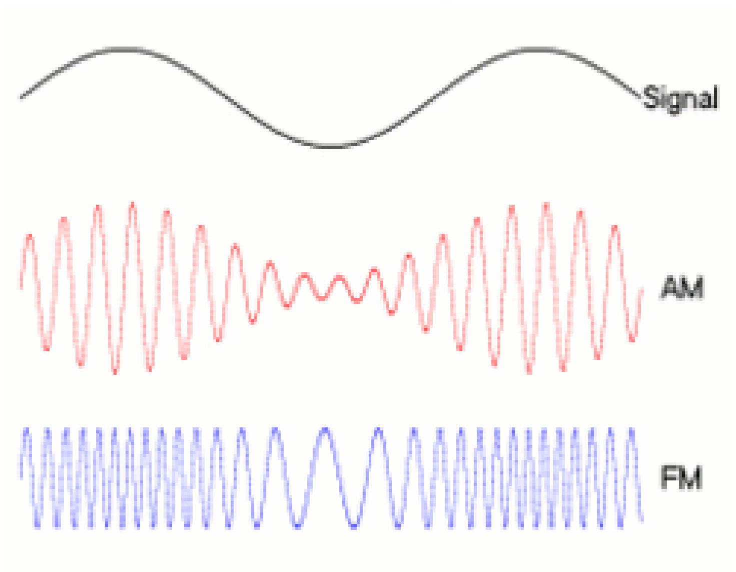
We refer to this scheme as amplitude modulation (AM).

Amplitude Modulation

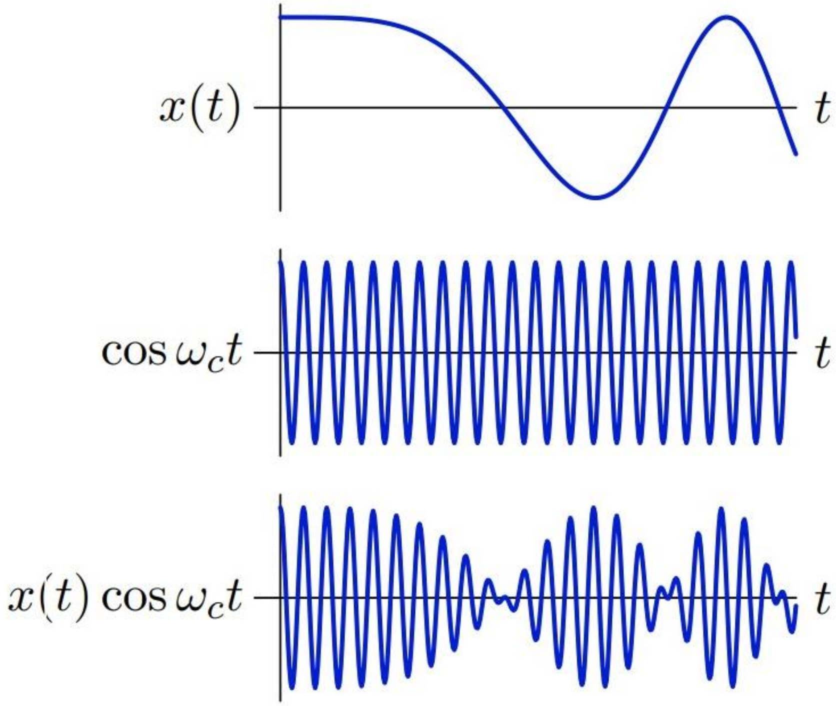
Multiplying a signal by a sinusoidal “carrier” is called amplitude modulation. The signal “modulates” the amplitude of the carrier.

In AM radio, the highest frequency in the message is 5 kHz and the carrier frequency is between 500 kHz and 1500 kHz.





https://en.wikipedia.org/wiki/Amplitude_modulation



- AM: the earliest modulation method transmitting audio in radio broadcasting
- remains in use in many forms of communication

Multiplication Property of Fourier Transform

Multiplication in time corresponds to convolution in frequency.

