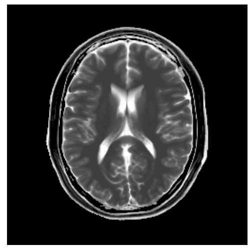
# 5 MRI (16 Points)

Recall that the measurements made by MRI machines are samples of the DFT $X[\cdot,\cdot]$ of some underlying image $x[\cdot,\cdot]$, which we recover via an inverse DFT. Recall also that, unlike many signals we have considered, this image's spatial domain representation is generally complex-valued.

Throughout this problem, we will consider the same 256×256 example image from lecture, for which $|x[\cdot,\cdot]|$ is shown below:



As we discussed in lecture and recitation, an important ongoing area of research involves attempting to faithfully reconstruct an image using as few samples of $X[\cdot,\cdot]$ as possible. In this problem, we will consider several different attempts to reduce the scan time of the image above.

## 5.1 Part 1

Consider reducing the scanning time by only sampling half of the rows of $X[\cdot,\cdot]$, creating a new image whose DFT is given by:

$$X_2[k_r,k_c] = \begin{cases} X[k_r,k_c] & \text{if } k_r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

Which image on the facing page most closely matches $|x_2[\cdot,\cdot]|$, the magnitude of the spatial domain representation of this image?

Enter a single letter: N

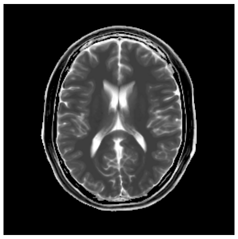
Also consider a different approach, where we still only sample half of the rows, but we fill in the missing rows via linear interpolation rather than leaving them as 0's:

$$X_3[k_r,k_c] = \begin{cases} X[k_r,k_c] & \text{if } k_r \text{ is even} \\ X[k_r+1,k_c]/2 + X[k_r-1,k_c]/2 & \text{otherwise} \end{cases}$$

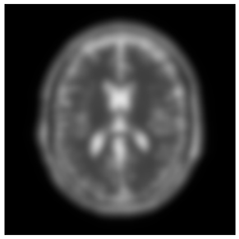
Which image on the facing page most closely matches $|x_3[\cdot,\cdot]|$, the magnitude of the spatial domain representation of this image?

