6.3000: Signal Processing

Sampling and Aliasing

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A “tone” is a pressure that changes sinusoidally with time.

In 6.3000, we will think of this as a “continuous-time” (CT) signal. In contrast, a “discrete-time” (DT) signal is a sequence of numbers.

Mathematically:

\[ x(t) = A \cos(\omega t) \]

\[ x[n] = A \cos(\Omega n) \]
Assume that \( x[n] \) represents “samples” of \( x(t) \):

\[
x(t), x[n]
\]

\[
x(t) = A \cos(\omega t)
\]

\[
x[n] = A \cos(\Omega n)
\]

• What are the units of \( \omega \), \( t \), \( \Omega \), and \( n \)?

Let \( f \) represent the “frequency” of the tone in cycles/second.

• Determine \( \omega \) in terms of \( f \).
• Determine \( \Omega \) in terms of \( \omega \).
• Determine \( \Omega \) in terms of \( f \).
Check Yourself

Compare two signals:

\[ x_1[n] = \cos \frac{3\pi n}{4} \]

\[ x_2[n] = \cos \frac{5\pi n}{4} \]

How many of the following statements are true?

- \( x_1[n] \) has period \( N=8 \).
- \( x_2[n] \) has period \( N=8 \).
- \( x_1[n] = x_2[n] \).
Frequencies

Consider the following CT signal:

\[ f(t) = 6 \cos(42\pi t) + 4 \cos(18\pi t - 0.5\pi) \]

What is the fundamental period of this signal?
Frequencies

Now imagine that this same signal
\[ f(t) = 6 \cos(42\pi t) + 4 \cos(18\pi t - 0.5\pi) \]
is sampled with a sampling rate of \( f_s = 60 \) Hz to obtain a discrete-time signal \( f[n] \), which is periodic in \( n \) with fundamental period \( N \). Determine the DT frequency components of \( f[n] \).
The DT signal
\[ f[n] = 6 \cos\left(42\pi \frac{n}{60}\right) + 4 \cos\left(18\pi \frac{n}{60} - 0.5\pi\right) \]
has a fundamental period of \( N = 20 \). However, this signal is also periodic in \( N = 80 \).

Which discrete frequencies are present if we reanalyze with \( N = 80 \)?
Determine `EXPR1` and `EXPR2` below to generate a 1000 Hz cosine tone using a sampling rate of 44,100 samples/second. The tone should last 2.5 seconds.

```python
import math
from lib6003.audio import wav_write
f = [cos(EXPR1 * n) for n in range(0, EXPR2)]
wav_write(f, 44100, 'output.wav')
```