Let $z(t) = x(t)y(t)$ then $Z(\omega) = \int_{-\infty}^{\infty} x(t)y(t) \cdot e^{-j\omega t}\, dt$

Substitute

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega) \cdot e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\lambda) \cdot e^{j\lambda t}\, d\lambda$$

$$Z(\omega) = \int_{-\infty}^{\infty} x(t)\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\lambda) \cdot e^{j\lambda t}\, d\lambda\right) \cdot e^{-j\omega t}\, dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\lambda)\left(\int_{-\infty}^{\infty} x(t) \cdot e^{-j(\omega-\lambda)t}\, dt\right) d\lambda$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\lambda)\, X(\omega-\lambda)\, d\lambda = \frac{1}{2\pi}(Y * X)(\omega)$$

This result is the dual of filtering, where convolution in time $\Longleftrightarrow$ multiplication in frequency.

Multiplication Property of Fourier Transform

Multiplication in time corresponds to convolution in frequency.

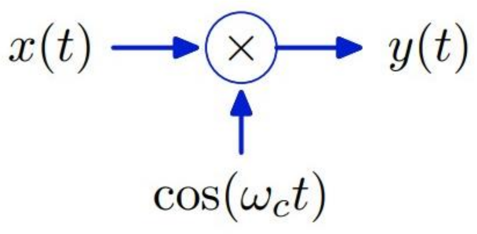
If $\quad z(t) = x(t)y(t)$

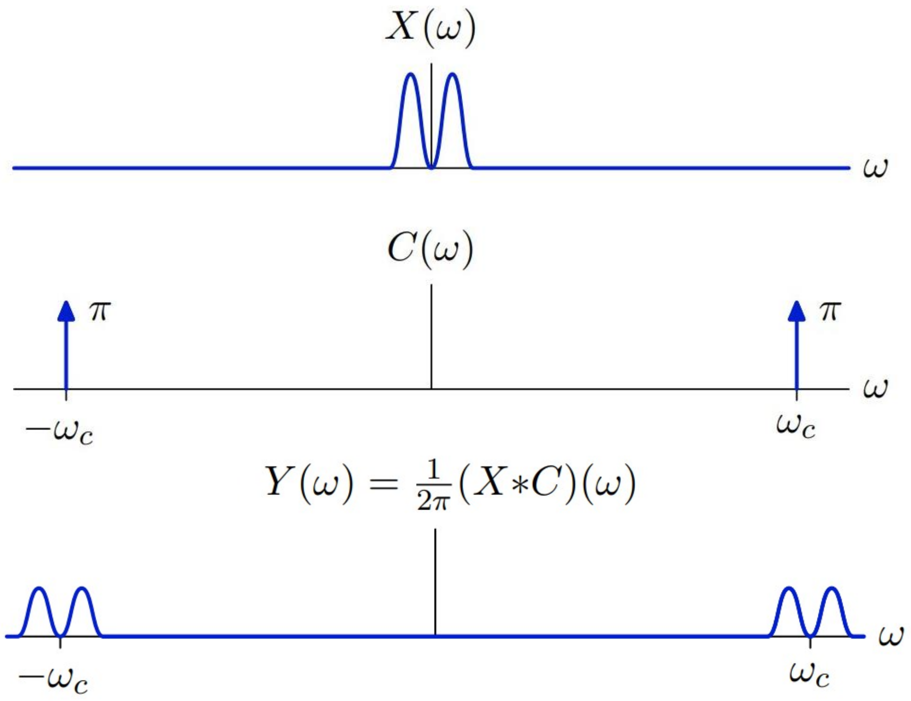
then $\quad Z(\omega) = \dfrac{1}{2\pi}(Y * X)(\omega)$

Let $X(\omega)$ represent the Fourier transform of the signal to be transmitted.

Let $C(\omega)$ represent the Fourier transform of $\cos(\omega_c t) = \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)$

Then $Y\ (\omega)$ is the result of convolving $X(\omega)$ with $C(\omega)$





What is $C(\omega)$?