Enter a single letter: S

**A**

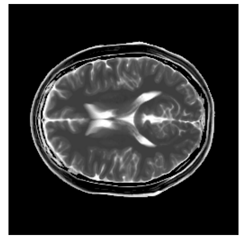


**B**

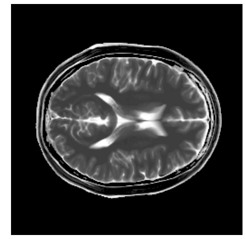


**C**

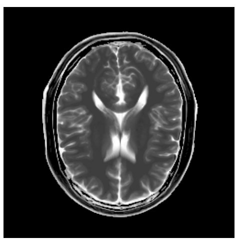
**D**



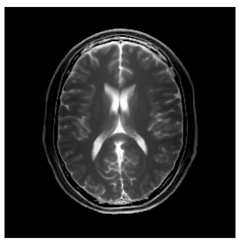
**E**



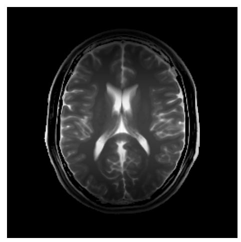
**F**



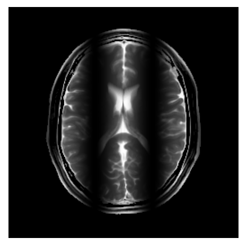
**G**



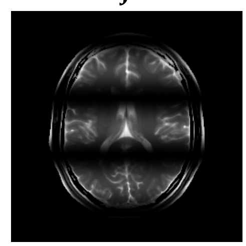
**H**



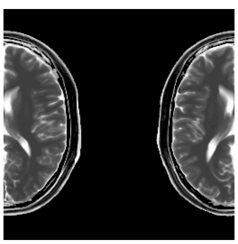
**I**



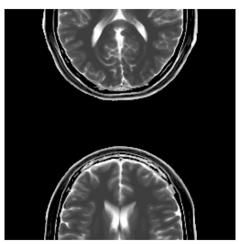
**J**



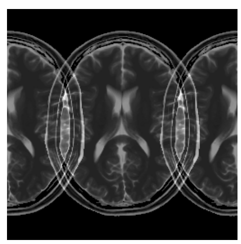
**K**



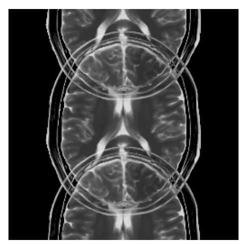
**L**



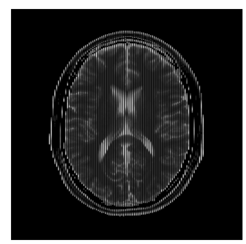
**M**



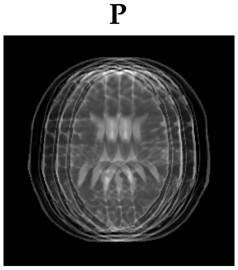
**N**



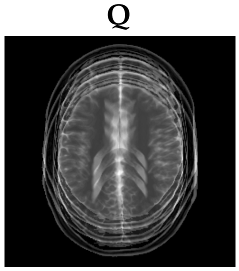
**O**



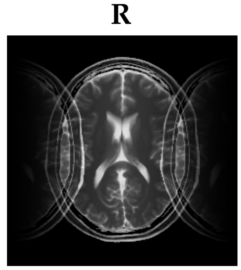
**P**



**Q**

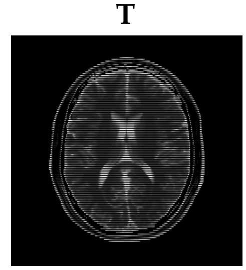


**R**



**S**

**T**



## 5.2 Part 2

Ben Bitdiddle suggests that a better way to cut down on scanning time would be to sample only half of the rows, but, particularly, to sample only where $0 \leqslant k_r \leqslant 128$, and then to use the conjugate symmetry of the DFT to fill in the missing values, i.e., for $-127 \leqslant k_r < 0$, set $X[k_r, k_c] = X^*[-k_r, -k_c]$.

Ben asserts that this approach will allow him to reconstruct $x[\cdot,\cdot]$ exactly, while only explicitly sampling half of the DFT coefficients.

Is Ben's assertion true? **Yes** or **No**: **No**

Briefly explain your reasoning:

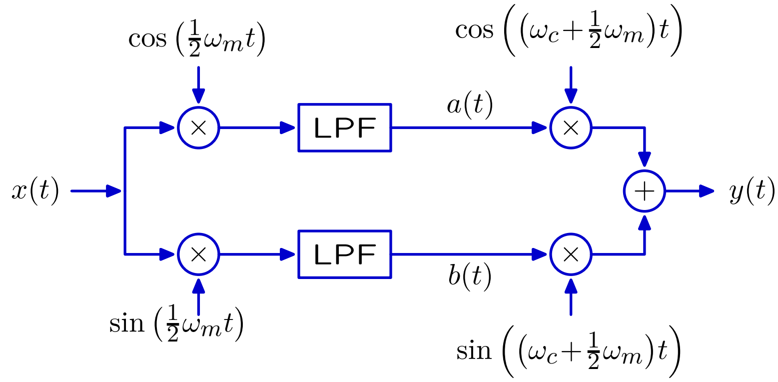
The assumption that the DFT $X[\cdot,\cdot]$ is conjugate symmetric is true if any only if the spatial-domain representation of the image $x[\cdot,\cdot]$ is purely real.

So this approach *would* work if $x[\cdot,\cdot]$ were real (regardless of the shape of $x[\cdot,\cdot]$). However, because $x[\cdot,\cdot]$ is actually complex valued, its DFT will not have this conjugate symmetry, and so Ben's approach will not properly reconstruct the image.

*Worksheet (intentionally blank)*

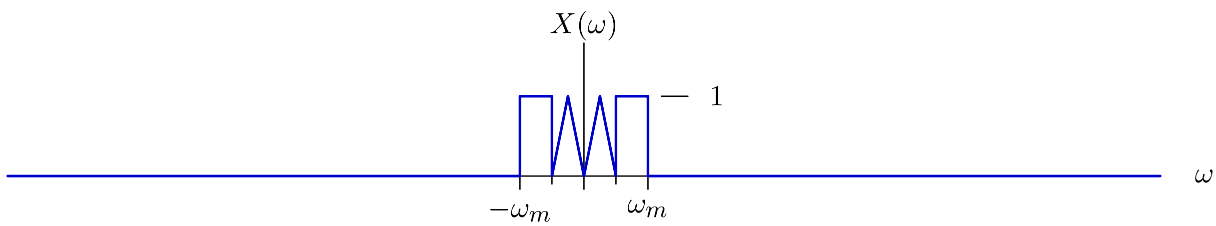
# 6 Modulation (14 Points)

Consider the following modulation scheme, where $\omega_c >> \omega_m$.

