Please enter all solutions in the boxes provided.
Work on other pages with QR codes will be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam (pages 13 and higher).

Trigonometric Identities Reference

\[
\begin{align*}
\cos(a+b) &= \cos(a) \cos(b) - \sin(a) \sin(b) & \cos(a-b) &= \cos(a) \cos(b) + \sin(a) \sin(b) \\
\sin(a+b) &= \sin(a) \cos(b) + \cos(a) \sin(b) & \sin(a-b) &= \sin(a) \cos(b) - \cos(a) \sin(b) \\
\cos(a) + \cos(b) &= 2\cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) & \cos(a) - \cos(b) &= -2\sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\
\sin(a) + \sin(b) &= 2\sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) & \sin(a) - \sin(b) &= 2\cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\
\cos(a+b) + \cos(a-b) &= 2\cos(a)\cos(b) & \cos(a+b) - \cos(a-b) &= -2\sin(a)\sin(b) \\
\sin(a+b) + \sin(a-b) &= 2\sin(a)\cos(b) & \sin(a+b) - \sin(a-b) &= 2\cos(a)\sin(b) \\
2\cos(a)\cos(b) &= \cos(a-b) + \cos(a+b) & 2\sin(a)\sin(b) &= \cos(a-b) - \cos(a+b) \\
2\sin(a)\cos(b) &= \sin(a+b) + \sin(a-b) & 2\cos(a)\sin(b) &= \sin(a+b) - \sin(a-b)
\end{align*}
\]
1 Complex Exponentials (10 points)

Part a. Find a complex constant \( c_1 \) so that \( \text{Re} \left( c_1 e^{j\omega t} \right) = \sin(\omega t) \) for all real numbers \( \omega \) and \( t \).

\[ c_1 = \]

Part b. Find a complex constant \( c_2 \) so that \( \text{Re} \left( c_2 e^{j\omega t} \right) = \cos(\omega t) + \sin(\omega t) \) for all real numbers \( \omega \) and \( t \).

\[ c_2 = \]

Part c. Find a complex constant \( c_3 \) so that \( \text{Re} \left( c_3 e^{j\omega t} \right) = A \cos(\omega t) + B \sin(\omega t) \) for all real numbers \( \omega \) and \( t \).

\[ c_3 = \]

Part d. Find a complex constant \( c_4 \) so that \( \text{Re} \left( c_4 e^{j\omega t} \right) = \cos(\omega t - \phi) \) for all real numbers \( \omega \) and \( t \).

\[ c_4 = \]

Part e. Find a complex constant \( c_5 \) so that \( \text{Re} \left( c_5 e^{j\Omega n} \right) = \cos \left( (2\pi - \Omega) n \right) \) for all real numbers \( \Omega \) and integers \( n \).

\[ c_5 = \]
2 Alternate Forms (20 points)

Consider a signal $f(t)$ that is given by the following series representation:

$$f(t) = \sum_{k=0}^{\infty} \left( \frac{1}{2} \right)^k \cos(2\pi kt) + \sum_{k=1}^{\infty} \left( \frac{1}{3} \right)^k \sin(\pi kt)$$

Determine the constants $a_k$ and $T$ to express $f(t)$ as a Fourier series in complex exponential form:

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi k}{T}t}$$

Enter the value of $T$ in the box below:

$$T =$$

Enter numerical expressions for $a_k$ in the table below:

<table>
<thead>
<tr>
<th>$k$</th>
<th>$a_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-4$</td>
<td></td>
</tr>
<tr>
<td>$-3$</td>
<td></td>
</tr>
<tr>
<td>$-2$</td>
<td></td>
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<tr>
<td>$-1$</td>
<td></td>
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<tr>
<td>$0$</td>
<td></td>
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<tr>
<td>$1$</td>
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<tr>
<td>$2$</td>
<td></td>
</tr>
<tr>
<td>$3$</td>
<td></td>
</tr>
<tr>
<td>$4$</td>
<td></td>
</tr>
</tbody>
</table>
3 Find All (24 points)