Participation question for Lecture

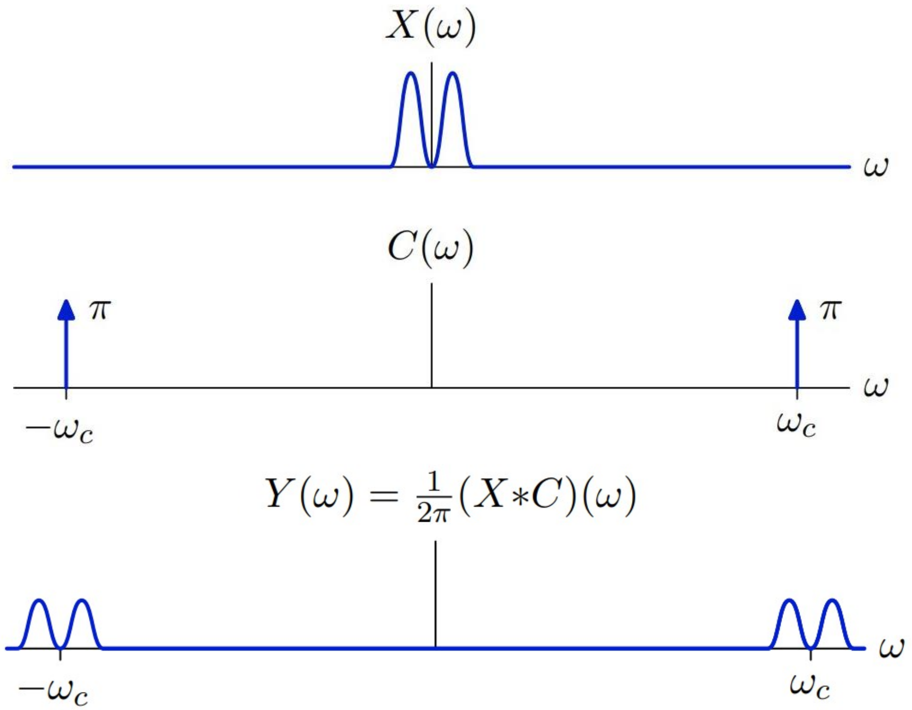
$$C(\omega) = \pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c)$$

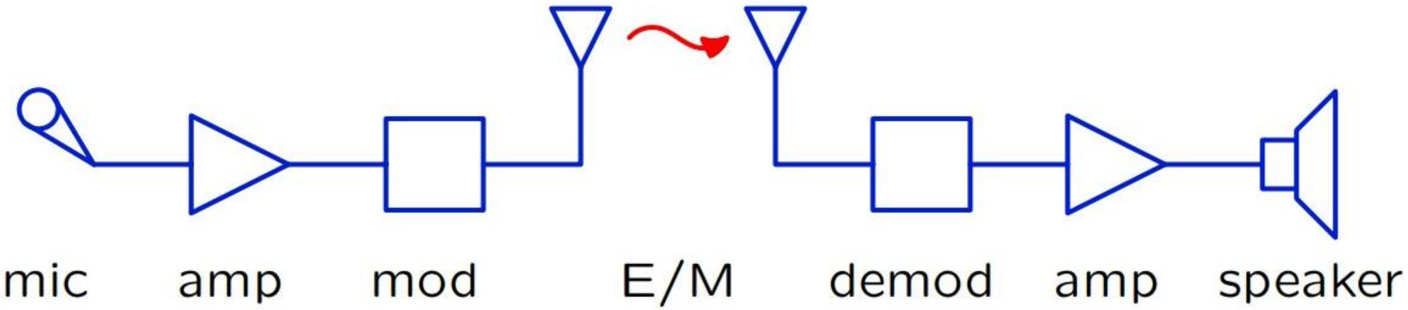
Slide #15 of Lect. 10A

Demodulating the Received Signal

We can match the signal to the medium with a modulator as shown by the "mod" box below.

But then we must demodulate the received signal to get back our original message (the "demod" box below).





How can we carry out demodulation (i.e., recover the original message from the modulated signal)?

Synchronous Demodulation

The original message can be recovered from an amplitude modulated signal by multiplying by the carrier and then low-pass filtering.

Assume that

$$y(t) = x(t)\cos(\omega_c t)$$

Then

$$z(t) = y(t)\cos(\omega_c t) = x(t)\cos(\omega_c t)\cos(\omega_c t) = x(t)\left(\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)\right)$$

This process is called synchronous demodulation.