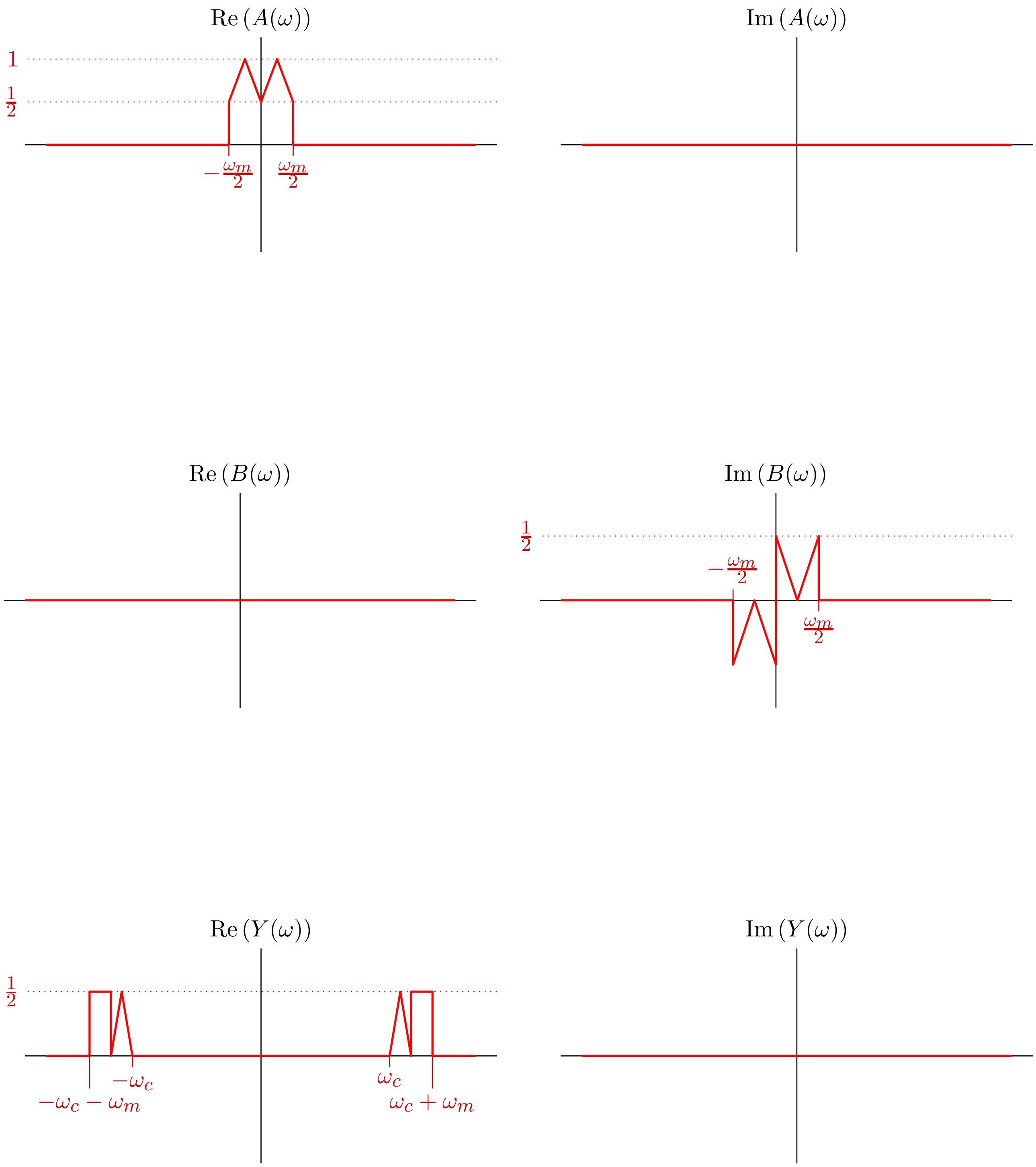



Assume that each lowpass filter (LPF) is ideal, with cutoff frequency $\omega_m/2$, and note that $a(\cdot)$ and $b(\cdot)$ represent the respective outputs of the two low-pass filters.

Also assume that the input signal has the following (purely real) Fourier transform:




On the facing page, sketch the real and imaginary parts of $A(\omega)$, $B(\omega)$, and $Y(\omega)$.
**Label all important magnitudes and frequencies.**


Re (A(ω))
1
1/2
−ωm/2
ωm/2
Im (A(ω))
Re (B(ω))
Im (B(ω))
1/2
−ωm/2
ωm/2
Re (Y(ω))
1/2
−ωc
−ωc − ωm
ωc
ωc + ωm
Im (Y(ω))