Fourier Series (Trigonometric Form)

If $f(t)$ is expressed as a Fourier series

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} \left( c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \right)$$

the Fourier coefficients are given by

$$c_0 = \frac{1}{T} \int_T f(t) \, dt$$

$$c_k = \frac{2}{T} \int_T f(t) \cos(k\omega_o t) \, dt; \quad k = 1, 2, 3, \ldots$$

$$d_k = \frac{2}{T} \int_T f(t) \sin(k\omega_o t) \, dt; \quad k = 1, 2, 3, \ldots$$
Let $f_1(t)$ represent the following function, which is periodic in $T = 7$:

$$f_1(t) = c_0 + \sum_{k=1}^{\infty} \left( c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t) \right)$$

Determine a Fourier series of the following form for $f_1(t)$. 

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Two Pulses

Let $f_1(t)$ represent the following function, which is periodic in $T = 7$:

$$f_1(t) = c_0 + \sum_{k=1}^{\infty} \left( c_k \cos(k\omega_0 t) + d_k \sin(k\omega_0 t) \right)$$

Determine a Fourier series of the following form for $f_1(t)$. 

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Let $f_2(t)$ represent the following function, which is periodic in $T = 7$:

$$f_2(t)$$

Find $\omega_o$ and the Fourier series coefficients $c_k$ and $d_k$ so that

$$f_2(t) = \sum_{k=0}^{\infty} \left( c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \right)$$
Let \( f_3(t) \) represent the following function, which is periodic in \( T = 7 \):

\[
\begin{align*}
f_3(t) &= 1, \quad -7 < t < 1 \quad \text{or} \quad 7 < t < 9 \\
&= 0, \quad 1 < t < 7 \\
&= \ldots
\end{align*}
\]

Determine the Fourier series coefficients for \( f_3(t) \).

Discuss the relation(s) among the Fourier series coefficients of \( f_1(t) \), \( f_2(t) \), and \( f_3(t) \).

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**Trig Table**

\[
\sin(a+b) = \sin(a) \cos(b) + \cos(a) \sin(b) \\
\sin(a-b) = \sin(a) \cos(b) - \cos(a) \sin(b) \\
\cos(a+b) = \cos(a) \cos(b) - \sin(a) \sin(b) \\
\cos(a-b) = \cos(a) \cos(b) + \sin(a) \sin(b) \\
\tan(a+b) = \frac{\tan(a)+\tan(b)}{1-\tan(a) \tan(b)} \\
\tan(a-b) = \frac{\tan(a)-\tan(b)}{1+\tan(a) \tan(b)}
\]

\[
\sin(A) + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \\
\sin(A) - \sin(B) = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \\
\cos(A) + \cos(B) = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \\
\cos(A) - \cos(B) = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)
\]

\[
\sin(a+b) + \sin(a-b) = 2 \sin(a) \cos(b) \\
\sin(a+b) - \sin(a-b) = 2 \cos(a) \sin(b) \\
\cos(a+b) + \cos(a-b) = 2 \cos(a) \cos(b) \\
\cos(a+b) - \cos(a-b) = -2 \sin(a) \sin(b)
\]

\[
2 \cos(A) \cos(B) = \cos(A-B) + \cos(A+B) \\
2 \sin(A) \sin(B) = \cos(A-B) - \cos(A+B) \\
2 \sin(A) \cos(B) = \sin(A+B) + \sin(A-B) \\
2 \cos(A) \sin(B) = \sin(A+B) - \sin(A-B)
\]