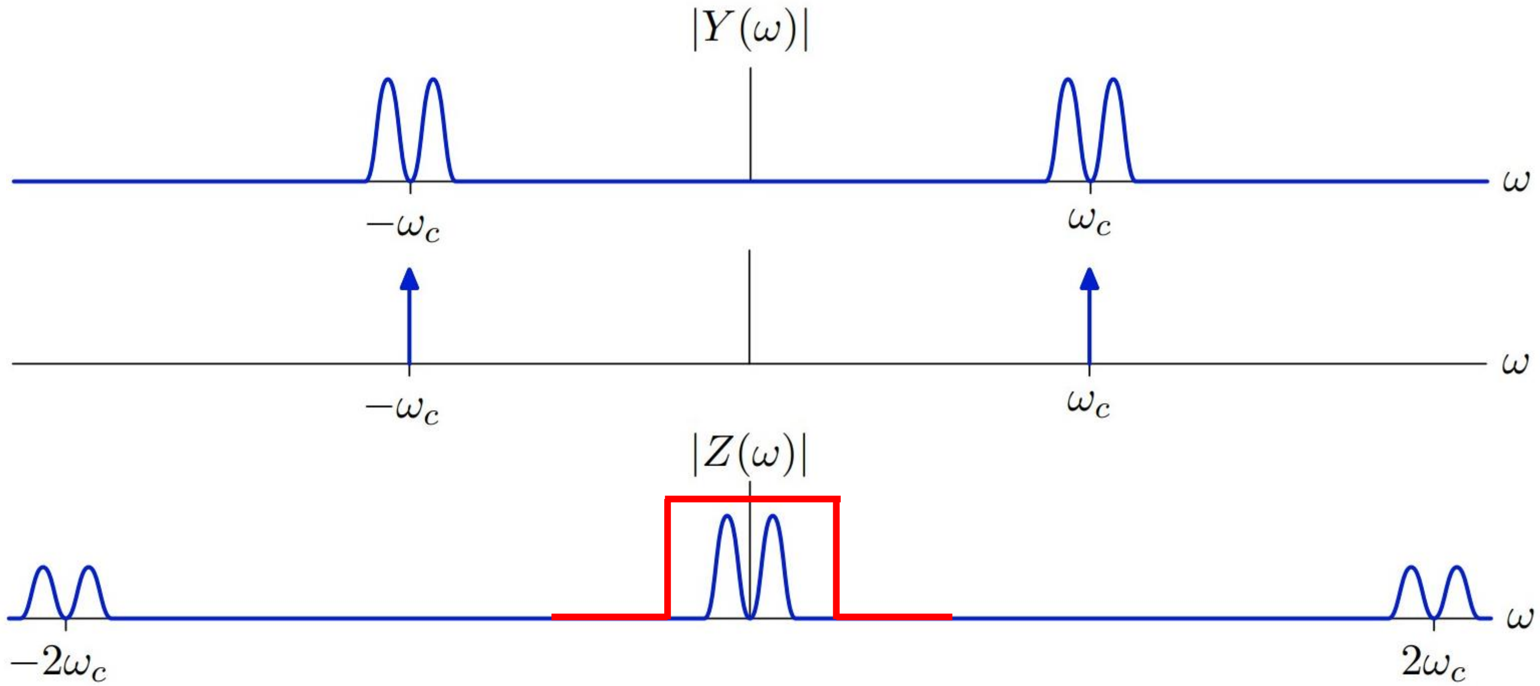
Synchronous Demodulation

Synchronous demodulation is equivalent to convolution in frequency.

$y(t) = x(t)\cos(\omega_c t)$ For $\cos(\omega_c t)$:

$z(t) = y(t)\cos(\omega_c t)$ $C(\omega) = \pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c)$