Consider the following signals:

\[ f_1[n] = \frac{1}{2} + 6 \cos(\pi n/2) + 4 \cos(\pi n/5 - \pi/2) \]
\[ f_2[n] = \cos(1.8\pi n) + 2 \sin(2.7\pi n) \]
\[ f_3[n] = |\sin(\pi n/10)| \quad \text{; where } |x| \text{ represents the magnitude of } x \]
\[ f_4[n] = \text{Im}\{e^{i(2\pi n/20+\pi/2)}\} \quad \text{; where Im}\{x\} \text{ represents the imaginary part } x \]

Part a. Determine which (if any) of signals \( f_1[n] \) through \( f_4[n] \) are symmetric about \( n=0 \), and circle all of those signals below. Circle None if no signal is symmetric about \( n=0 \).

\[ f_1[n] \quad f_2[n] \quad f_3[n] \quad f_4[n] \quad \text{None} \]

Part b. Determine which (if any) of signals \( f_1[n] \) through \( f_4[n] \) are periodic with a fundamental (smallest) period \( N=20 \), and circle all of those signals below. Circle None if no signal is periodic with fundamental period \( N=20 \).

\[ f_1[n] \quad f_2[n] \quad f_3[n] \quad f_4[n] \quad \text{None} \]
Part c. Determine which (if any) of signals $f_1[n]$ through $f_4[n]$ can be represented by Fourier series with purely imaginary coefficients, and circle all of those entries below. Circle None if none of these signals can be represented by Fourier series with purely imaginary coefficients.

\[
\begin{array}{cccc}
  f_1[n] & f_2[n] & f_3[n] & f_4[n] \\
  \text{None} & & & \\
\end{array}
\]

Part d. Determine which (if any) of signals $f_i[n]$ can be represented by Fourier series coefficients $F_i[k]$ that are symmetric functions of $k$ (i.e., $F_i[k] = F_i[-k]$) and circle all of those entries below. Circle None if none of the Fourier series coefficients are symmetric functions of $k$.

\[
\begin{array}{cccc}
  f_1[n] & f_2[n] & f_3[n] & f_4[n] \\
  \text{None} & & & \\
\end{array}
\]
4 Sampled Signals (26 points)

Consider sampling CT signals of the following form

\[ f(t) = \cos(\omega t) \]

at times that are integer multiples of a constant \( \Delta = 1 \). The resulting DT signals will have the following form

\[ f[n] = \cos(\Omega n) \]

where \( \Omega = \omega \Delta \).

**Part a.** Determine all the values of \( \omega \) in the range \( 0 < \omega < 2\pi \) for which the fundamental period of \( f[n] \) is 5. Enter these values of \( \omega \) in the box below.

**Part b.** Determine all the values of \( \omega \) in the range \( 0 < \omega < 2\pi \) for which the fundamental period of \( f[n] \) is 9. Enter these values of \( \omega \) in the box below.
Part c. Determine all the values of $\omega$ in the range $0<\omega<2\pi$ for which $f[n] = f[n+7]$.
Enter these values of $\omega$ in the box below.

Part d. Determine all the values of $\omega$ in the range $0<\omega<2\pi$ for which $f[n] = \cos\left(\frac{18\pi n}{8}\right)$.
Enter these values of $\omega$ in the box below.
5 Continuous-Time Fourier Series (24 points)

Each of the following six plots shows a one-second interval of a periodic continuous-time signal \( f_i(t) \). Each of these signals can be represented by its Fourier series coefficients \( F_i[k] \) computed with \( T = 1 \) as follows:

\[
f_i(t) = f_i(t+1) = \sum_{k=-\infty}^{\infty} F_i[k] e^{j2\pi kt}
\]

where \( F_i[k] = \int_{-0.5}^{0.5} f_i(t) e^{-j2\pi kt} \, dt \)

Determine which of the plots below (A–L) represents the real part of \( F_i[k] \) for \(-8 \leq k \leq 8\) and enter that letter in the box labeled "Re" (above). Do the same for the imaginary part and enter that letter in the box labeled "Im".
Worksheet (intentionally blank)
Worksheet (intentionally blank)