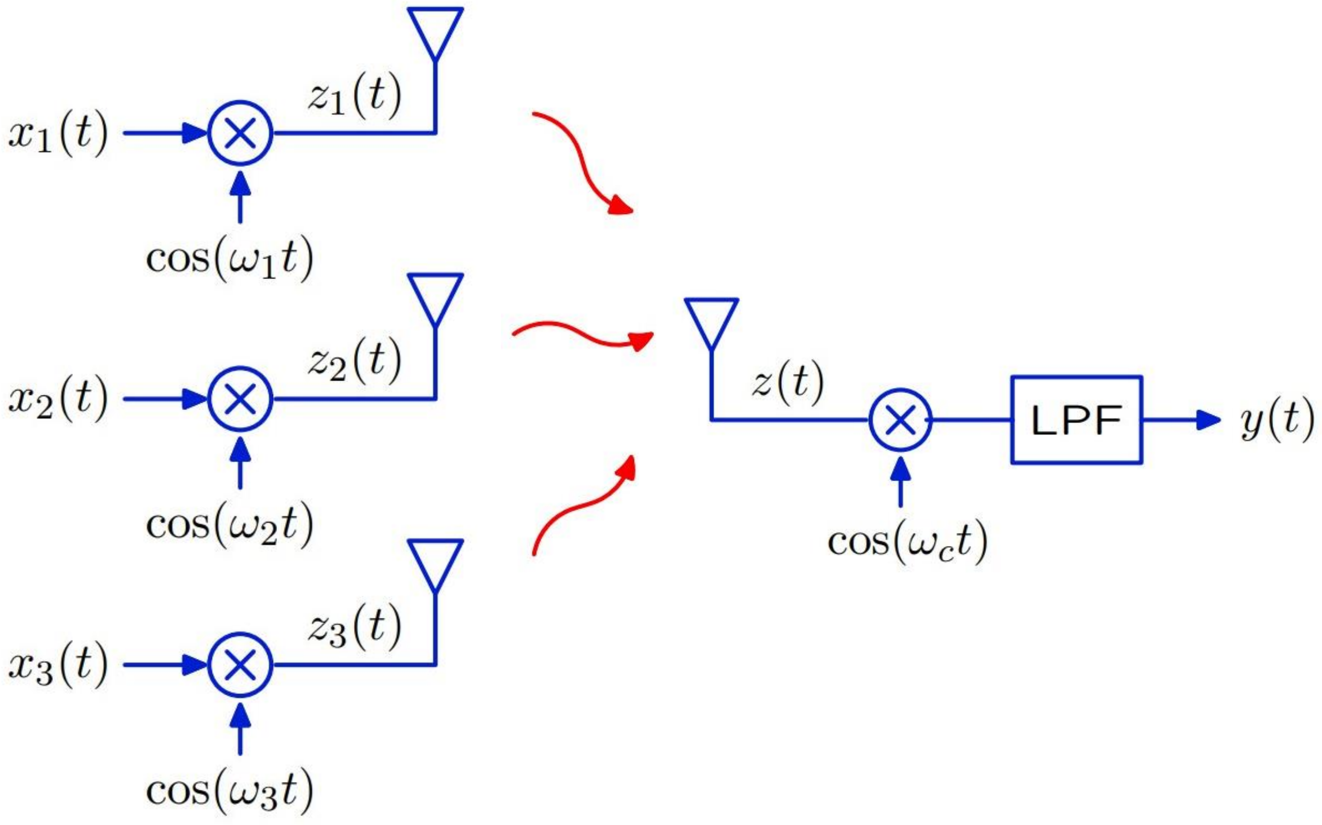
$$Z(\omega) = \frac{1}{2\pi}(Y * C)(\omega) = \frac{1}{2\pi}Y(\omega) * (\pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c))$$

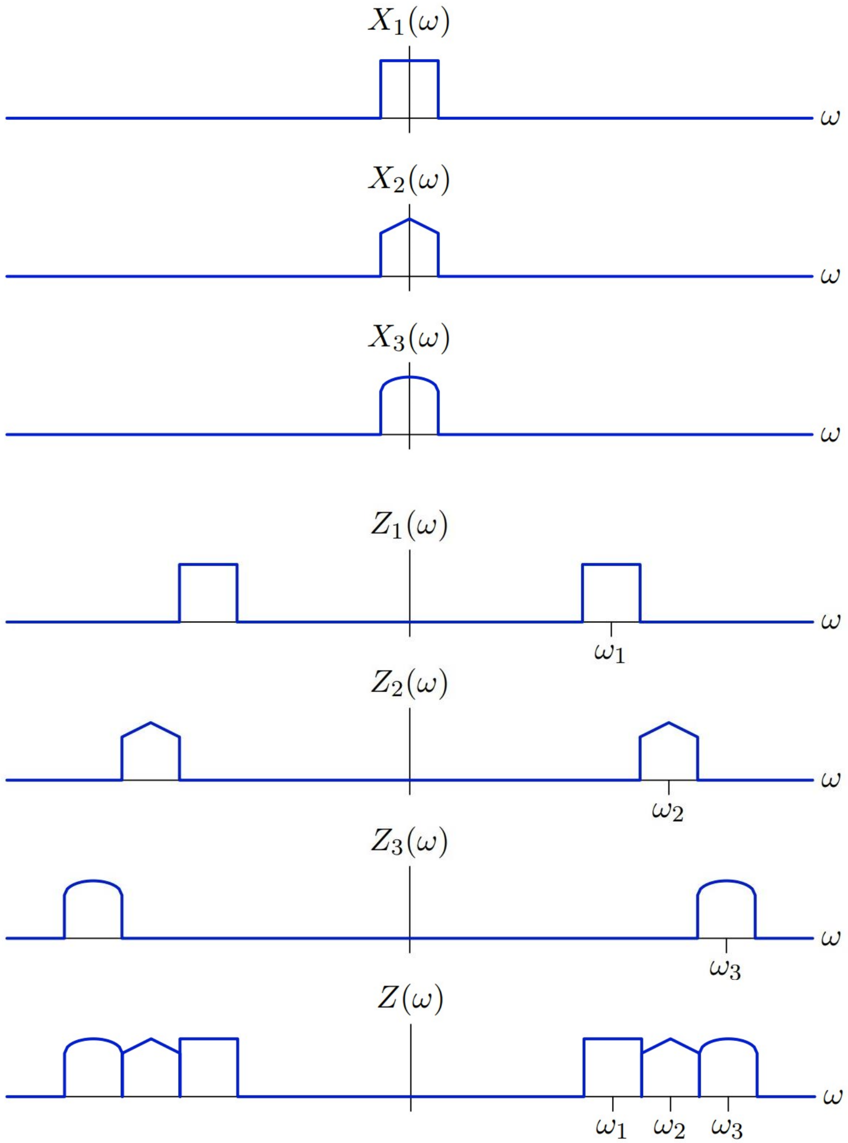


We can recover the original message by low-pass filtering:

Frequency-Division Multiplexing

Multiple transmitters can co-exist, as long as the frequencies that they transmit do not overlap.





Broadcast Radio

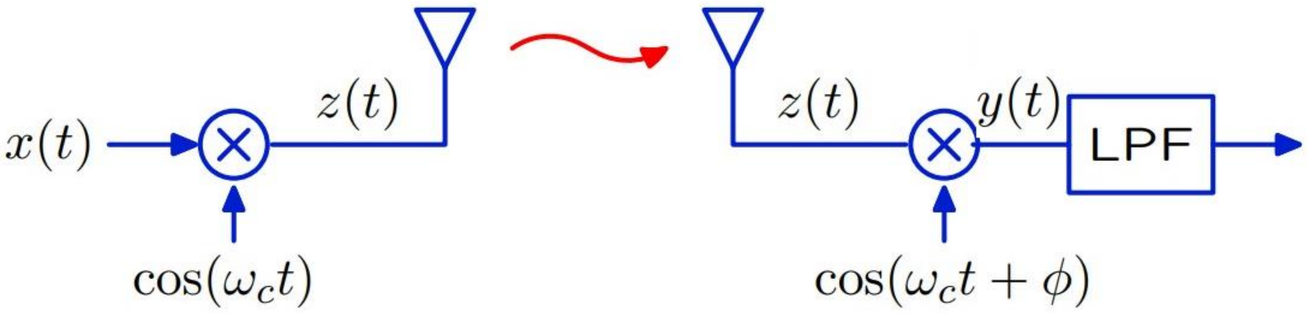
"Broadcast" radio was championed by David Sarnoff, who previously worked at Marconi Wireless Telegraphy Company (point-to-point).

- envisioned "radio music boxes"
- analogous to newspaper, but at speed of light
- receiver must be cheap (as with newsprint)
- transmitter can be expensive (as with printing press)

Sarnoff (left) and Marconi (right)

Challenge In Synchronous Demodulation

The problem with making the simple strategy we've discussed is that you must know the carrier signal exactly! Generally there is a phase difference between them which is unknown.



What happens if there is a phase shift ϕ between the signal used to modulate and the one used to demodulate?

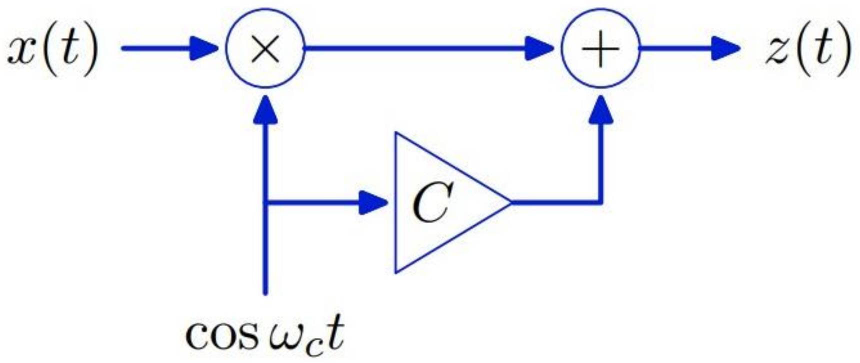
$$y(t) = x(t) \cdot \cos(\omega_c t) \cdot \cos(\omega_c t + \phi) \;\; = x(t) \cdot (\tfrac{1}{2}\cos(\phi) + \tfrac{1}{2}\cos(2\omega_c t + \phi))$$

$$= \tfrac{1}{2}x(t) \cdot \cos(\phi) + \tfrac{1}{2}x(t)\;\cos(2\omega_c t + \phi)$$

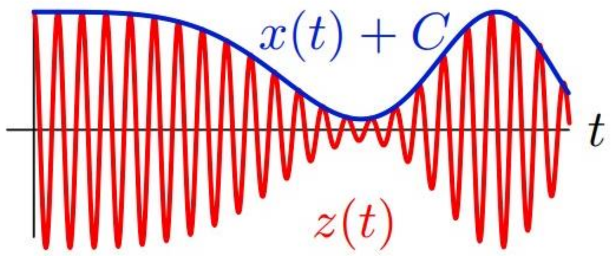
The second term is at a high frequency, so we can filter it out.
But multiplying by cos ϕ in the first term is a problem: the signal "fades."
Particularly, if ϕ = π/2 , there is no output at all!

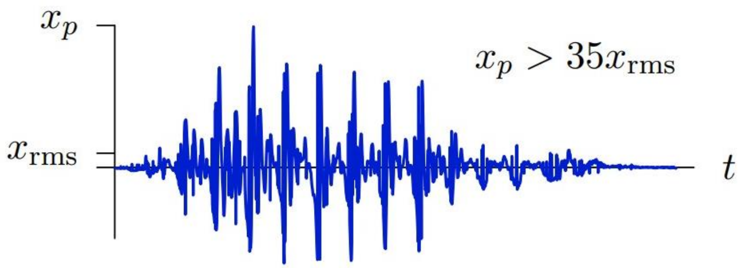
AM with Carrier

One way to synchronize the sender and receiver is to send the carrier along with the message.



$$z(t) = x(t) \cdot \cos(\omega_c t) + C\cos(\omega_c t) = (x(t) + C)\cos(\omega_c t)$$





Adding carrier is equivalent to shifting the DC value of the signal $x(t)$.

If we shift the DC value suffciently, the message is easy to decode: it is just the envelope (minus the DC shift)

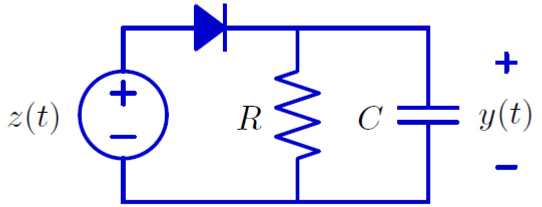
Speech sounds have high crest factors (peak value divided by rms value). Envelope detection will only work if the DC offset C is larger than x_p.

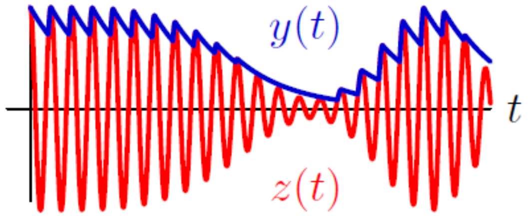
AM with carrier requires more power to transmit the carrier than to transmit the message!

Okay for broadcast radio (WBZ: 50 kwatts).
Not for point-to-point (cell phone batteries wouldn't last long!).

Radio Receiver

If the carrier frequency is much greater than the highest frequency in the message, AM with carrier can be demodulated with a peak detector.

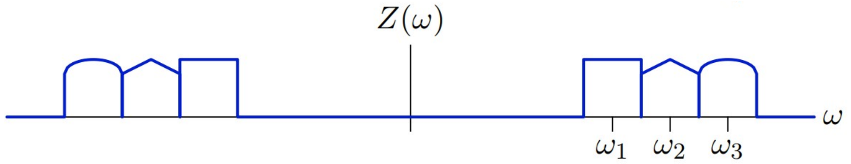




In AM radio, the highest frequency in the message is 5 kHz and the carrier frequency is between 500 kHz and 1500 kHz.

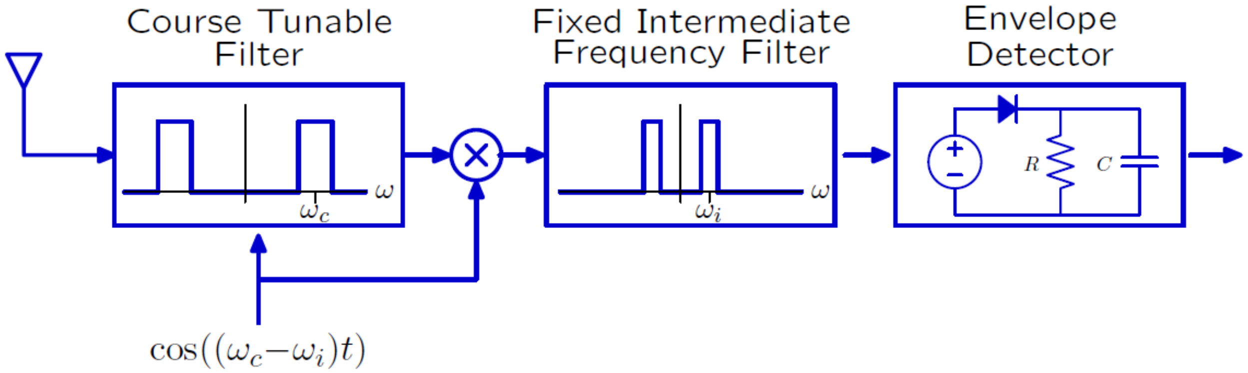
This circuit is simple and inexpensive. But there is a problem.

Envelope detection cannot separate multiple senders.



Superheterodyne Receiver

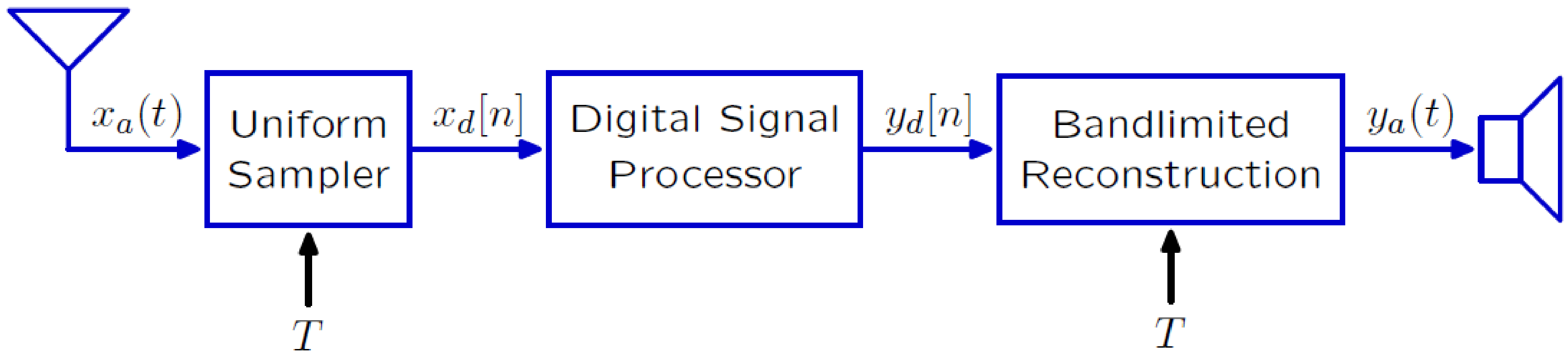
Edwin Howard Armstrong invented the superheterodyne receiver, which made broadcast AM practical.



Edwin Howard Armstrong also invented wideband FM, he also patented the “regenerative” (positive feedback) circuit for amplifying radio signals (while he was a junior at Columbia University).

Digital Radio

Today's radios are very different from those that launched broadcast radio.



Some issues remain the same:

- Power utilization
- Bandwidth limitations

Other issues are newer:

- More users
- More messages per user
- More different kinds of messages (audio, video, data)
- Privacy and security

Signal processing plays an important role in all of these areas.

Summary

A key problem in communications is matching the signal to the medium.

Modulation is an effective way to use frequencies in one band to communicate messages in a different band.

Amplitude modulation is based on the multiplication property of the Fourier transform:
– Multiplication in time corresponds to convolution in frequency.

=> Multiplication by a sinusoid shifts the frequency content of a signal.

We will now go to 4-370 for recitation & common hour